

Today 11.4, 11.5

L34



Today 11.4, 11.5

L34

Stress,  
Strain,  
Elastic  
moduli

Today 11.4, 11.5

L34

Stress,  
Strain,  
Elast.:l.  
moduli

Elasticity  
& plasticity

Today 11.4, 11.5

Friday 13.1, 13.2

L34

Today 11.4, 11.5

L34

Friday 13.1, 13.2

Newton's  
Law of  
gravitation

Today 11.4, 11.5

L34

Friday 13.1, 13.2

Newton's  
Law of  
gravitation: on  
weight

Today 11.4, 11.5

L34

Friday 13.1, 13.2

Monday 13.3, 13.4

Today 11.4, 11.5

L34

Friday 13.1, 13.2

Monday 13.3, 13.4

Gravitational  
potential  
energy

Today 11.4, 11.5

L34

Friday 13.1, 13.2

Monday 13.3, 13.4

Gravitational  
potential  
energy

Motion of  
satellites

Today 11.4, 11.5

L34

Friday 13.1, 13.2

Monday 13.3, 13.4

Wednesday Nov. 25<sup>th</sup> 13.5, 13.8

Today 11.4, 11.5

L34

Friday 13.1, 13.2

Monday 13.3, 13.4

Wednesday Nov. 25<sup>th</sup> 13.5, 13.8

Kepler's  
laws

Today 11.4, 11.5

L34

Friday 13.1, 13.2

Monday 13.3, 13.4

Wednesday Nov. 25<sup>th</sup> 13.5, 13.8

Kepler's  
laws

Black  
holes

Today 11.4, 11.5

L34

Friday 13.1, 13.2

Monday 13.3, 13.4

Wednesday Nov. 25<sup>th</sup> 13.5, 13.8

Friday Nov. 27<sup>th</sup> Holiday

Today 11.4, 11.5

L34

Friday 13.1, 13.2

Monday 13.3, 13.4

Wednesday Nov. 25<sup>th</sup> 13.5, 13.8

Friday Nov. 27<sup>th</sup> Holiday

Monday Nov. 30<sup>th</sup> Exam #4

Today 11.4, 11.5

L34

Friday 13.1, 13.2

Monday 13.3, 13.4

Wednesday Nov. 25<sup>th</sup> 13.5, 13.8

Friday Nov. 27<sup>th</sup> Holiday

Monday Nov. 30<sup>th</sup> Exam #4

Wednesday Dec. 2<sup>nd</sup> Day of Reckoning

Today 11.4, 11.5

L34

Friday 13.1, 13.2

Monday 13.3, 13.4

Wednesday Nov. 25<sup>th</sup> 13.5, 13.8

Friday Nov. 27<sup>th</sup> Holiday

Monday Nov. 30<sup>th</sup> Exam #4

Wednesday Dec. 2<sup>nd</sup> Day of Reckoning

Friday Dec. 4<sup>th</sup> Final exam

# Types of stress



# Types of stress



Guitar strings under tensile stress,

# Types of stress



Guitar strings under tensile stress, being stretched by forces at the ends

# Types of stress



# Types of stress



Diver under bulk stress

# Types of stress



Diver under bulk stress,  
being squeezed from all sides

# Types of stress



# Types of stress



A ribbon under shear stress,

# Types of stress

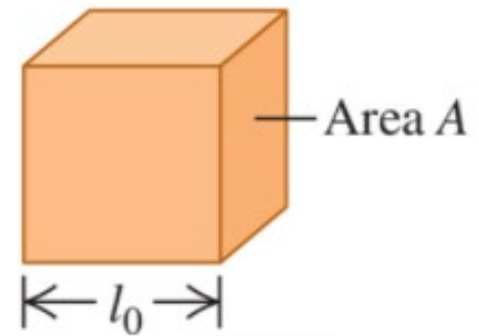


A ribbon under shear stress, being deformed and eventually cut by forces exerted by the scissors

