Today 13.5, 13.8

$\boxed{L37}$

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$\boxed{L37}$

Kepler's Laws

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$\boxed{\text{L37}}$

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Black holes

$\boxed{L37}$

Today 13.5, 13.8

Friday Holiday

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Monday Exam #4

L37

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Wednesday Dec. 2nd Day of Reckoning

Today 13.5, 13.8

$\boxed{L37}$

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Friday Dec 4th Final exam

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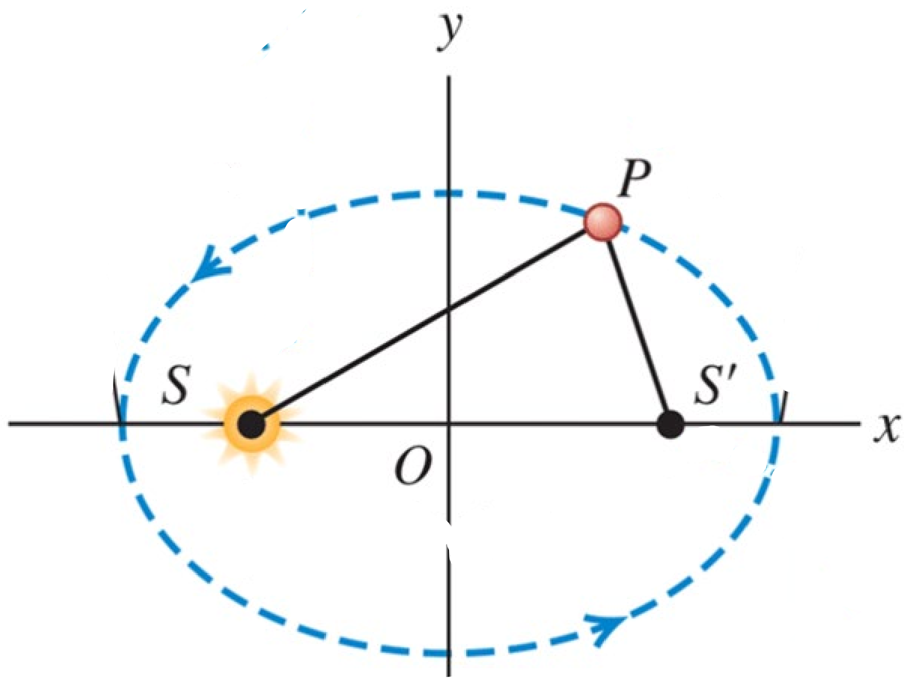

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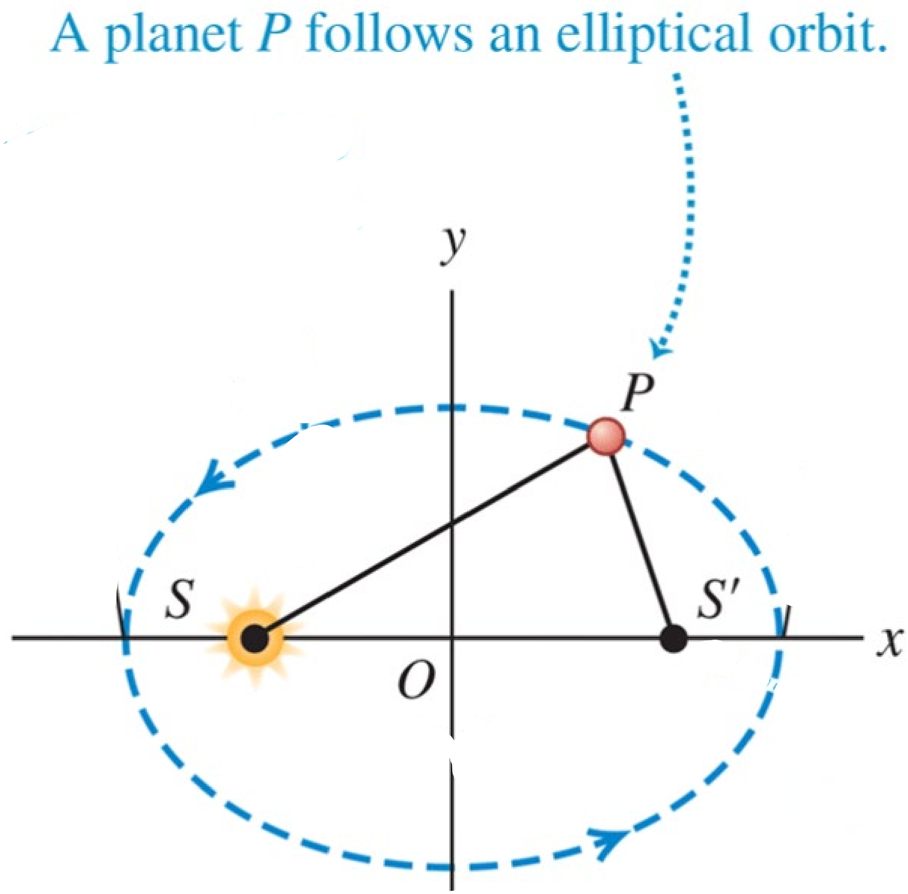

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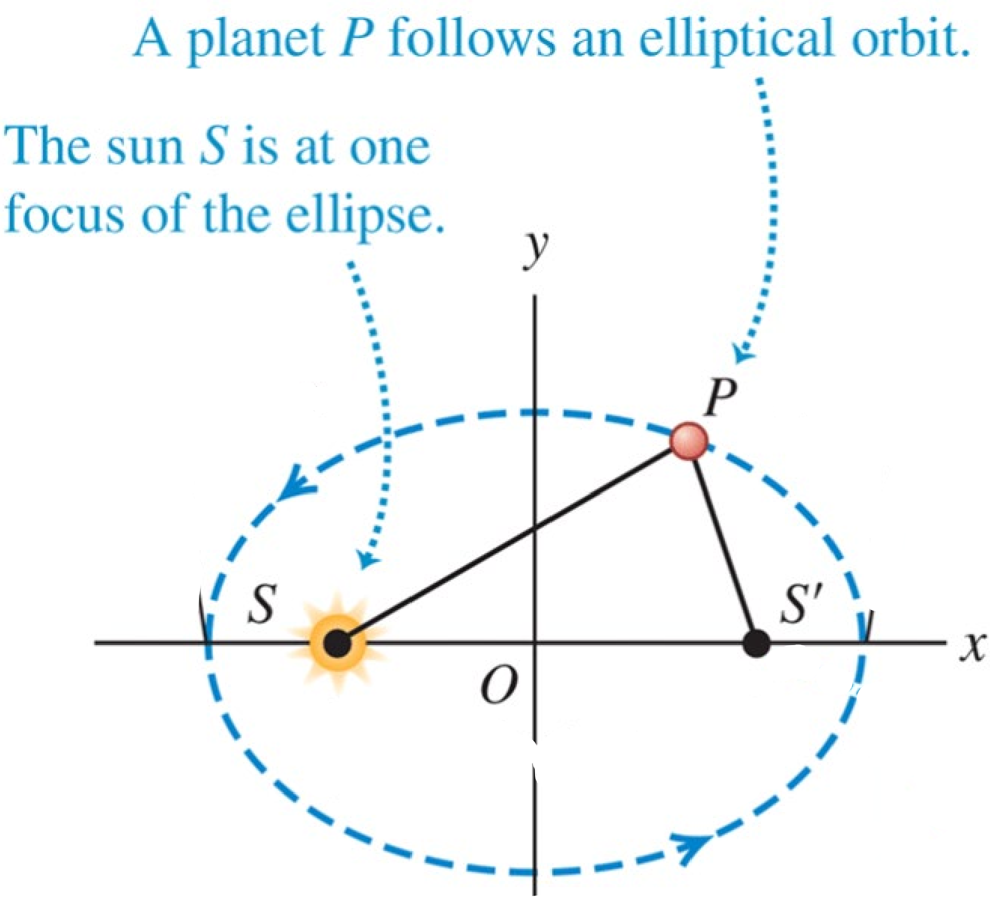

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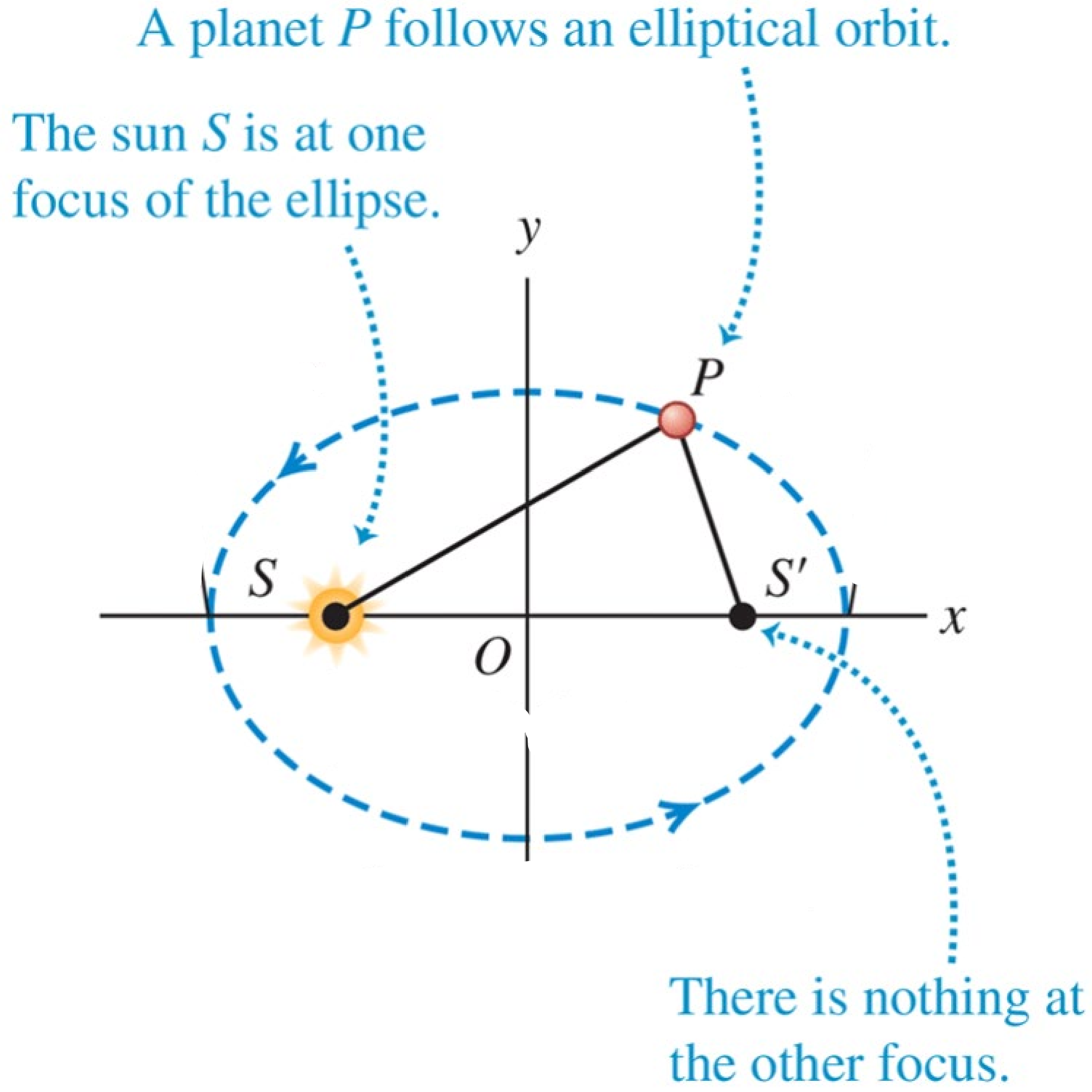