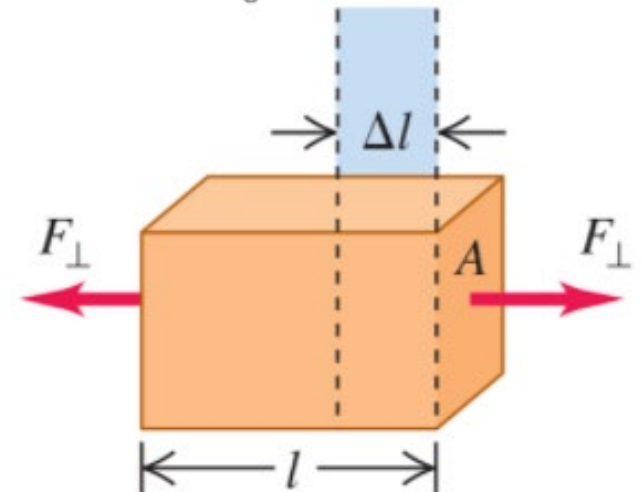
# Hook's law

# Hook's law

Initial state  
of the object



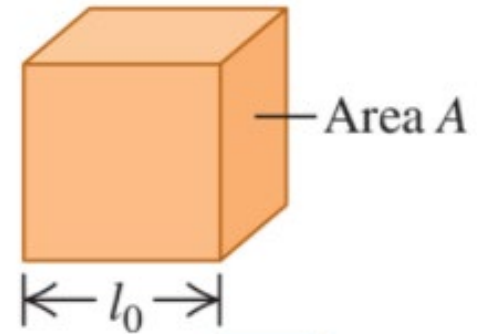
Object under  
tensile stress



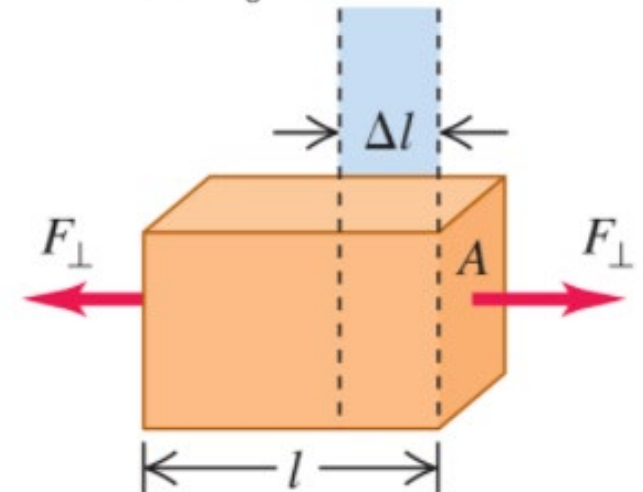
# Hook's law

$$\frac{\text{Stress}}{\text{Strain}} = \text{Elastic modulus, where}$$

Initial state  
of the object



Object under  
tensile stress

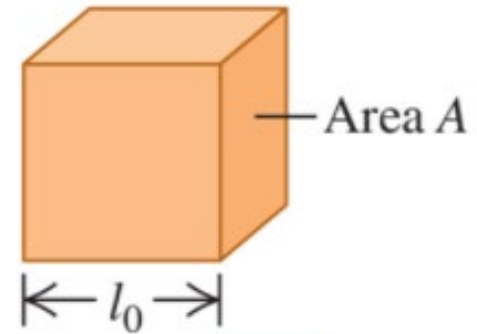


# Hook's law

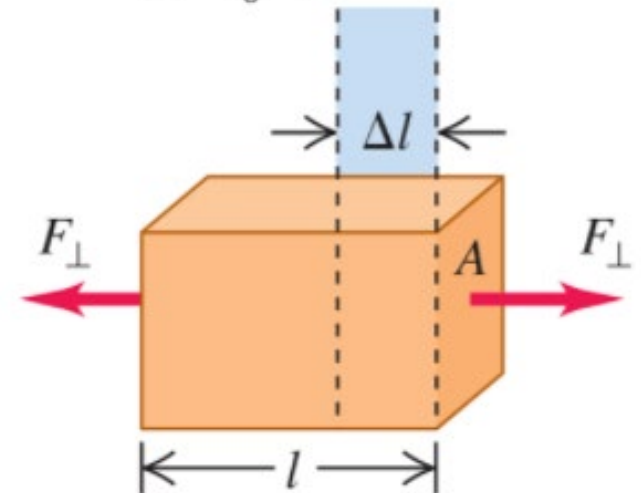
$\frac{\text{Stress}}{\text{Strain}} = \text{Elastic modulus}$ , where

$$\text{Tensile stress} = \frac{F_{\perp}}{A}$$

Initial state  
of the object



Object under  
tensile stress



# Hook's law

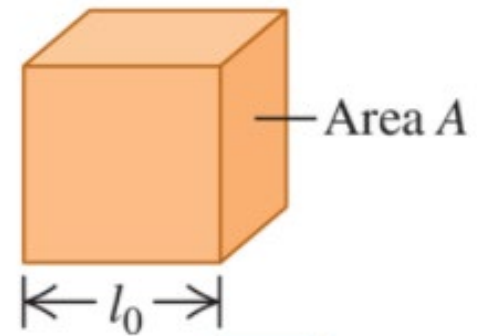
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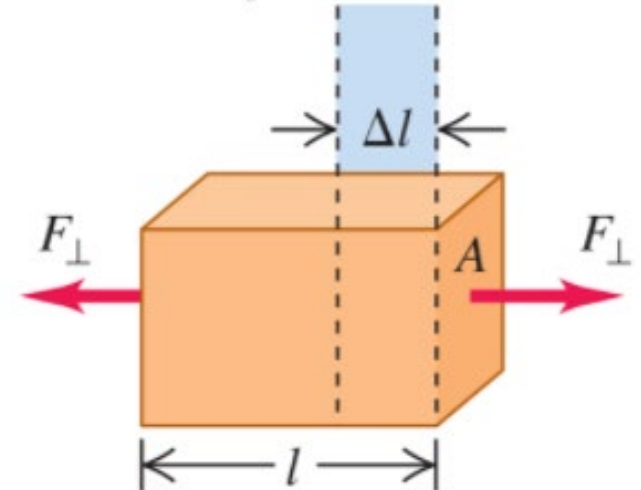
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$$\text{Tensile strain} = \frac{\Delta l}{l_0}$$

Initial state  
of the object



Object under  
tensile stress



# Hook's law

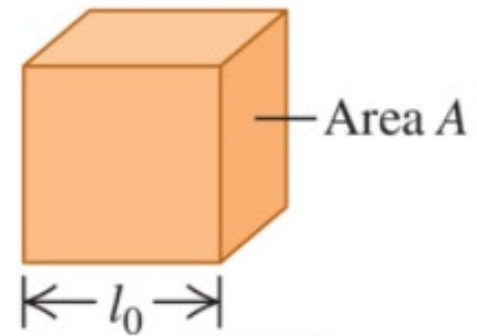
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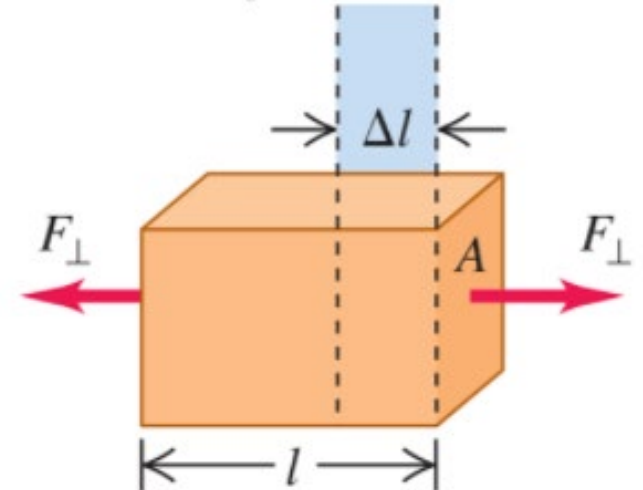
$\&$