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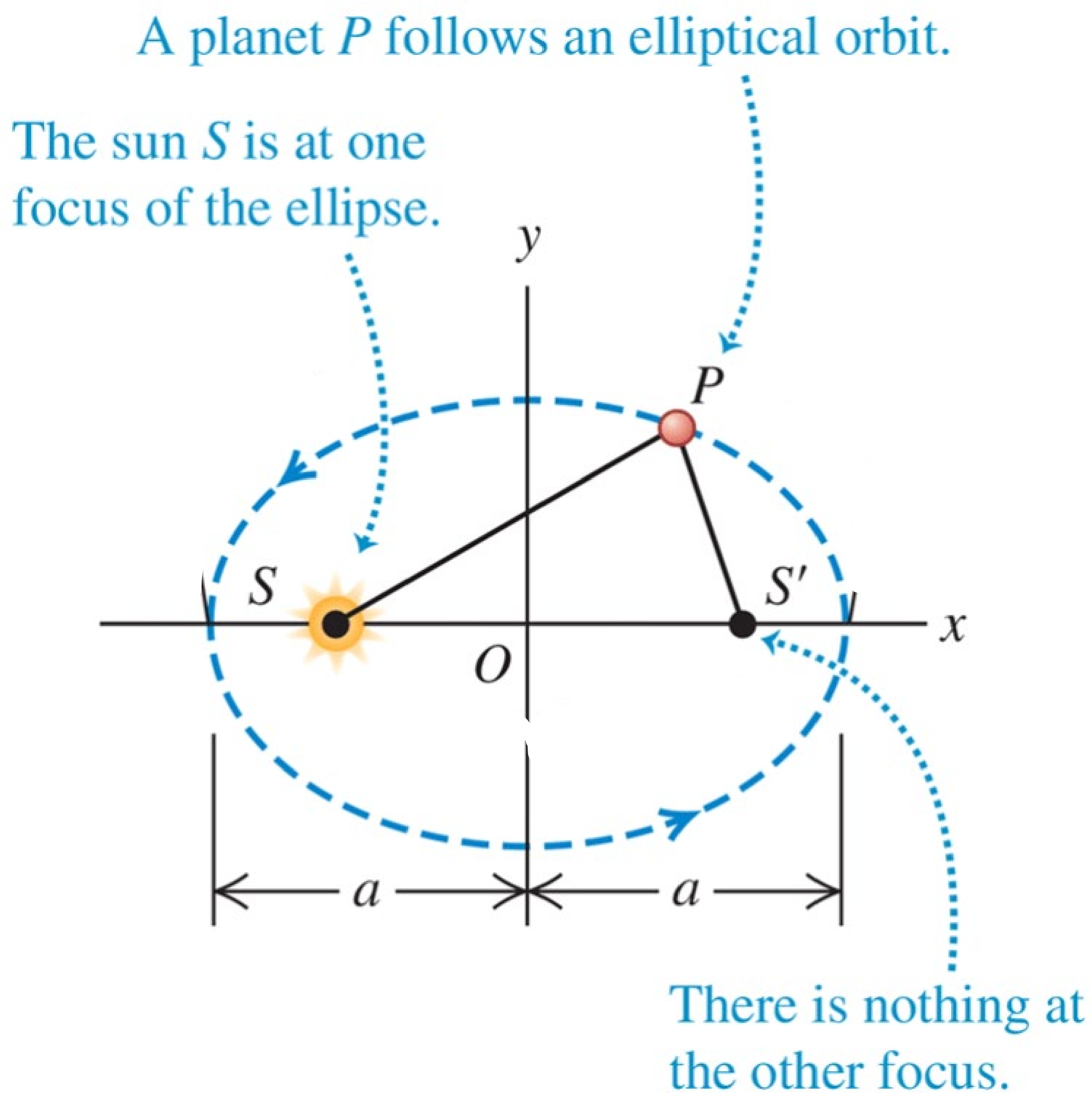

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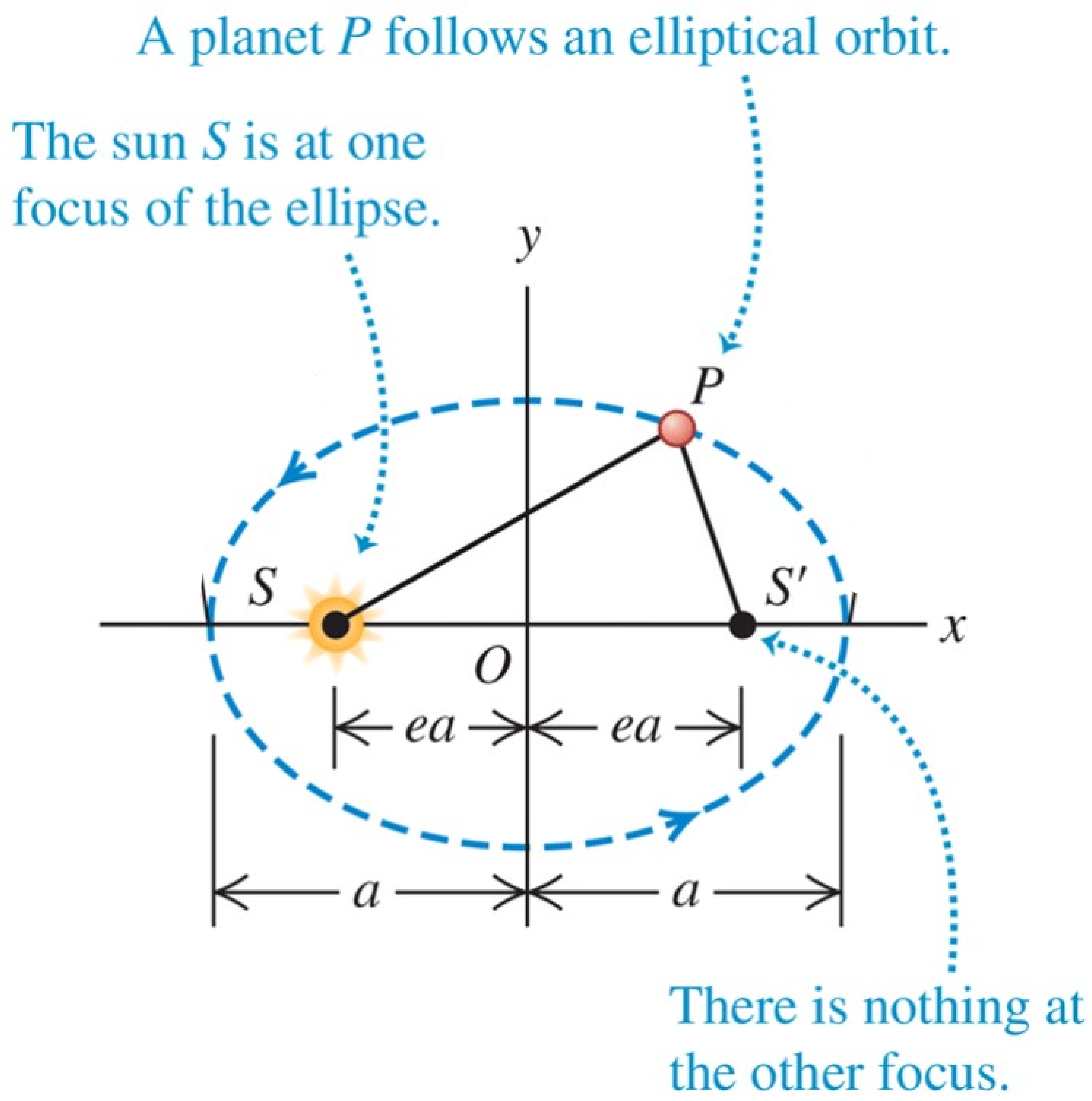

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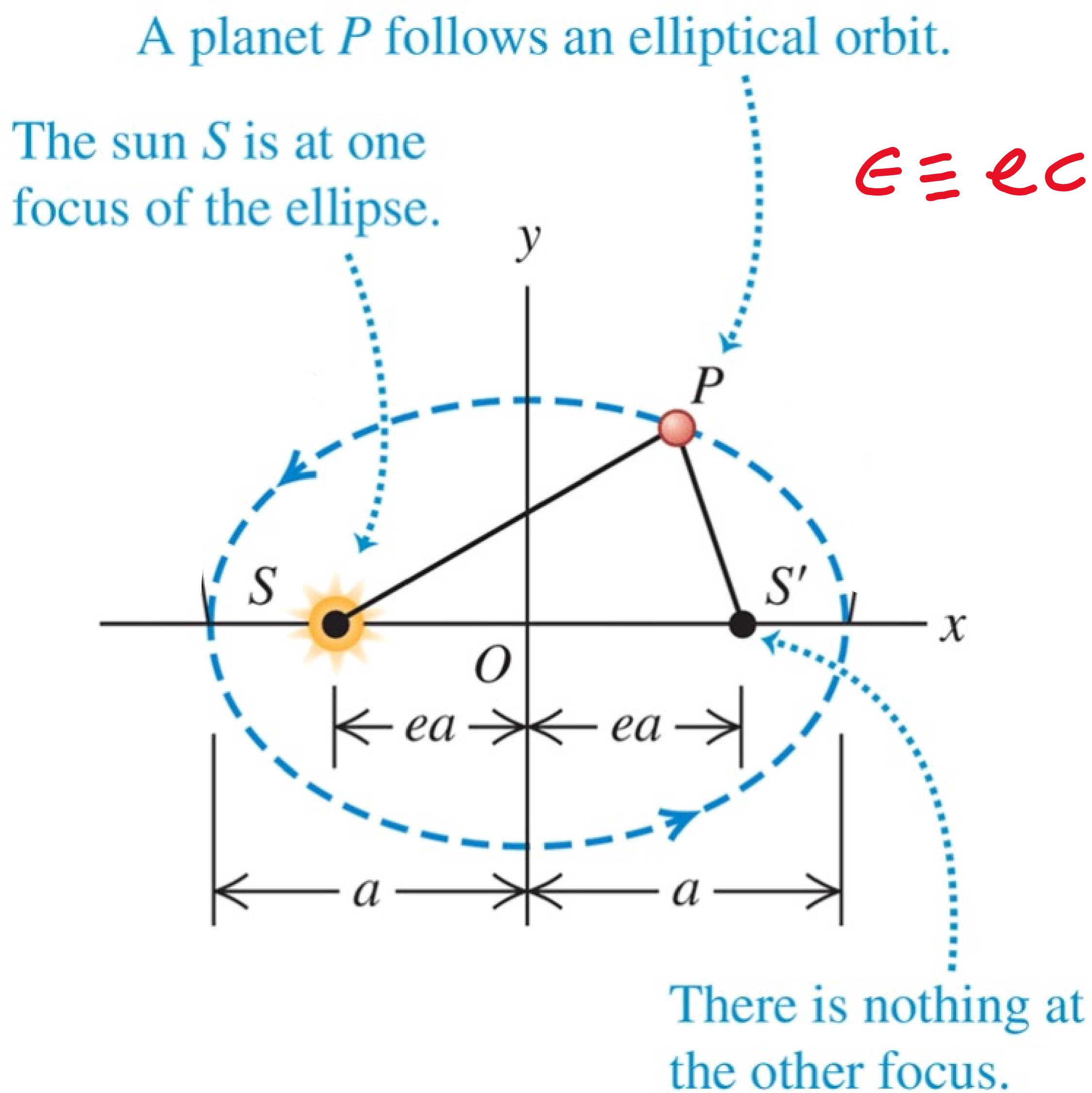