$$\text{Tensile strain} = \frac{\Delta l}{l_0}$$

Initial state  
of the object



Object under  
tensile stress



Young's modulus  $\equiv Y$

Young's modulus  $\equiv Y$  †

$$Y \equiv \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

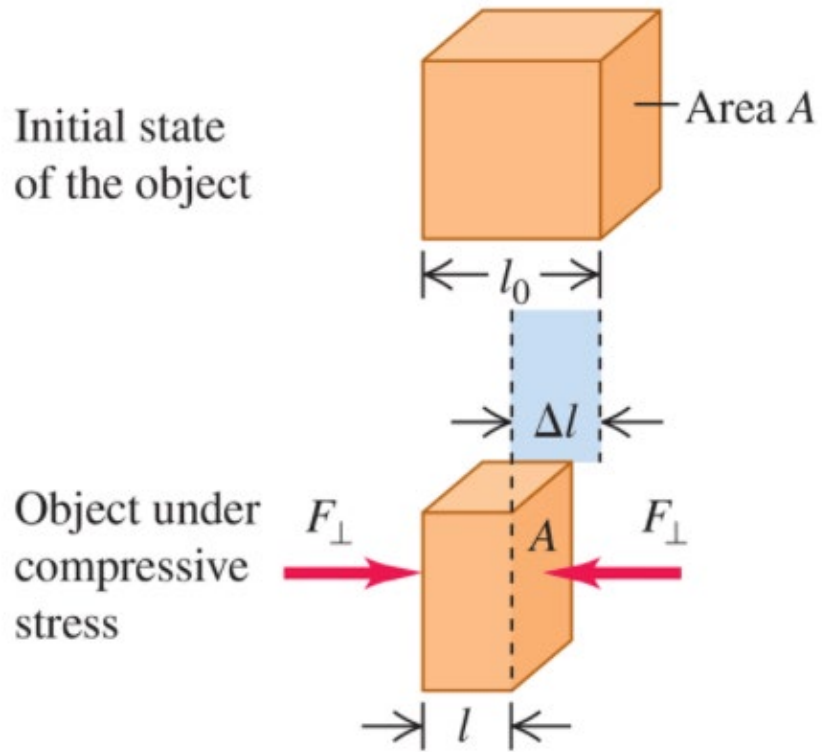
Young's modulus  $\equiv Y$  †

$$Y \equiv \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{(F_t/A)}{(\Delta l/l_0)}$$

**Table 11.1 Approximate Elastic Moduli**

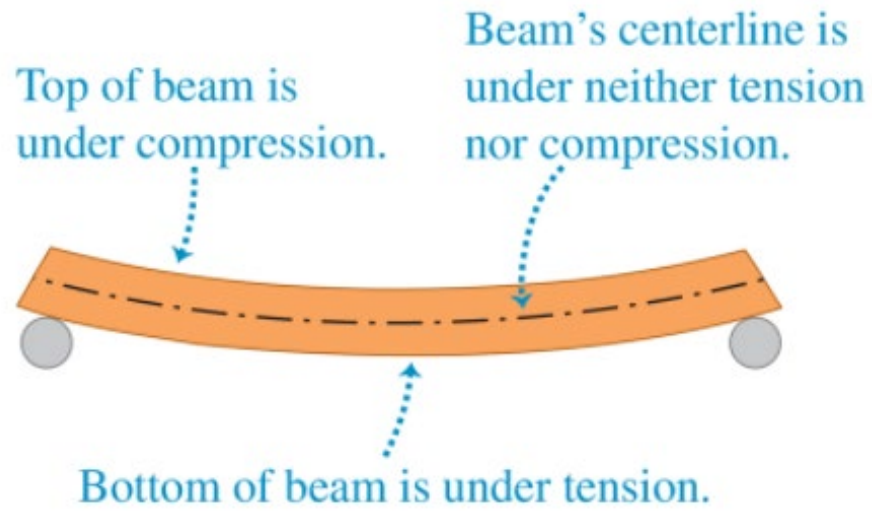
<b>Material</b>	<b>Young's Modulus, <math>Y</math> (Pa)</b>	<b>Bulk Modulus, <math>B</math> (Pa)</b>	<b>Shear Modulus, <math>S</math> (Pa)</b>
Aluminum	$7.0 \times 10^{10}$	$7.5 \times 10^{10}$	$2.5 \times 10^{10}$
Brass	$9.0 \times 10^{10}$	$6.0 \times 10^{10}$	$3.5 \times 10^{10}$
Copper	$11 \times 10^{10}$	$14 \times 10^{10}$	$4.4 \times 10^{10}$
Iron	$21 \times 10^{10}$	$16 \times 10^{10}$	$7.7 \times 10^{10}$
Lead	$1.6 \times 10^{10}$	$4.1 \times 10^{10}$	$0.6 \times 10^{10}$
Nickel	$21 \times 10^{10}$	$17 \times 10^{10}$	$7.8 \times 10^{10}$
Silicone rubber	$0.001 \times 10^{10}$	$0.2 \times 10^{10}$	$0.0002 \times 10^{10}$
Steel	$20 \times 10^{10}$	$16 \times 10^{10}$	$7.5 \times 10^{10}$
Tendon (typical)	$0.12 \times 10^{10}$	—	—

Compressive stress & strain defined in same way except  $\Delta l$  is now is distance contracted

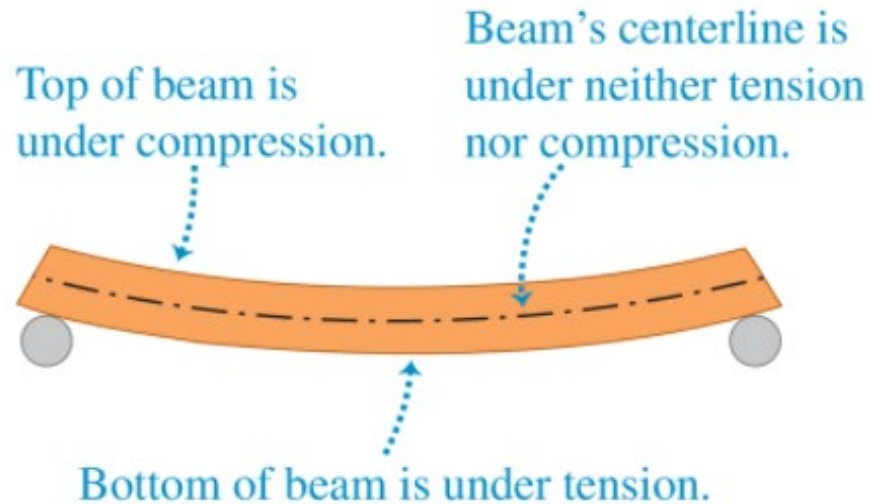


$$\text{Compressive stress} = \frac{F_{\perp}}{A} \quad \text{Compressive strain} = \frac{\Delta l}{l_0}$$

(a)

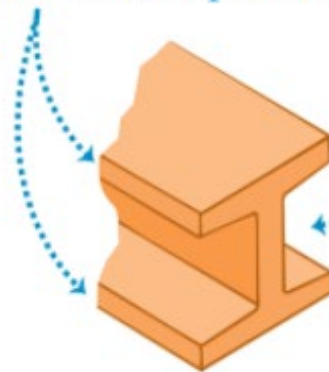


(a)



(b)

The top and bottom of an I-beam are broad to minimize the compressive and tensile stresses.



The beam can be narrow near its centerline, which is under neither compression nor tension.

A steel rod 2.0 m long has a cross-sectional area of  $0.30 \text{ cm}^2$ . It is hung by one end from a support, and a 550 kg milling machine is hung from its other end. Determine the stress on the rod and the resulting strain and elongation.



A steel rod 2.0 m long has a cross-sectional area of  $0.30 \text{ cm}^2$ . It is hung by one end from a support, and a 550 kg milling machine is hung from its other end. Determine the stress on the rod and the resulting strain and elongation.  $L = 2 \text{ m}$

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$$L = 2\text{ m}$$

$$A = 0.3\text{ cm}^2$$

A steel rod 2.0 m long has a cross-sectional area of 0.30 cm<sup>2</sup>. It is hung by one end from a support, and a 550 kg milling machine is hung from its other end. Determine the stress on the rod and the resulting strain and elongation.