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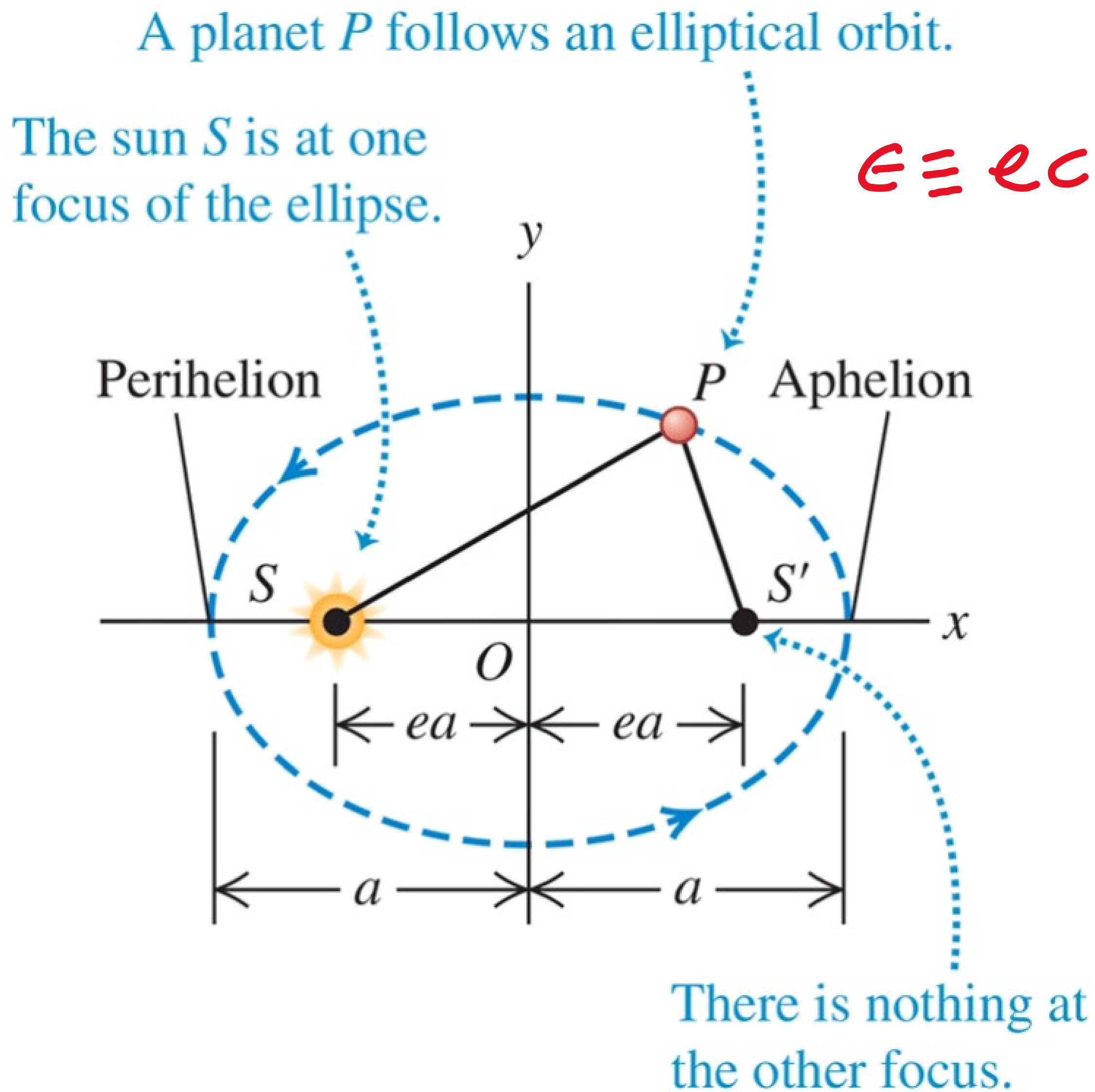

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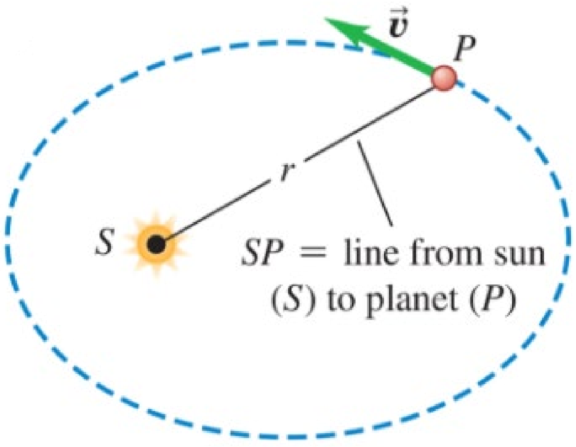

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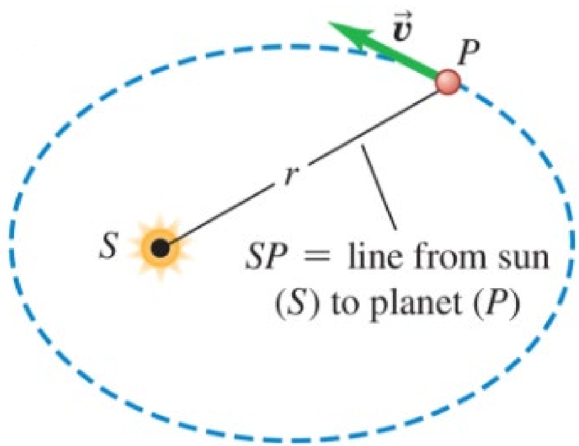

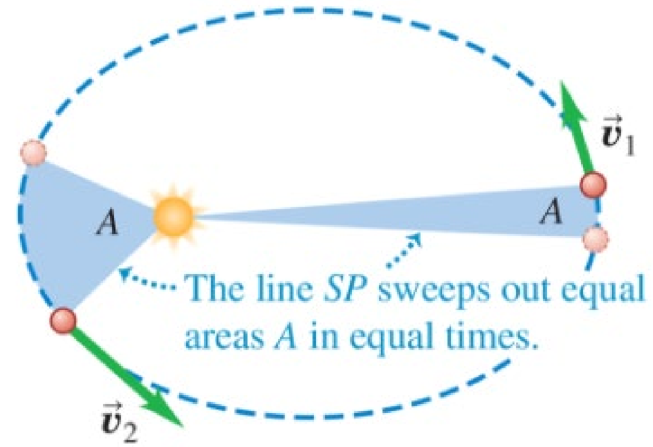

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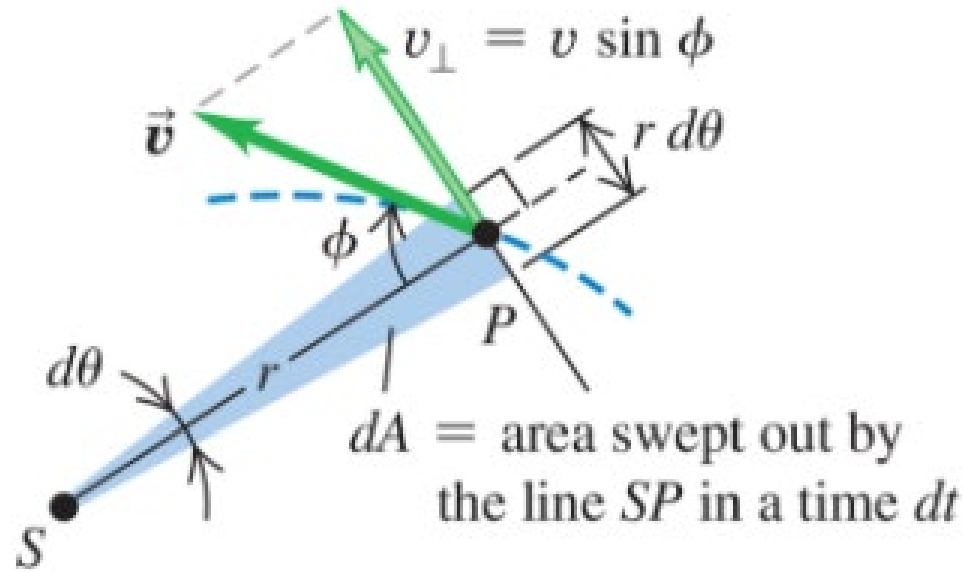

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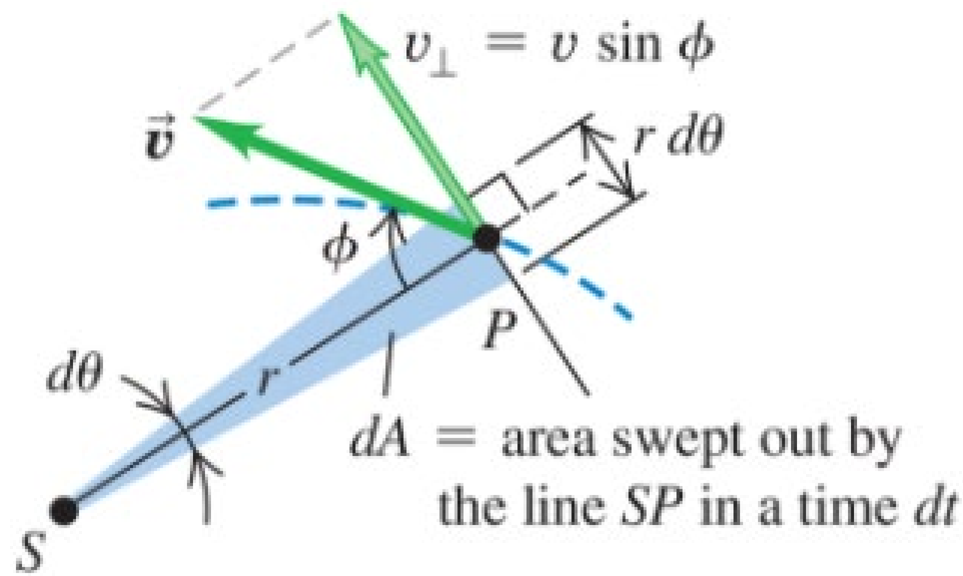

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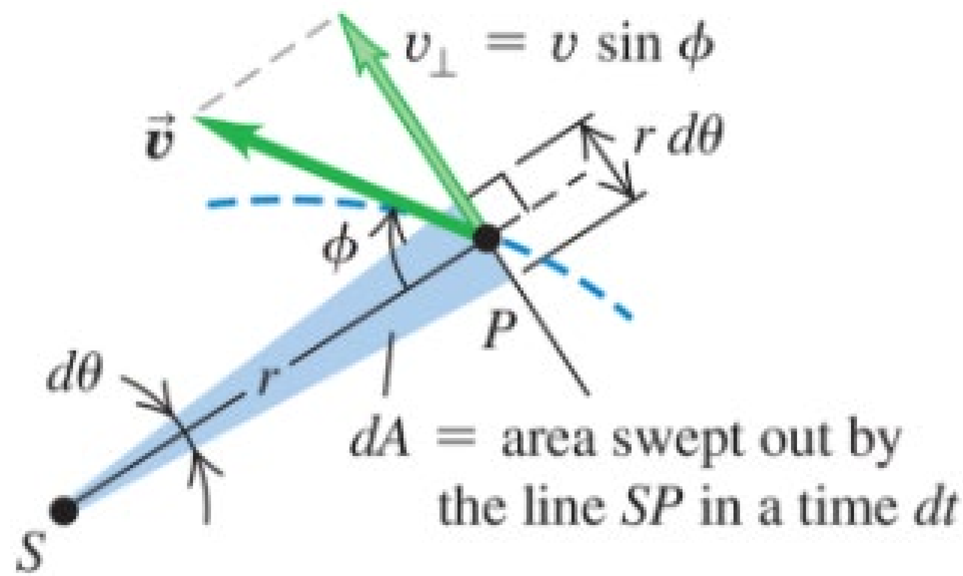

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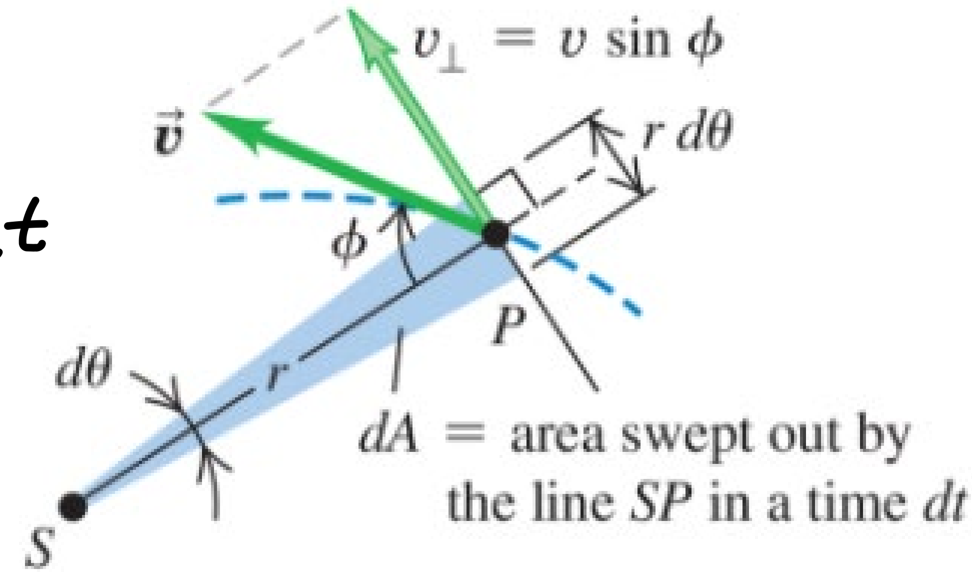

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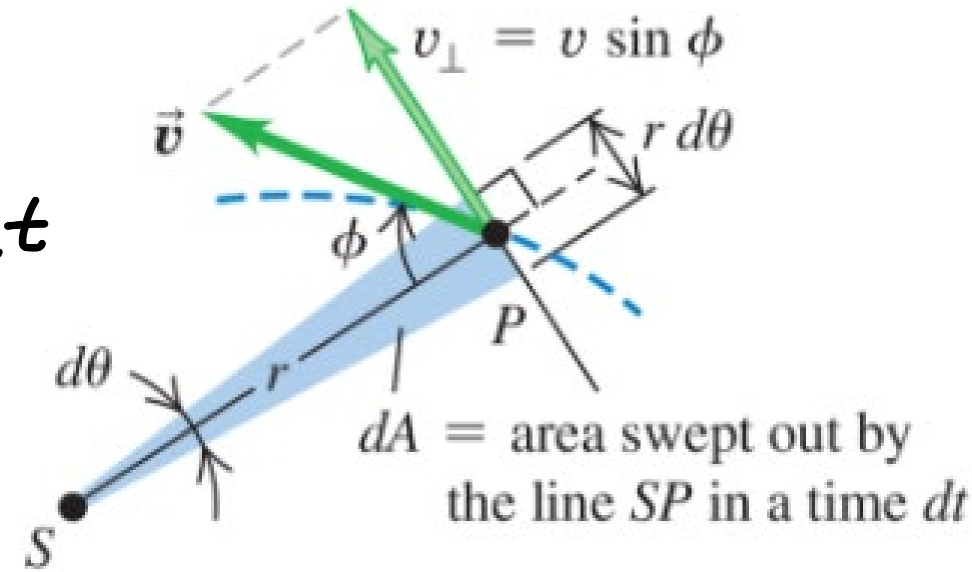

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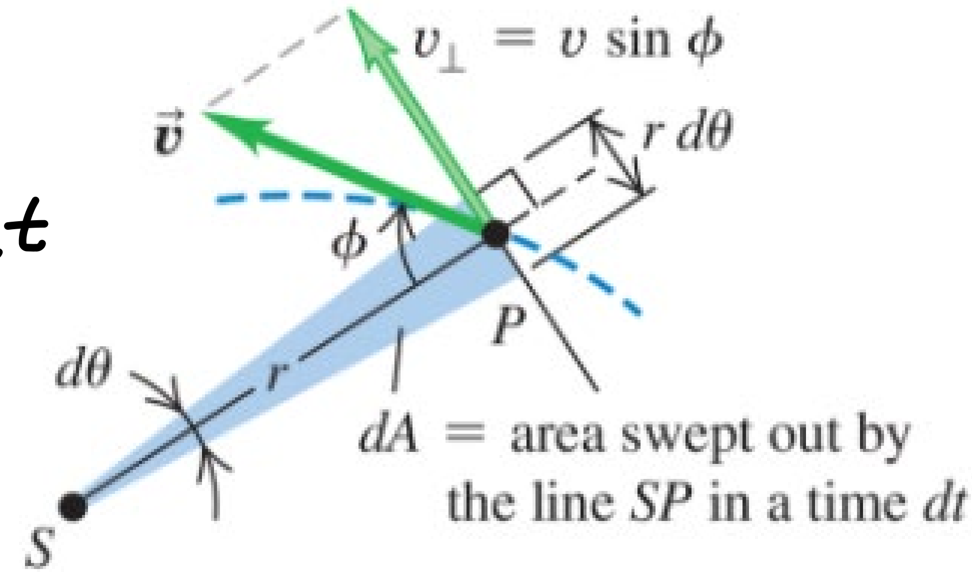

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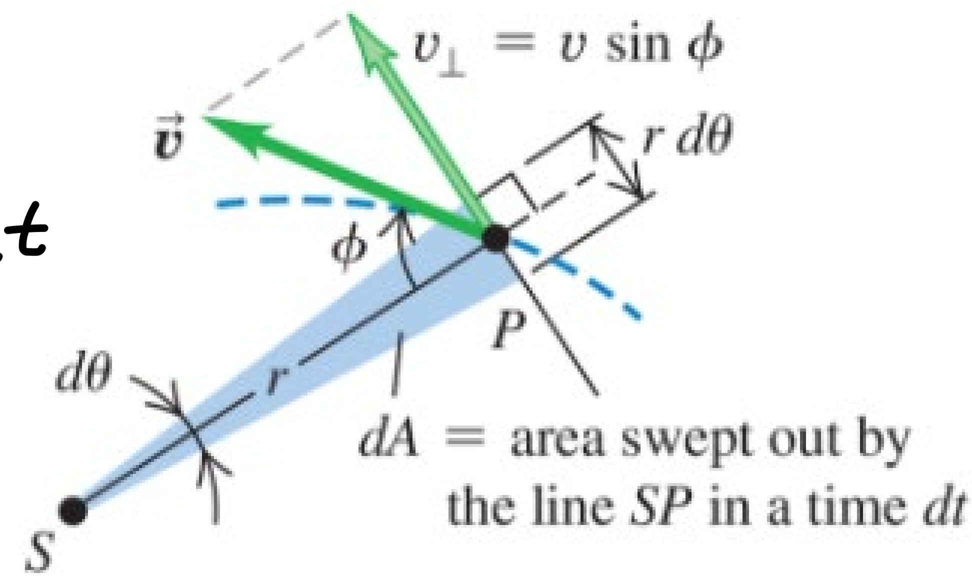

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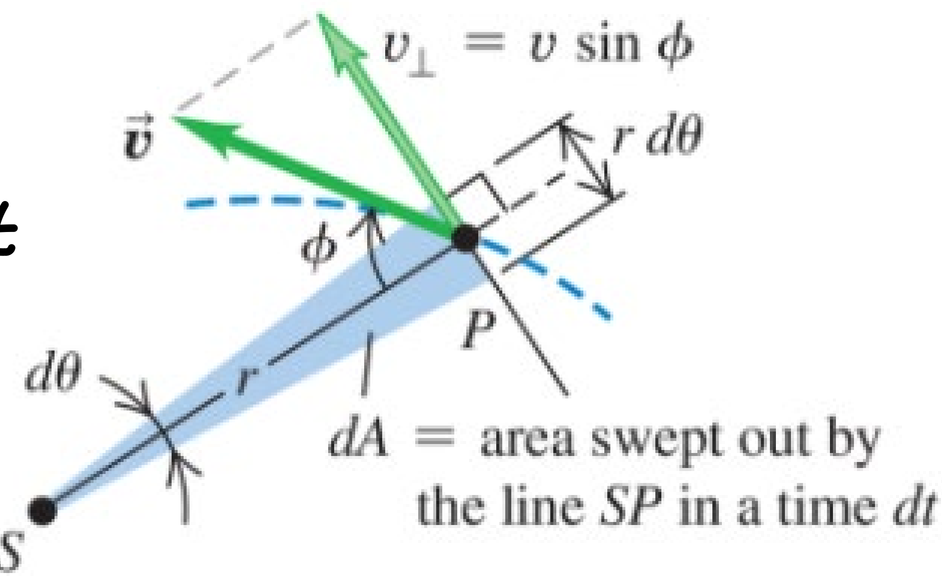

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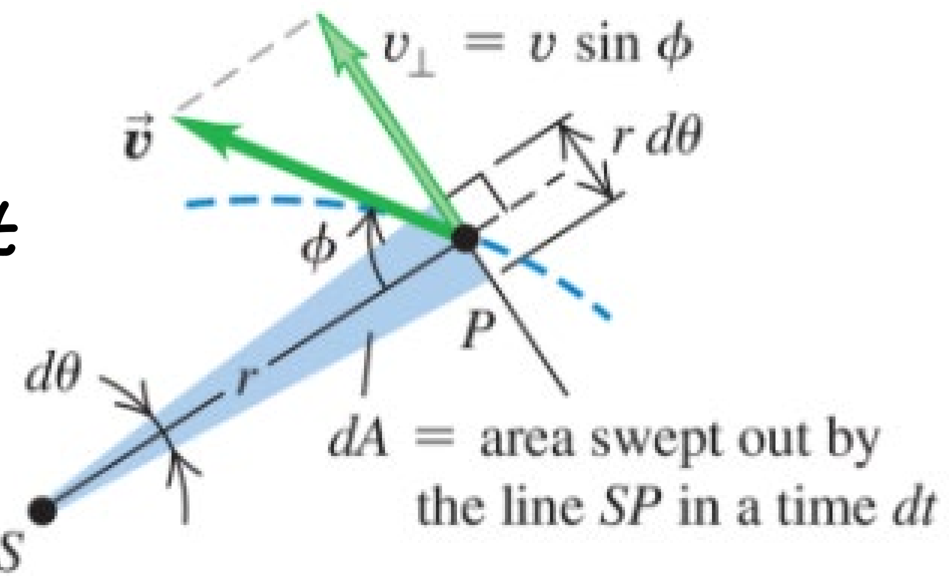

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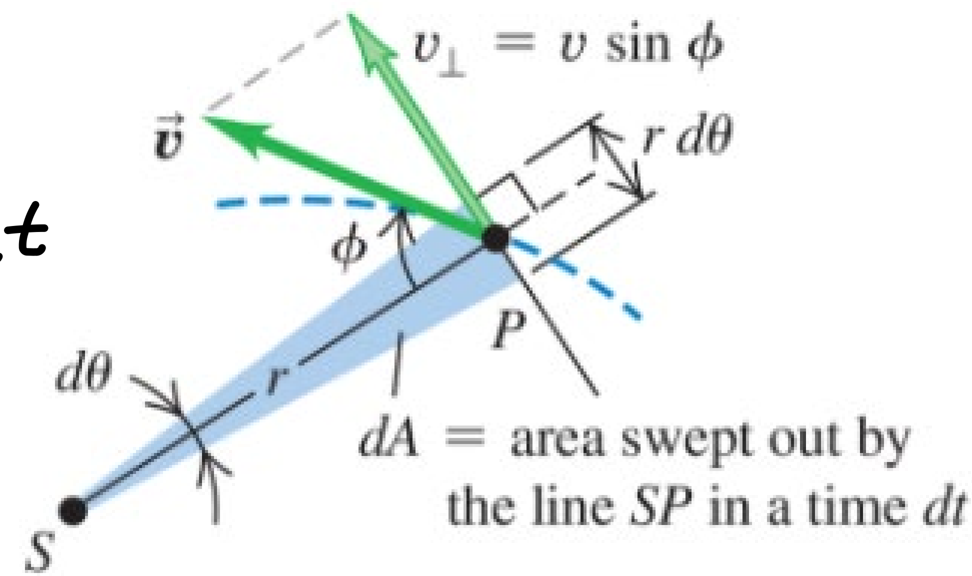

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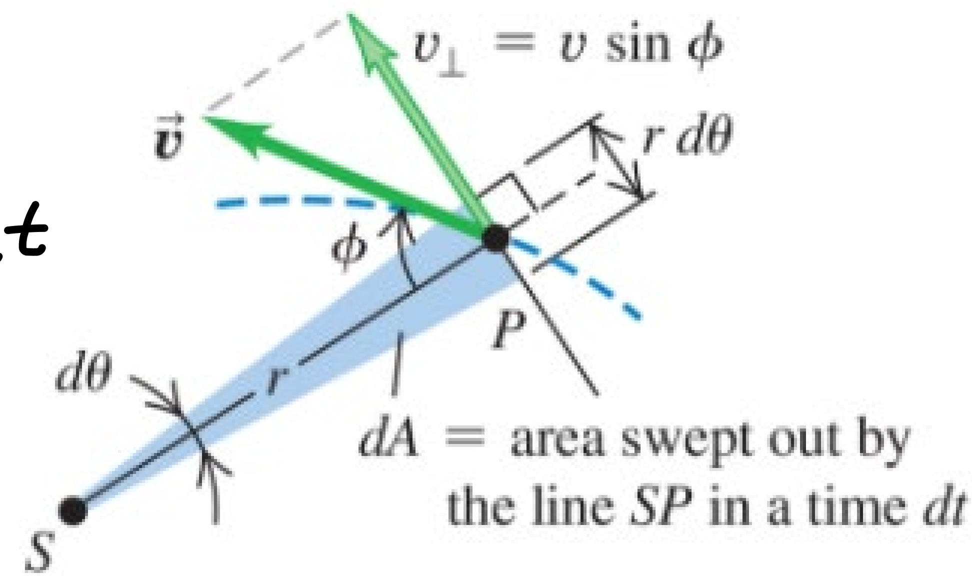