$$L = 2\text{ m}$$

$$A = 0.3\text{ cm}^2 \left(\frac{\text{m}}{100\text{ cm}}\right)^2$$

A steel rod 2.0 m long has a cross-sectional area of 0.30 cm<sup>2</sup>. It is hung by one end from a support, and a 550 kg milling machine is hung from its other end. Determine the stress on the rod and the resulting strain and elongation.

$$L = 2 \text{ m}$$
$$A = 0.3 \text{ cm}^2 \left( \frac{\text{m}}{100 \text{ cm}} \right)^2 = 3 \times 10^{-5} \text{ m}^2$$

A steel rod 2.0 m long has a cross-sectional area of 0.30 cm<sup>2</sup>. It is hung by one end from a support, and a 550 kg milling machine is hung from its other end. Determine the stress on the rod and the resulting strain and elongation.  $L = 2\text{ m}$

$$A = 0.3 \text{ cm}^2 \left(\frac{\text{m}}{100\text{cm}}\right)^2 = 3 \times 10^{-5} \text{ m}^2, M = 550 \text{ kg}$$

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$$\text{Stress} = \frac{F}{A}$$

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$$\text{Stress} = \frac{F}{A} = \frac{(550 \text{ kg})(9.8 \text{ m/s}^2)}{3 \times 10^{-5} \text{ m}^2}$$

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$$\text{Stress} = \frac{F}{A} = \frac{(550 \text{ kg})(9.8 \text{ m/s}^2)}{3 \times 10^{-5} \text{ m}^2} = 1.8 \times 10^8 \text{ Pa}$$

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$$\text{Stress} = \frac{F_{\perp}}{A} = \frac{(550 \text{ kg})(9.8 \text{ m/s}^2)}{3 \times 10^{-5} \text{ m}^2} = 1.8 \times 10^8 \text{ Pa}$$

$$Y = \frac{(F_{\perp}/A)}{(\Delta l/l)} = \frac{\text{Stress}}{\text{Strain}}$$

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$$Y = \frac{(F_{\perp}/A)}{(\Delta l/l)} = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow \frac{\Delta l}{l} = \frac{1.8 \times 10^8}{20 \times 10^{10}}$$

Table 11.1 Approximate Elastic Moduli

Material	Young's Modulus, Y (Pa)
Aluminum	$7.0 \times 10^{10}$
Brass	$9.0 \times 10^{10}$
Copper	$11 \times 10^{10}$
Iron	$21 \times 10^{10}$
Lead	$1.6 \times 10^{10}$
Nickel	$21 \times 10^{10}$
Silicone rubber	$0.001 \times 10^{10}$
Steel	$20 \times 10^{10}$
Tendon (typical)	$0.12 \times 10^{10}$



A steel rod 2.0 m long has a cross-sectional area of 0.30 cm<sup>2</sup>. It is hung by one end from a support, and a 550 kg milling machine is hung from its other end. Determine the stress on the rod and the resulting strain and elongation.  $L = 2\text{ m}$

$$A = 0.3 \text{ cm}^2 \left(\frac{\text{m}}{100\text{cm}}\right)^2 = 3 \times 10^{-5} \text{ m}^2, \quad M = 550 \text{ kg}$$

$$\text{Stress} = \frac{F_{\perp}}{A} = \frac{(550 \text{ kg})(9.8 \text{ m/s}^2)}{3 \times 10^{-5} \text{ m}^2} = 1.8 \times 10^8 \text{ Pa}$$

$$Y = \frac{(F_{\perp}/A)}{(\Delta l/l)} = \frac{\text{Stress}}{\text{Strain}} \Rightarrow \text{Strain} = \frac{\Delta l}{l} = \frac{(F_{\perp}/A)}{Y}$$

$$\Rightarrow \frac{\Delta l}{l} = \frac{1.8 \times 10^8}{20 \times 10^{10}} = 0.9 \times 10^{-3}$$

A steel rod 2.0 m long has a cross-sectional area of 0.30 cm<sup>2</sup>. It is hung by one end from a support, and a 550 kg milling machine is hung from its other end. Determine the stress on the rod and the resulting strain and elongation.  $L = 2\text{ m}$