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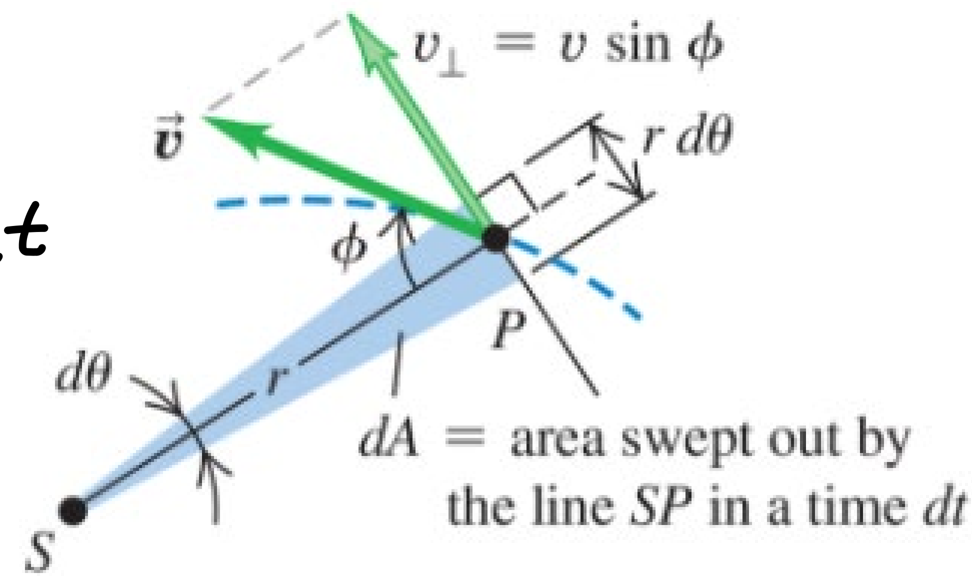

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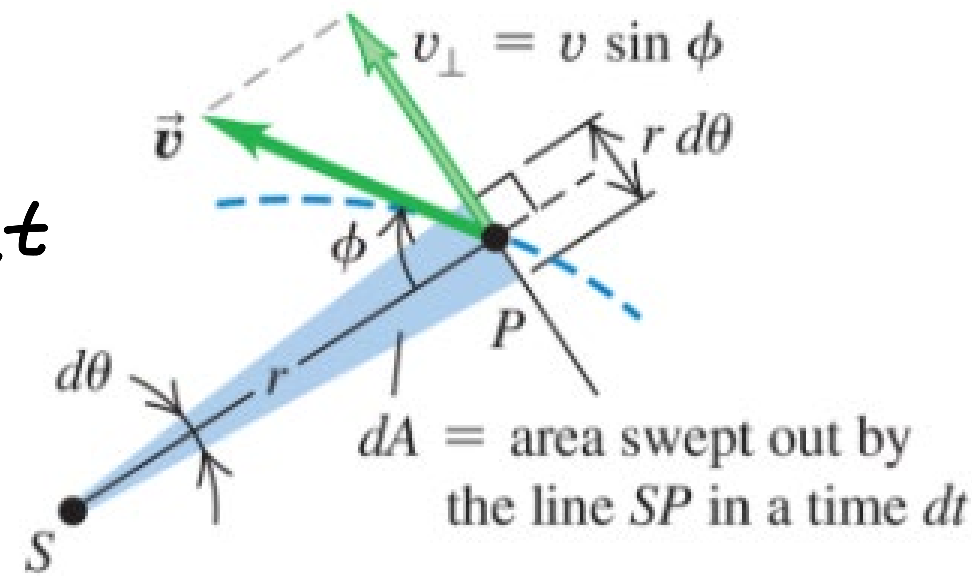

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thus $$\frac{dA}{dt} = \frac{L}{2m} = \text{const.}$$

ASU

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$$T = \frac{2\pi a^{3/2}}{\sqrt{G M_s}}$$

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we will not prove this for eliptical orbit

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For circular orbit of radius a

$F = ma$

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$$\Rightarrow \frac{4\pi^2 a^2}{T^2} = \frac{GM_s}{a}$$

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$$\Rightarrow \quad T = \frac{2\pi a^{3/2}}{\sqrt{GM_s}}$$

ASU

At what point in an elliptical orbit (see Fig. 13.19) does a planet move the fastest? The slowest?

The asteroid Pallas has an orbital period of 4.62 years and an orbital eccentricity of 0.233. Find the semi-major axis of its orbit.

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Find a:

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Find a: $T = \frac{2\pi a^{3/2}}{\sqrt{GM_s}}$

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Note: Do not need value of ϵ

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Find a:

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM_s}} \Rightarrow a^{3/2} = \frac{T}{2\pi}\sqrt{GM_s}$$

$$\Rightarrow \quad a = \left[\frac{T}{2\pi}\right]^{2/3}\left[GM_s\right]^{1/3}$$

The asteroid Pallas has an orbital period of 4.62 years and an orbital eccentricity of 0.233. Find the semi-major axis of its orbit.

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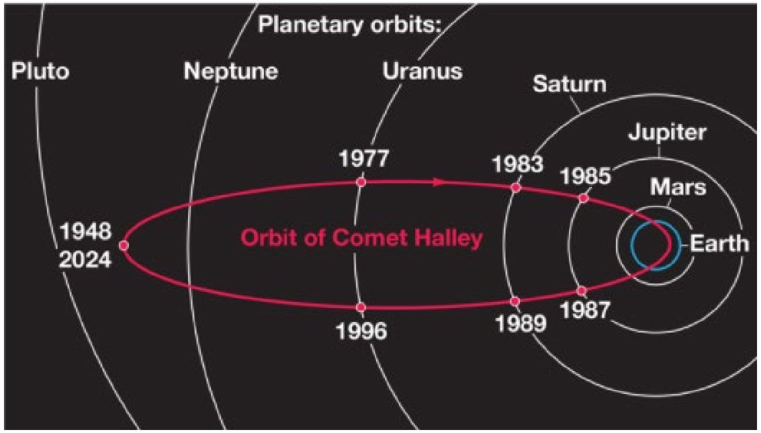
Comet Halley (Fig. 13.21) moves in an elongated elliptical orbit around the sun. Its distances from the sun at perihelion and aphelion are 8.80×10^7 km and 5.25×10^9 km, respectively. Find the orbital semi-major axis, eccentricity, and period in years.



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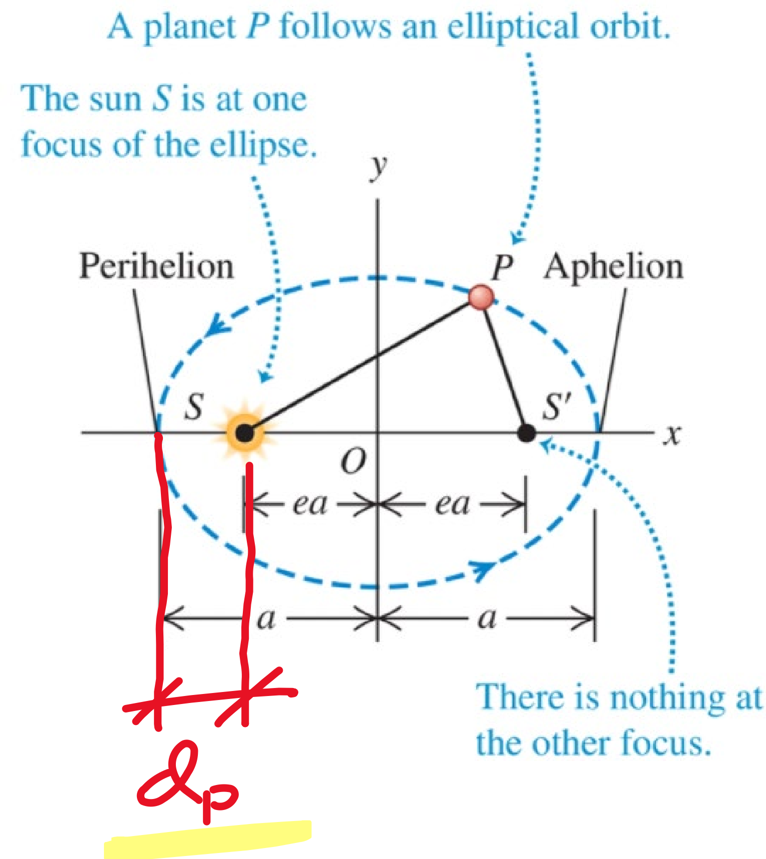

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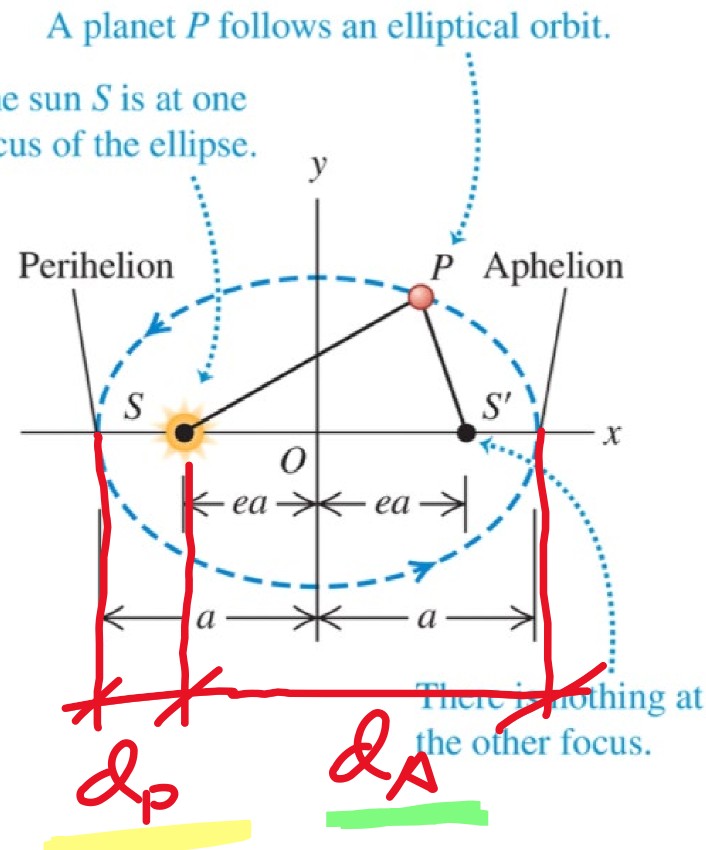

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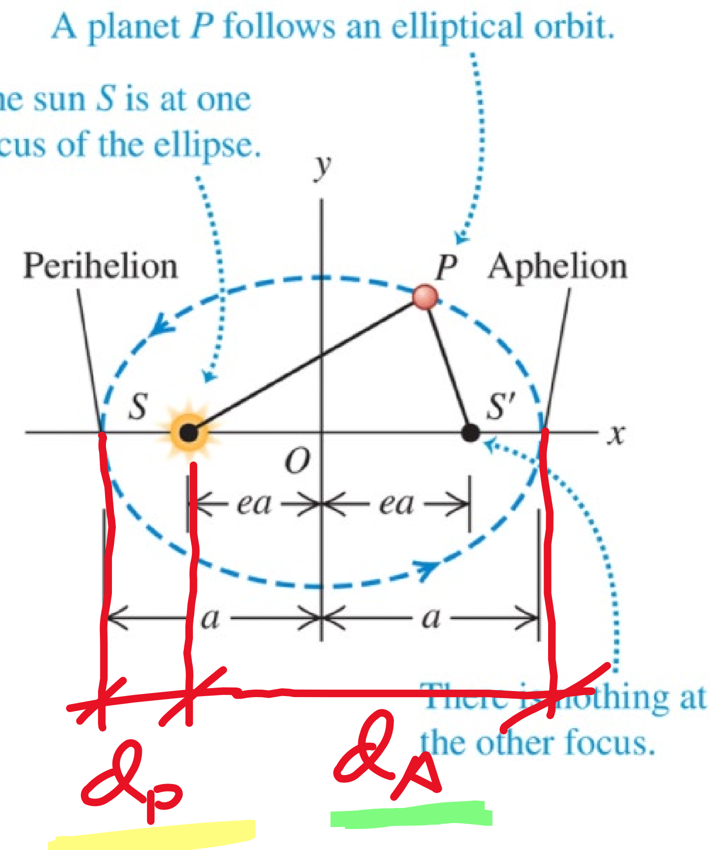

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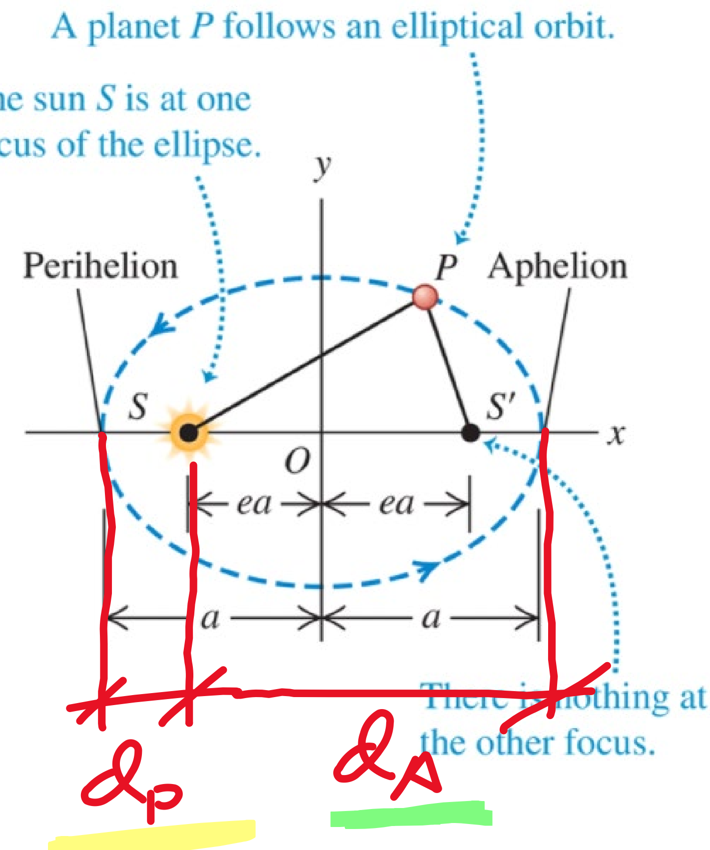

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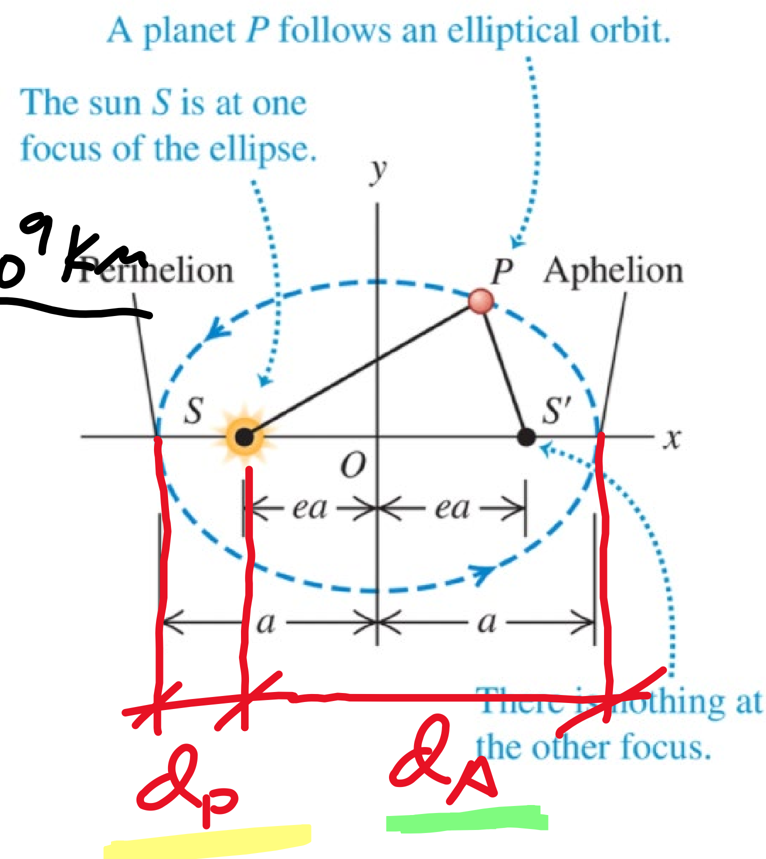

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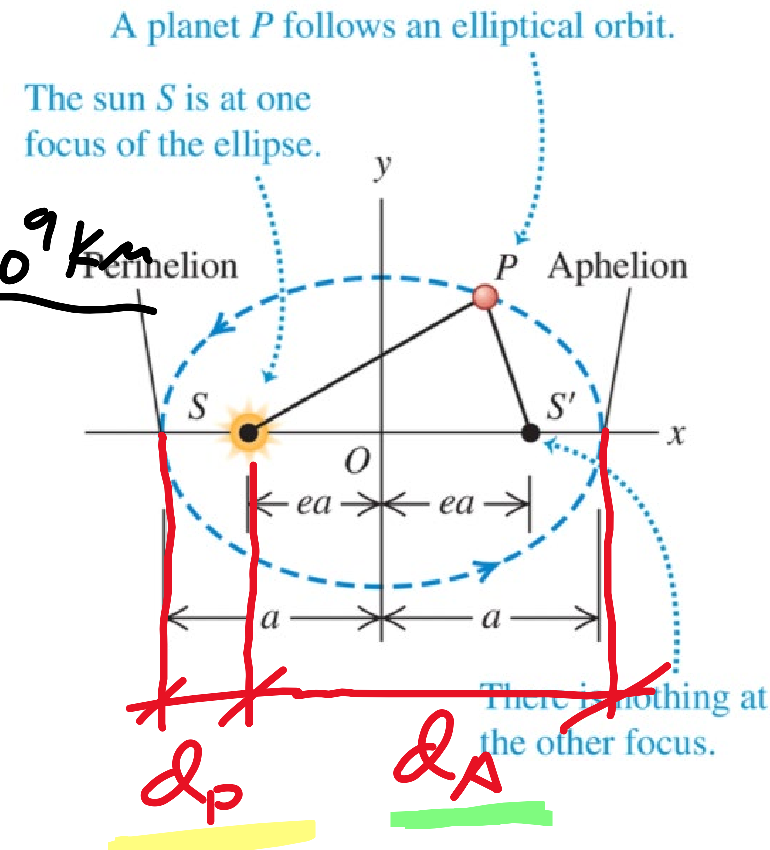

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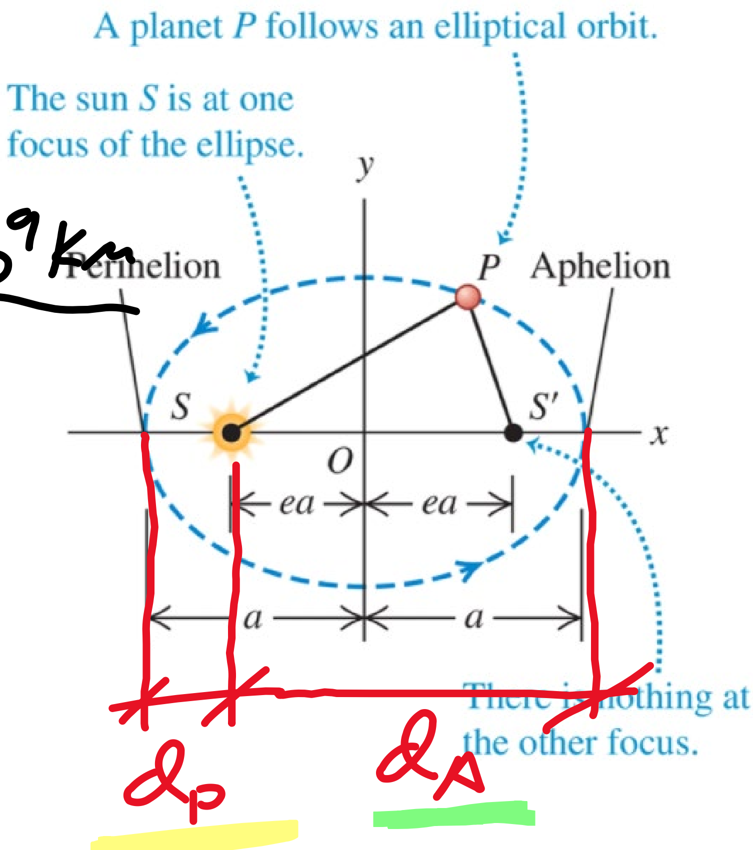

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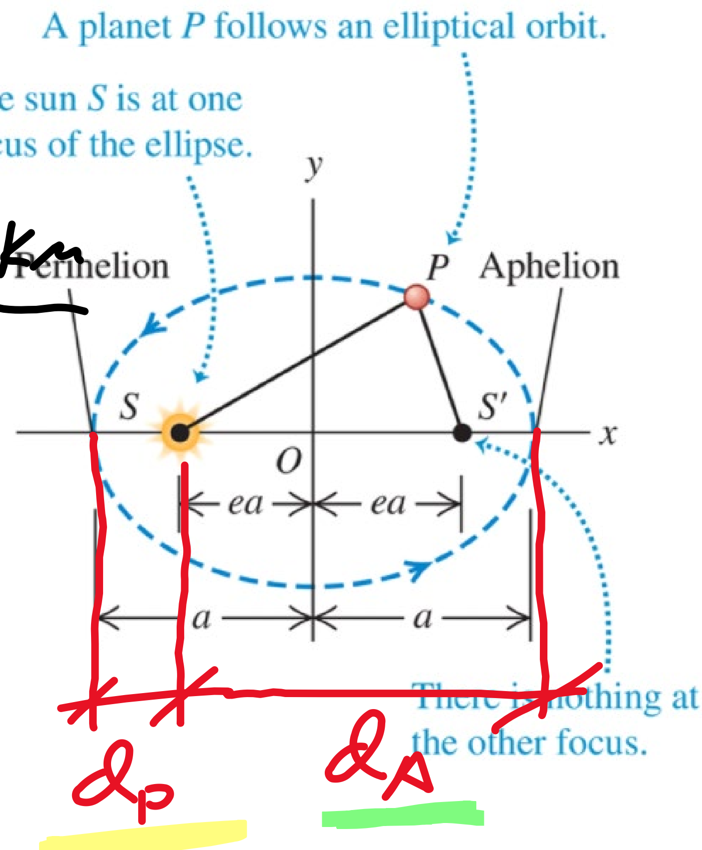

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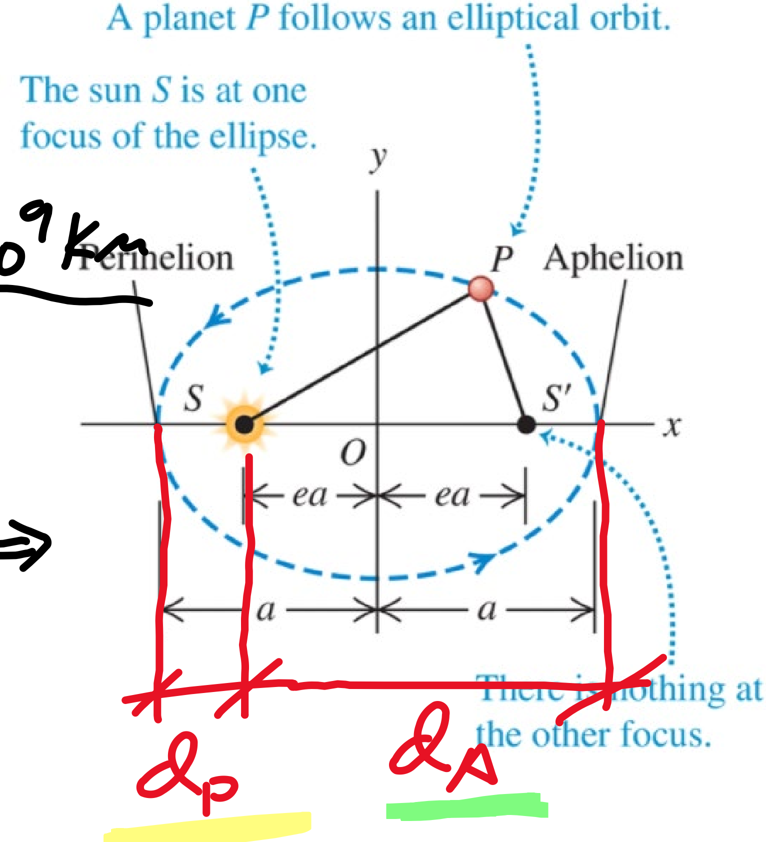

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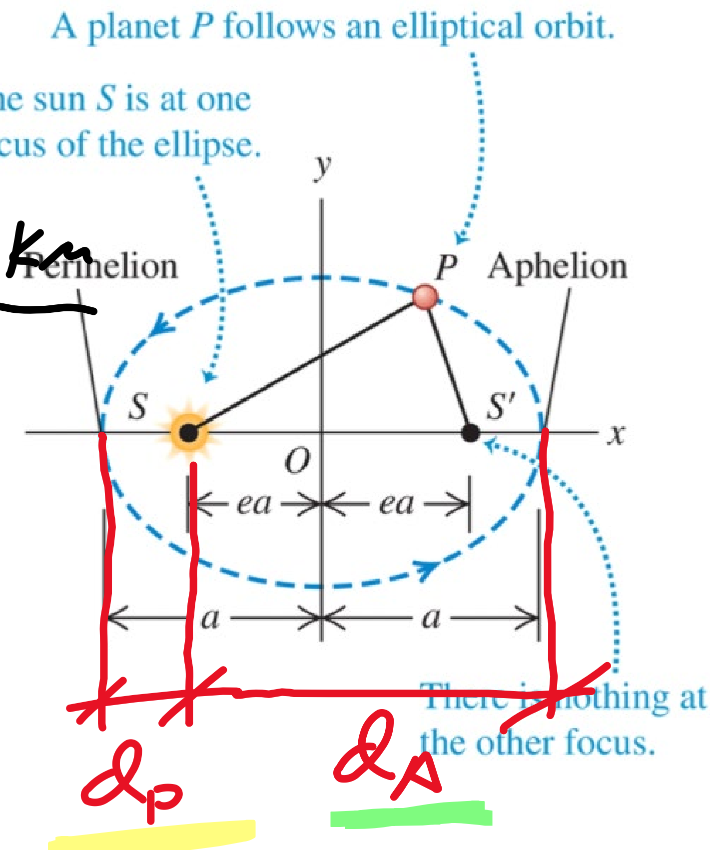

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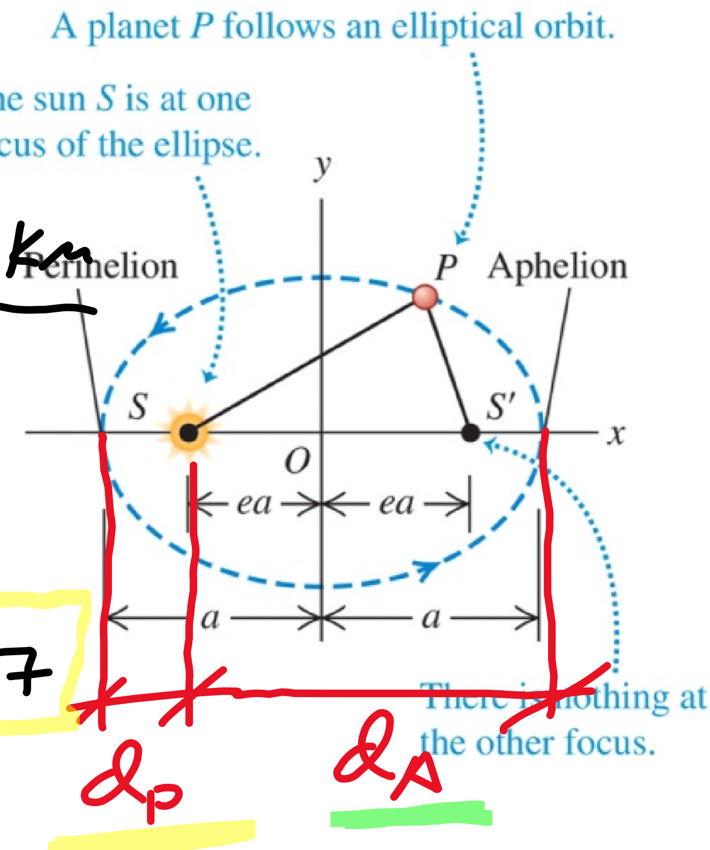

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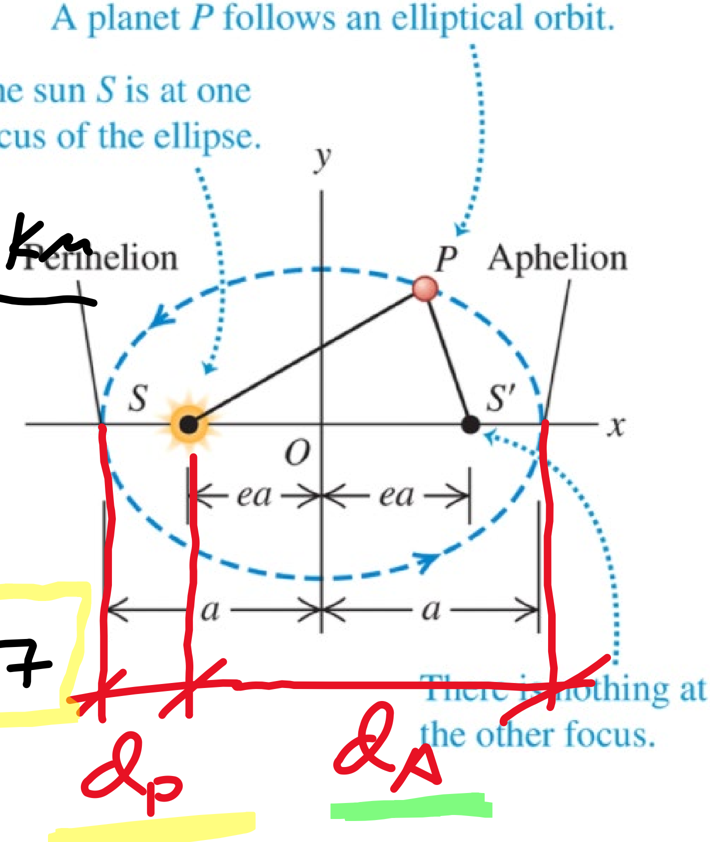

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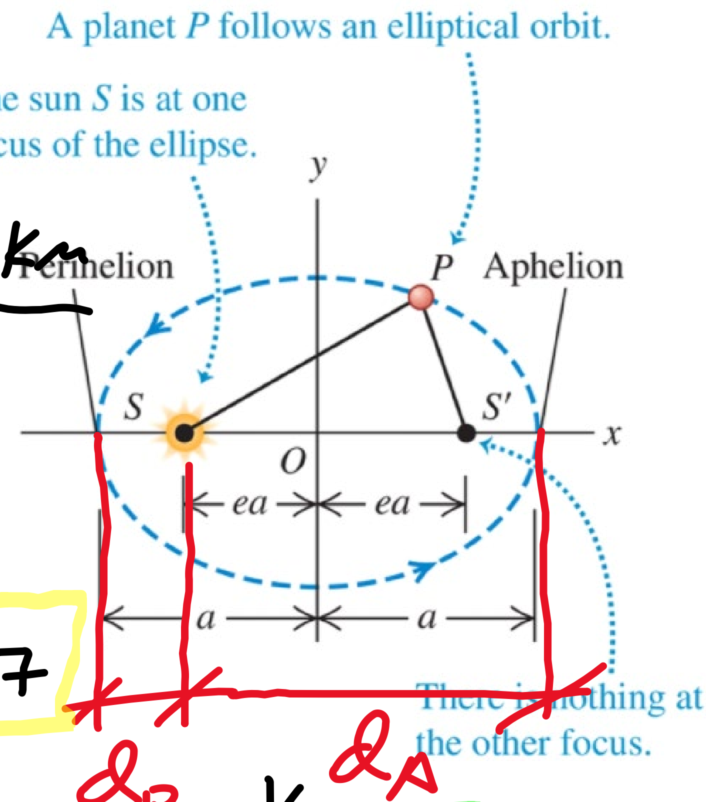

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM_s}} = 2\pi \left[\frac{(2.67 * 10^{12})^3}{6.67 * 10^{-11} * 1.99 * 10^{30}}\right]^{1/2} s$$

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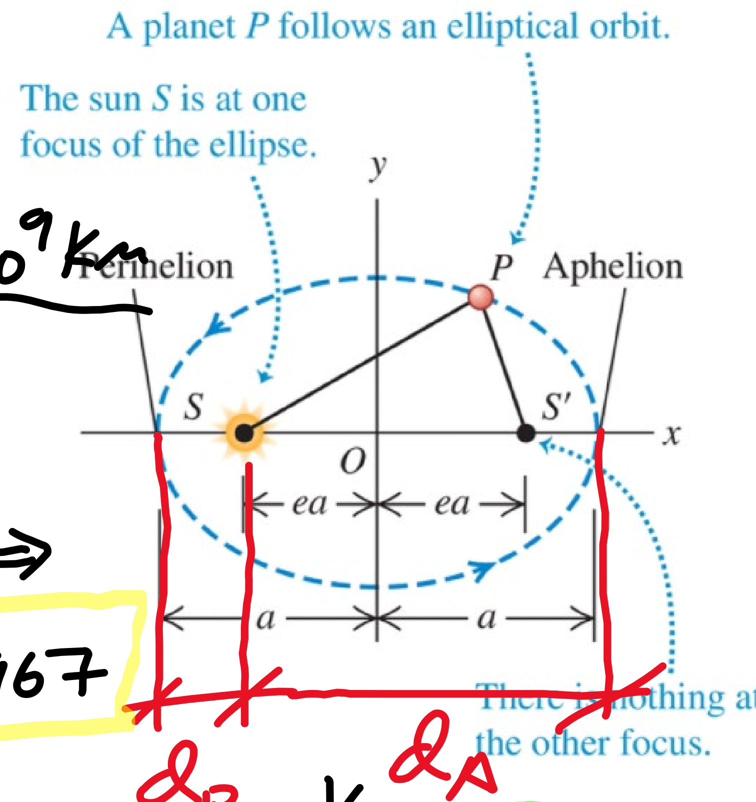
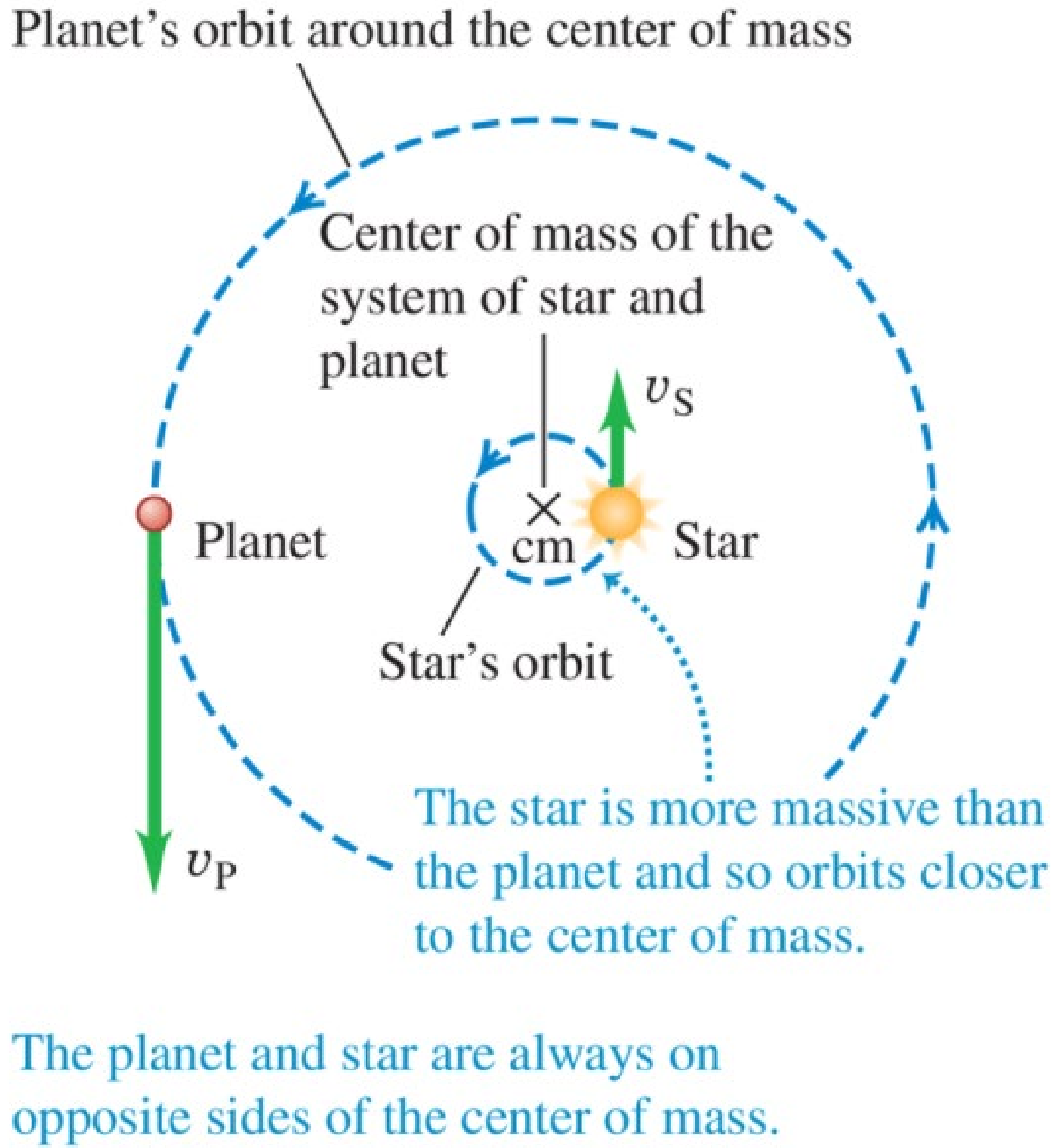


$$T = \frac{2\pi a^{3/2}}{\sqrt{GM_S}} = 2\pi\left[\frac{(2.67\times 10^{12})^3}{6.67\times 10^{-11}\times 1.99\times 10^{30}}\right]^{1/2} s = 2.38\times 10^{9}\ s$$


Planet's orbit around the center of mass
Center of mass of the system of star and planet
v_S
Planet
cm
Star
Star's orbit
v_P
The star is more massive than the planet and so orbits closer to the center of mass.
The planet and star are always on opposite sides of the center of mass.

§13.8

Black holes

Escape speed from star

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$$U_1 + K_1 = U_2 + K_2$$

Escape speed from star

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$$\boxed{v = R\sqrt{\frac{8}{3}\pi G \rho}}$$

Schwarzschild radius & Event horizon

If we set $v = c$ then we get

$$c = \sqrt{\frac{2GM}{R}}$$

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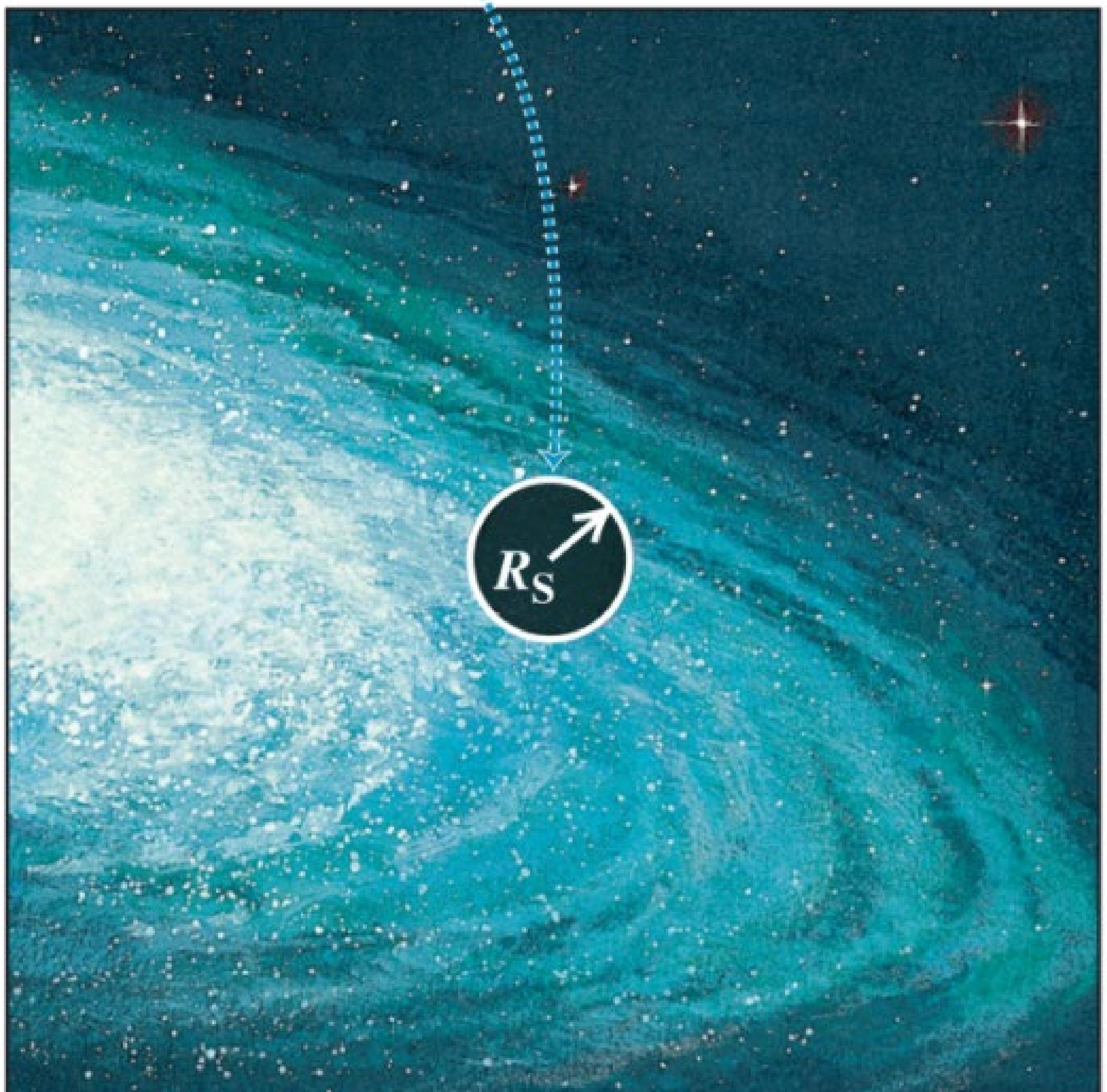
Spherical surface with R_s is "Event horizon"

(a) When the radius R of an object is greater than the Schwarzschild radius R_S, light can escape from the surface of the object.

Gravity acting on the escaping light "red shifts" it to longer wavelengths.

(b) If all the mass of the object lies inside radius R_S, the object is a black hole: No light can escape from it.



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$$\Rightarrow R = \frac{6GM_s}{V^2}$$

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Now, set $v = c$

Astrophysical theory suggests that a burned-out star whose mass is at least three solar masses will collapse under its own gravity to form a black hole. If it does, what is the radius of its event horizon?

$$K_1 + U_1 = \cancel{K_2}^{0} + \cancel{U_2}^{0} \Rightarrow K_1 = -U_1$$

$$\Rightarrow \frac{1}{2}\cancel{m}v^2 = \frac{G(3M_s)\cancel{m}}{R} \Rightarrow v^2 = \frac{6GM_s}{R}$$

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to get $R_s = \dfrac{6GM_s}{c^2}$

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to get $$R_s = \frac{6GM_s}{c^2} = \frac{6 * 6.67 * 10^{-11} * 1.99 * 10^{30}}{9 * 10^{16}} m$$

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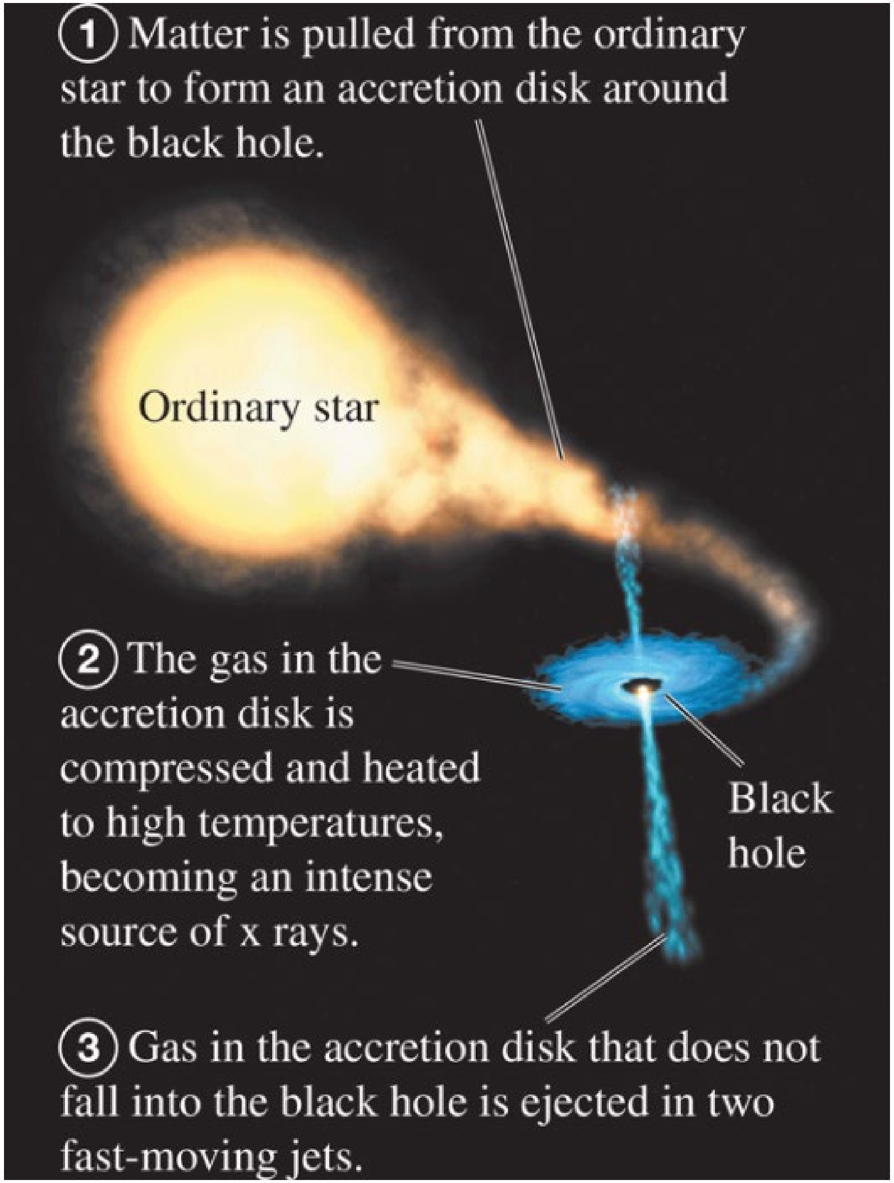
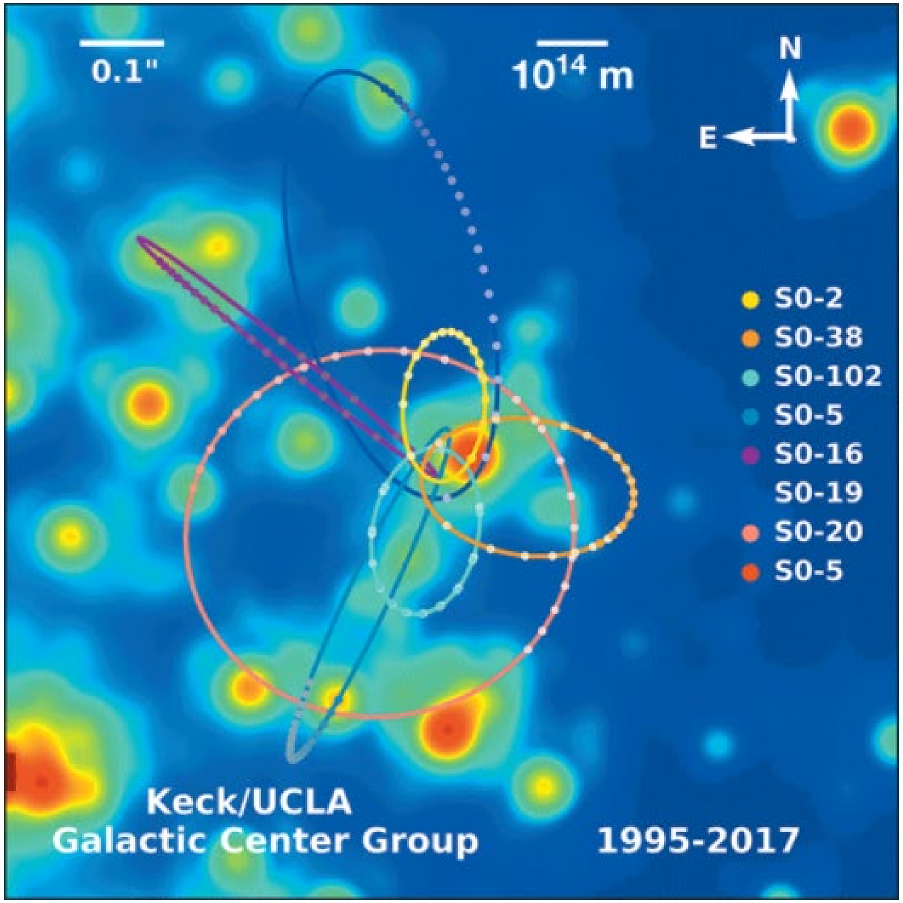
Detecting Black holes





This false-color image shows the motions of stars at the center of our galaxy over a 17-year period. Analysis of these orbits by using Kepler's third law indicates that the stars are moving about an unseen object that is some 4.1×10^6 times the mass of the sun. The scale bar indicates a length of 10^{14} m (670 times the distance from the earth to the sun).

The End