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$$\Rightarrow \frac{\Delta l}{l} = \frac{1.8 \times 10^8}{20 \times 10^{10}} = 0.9 \times 10^{-3} = 9 \times 10^{-4}$$

A steel rod 2.0 m long has a cross-sectional area of 0.30 cm<sup>2</sup>. It is hung by one end from a support, and a 550 kg milling machine is hung from its other end. Determine the stress on the rod and the resulting strain and elongation.  $L = 2\text{ m}$

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$$\Rightarrow \Delta l = l \times 9 \times 10^{-4}$$

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$$\Rightarrow \frac{\Delta l}{l} = \frac{1.8 \times 10^8}{20 \times 10^{10}} = 0.9 \times 10^{-3} = 9 \times 10^{-4}$$

$$\Rightarrow \Delta l = l \times 9 \times 10^{-4} = 18 \times 10^{-4} \text{ m}$$

A steel rod 2.0 m long has a cross-sectional area of 0.30 cm<sup>2</sup>. It is hung by one end from a support, and a 550 kg milling machine is hung from its other end. Determine the stress on the rod and the resulting strain and elongation.  $L = 2\text{ m}$

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$$\Rightarrow \frac{\Delta l}{l} = \frac{1.8 \times 10^8}{20 \times 10^{10}} = 0.9 \times 10^{-3} = 9 \times 10^{-4}$$

$$\Rightarrow \Delta l = l \times 9 \times 10^{-4} = 18 \times 10^{-4} \text{ m} = 1.8 \times 10^{-3} \text{ m}$$

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$$Y = \frac{(F_{\perp}/A)}{(\Delta l/l)} = \frac{\text{Stress}}{\text{Strain}} \Rightarrow \text{Strain} = \frac{\Delta l}{l} = \frac{(F_{\perp}/A)}{Y}$$

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$$\Rightarrow \Delta l = l \times 9 \times 10^{-4} = 18 \times 10^{-4} \text{ m} = 1.8 \times 10^{-3} \text{ m} \\ = 1.8 \text{ mm}$$

# Bulk stress and strain

$$P = \frac{F_{\perp}}{A}$$

# Bulk stress and strain

$$P = \frac{F_{\perp}}{A}$$

pressure  
in fluid

# Bulk stress and strain

$$P = \frac{F_{\perp}}{A}$$

Force fluid applies  
to immersed  
object

pressure  
in fluid

# Bulk stress and strain

$$P = \frac{F_{\perp}}{A}$$

Force fluid applies  
to immersed  
object

pressure  
in fluid

Area that  
force is exerted  
over

# Bulk stress and strain

$$P = \frac{F_{\perp}}{A}$$

Force fluid applies  
to immersed  
object

pressure  
in fluid

Area that  
force is exerted  
over

$$\text{Bulk (volume) strain} = \frac{\Delta V}{V_0}$$

# Bulk stress and strain

$$P = \frac{F_{\perp}}{A}$$

Force fluid applies  
to immersed  
object

pressure  
in fluid

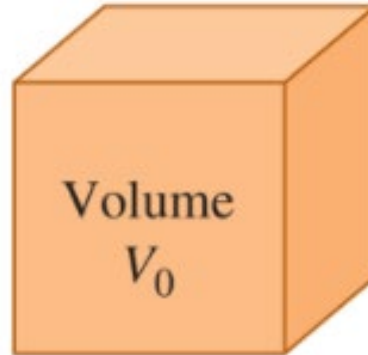
Area that  
force is exerted  
over

$$\text{Bulk (volume) strain} = \frac{\Delta V}{V_0}$$

$$\text{Bulk stress} = \Delta p$$

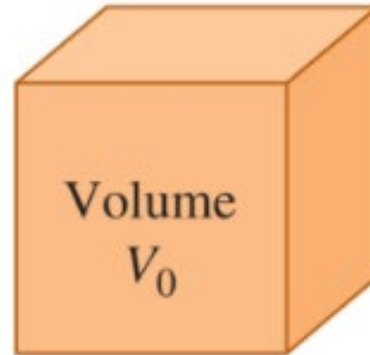
Pressure =  $p_0$

Initial state  
of the object



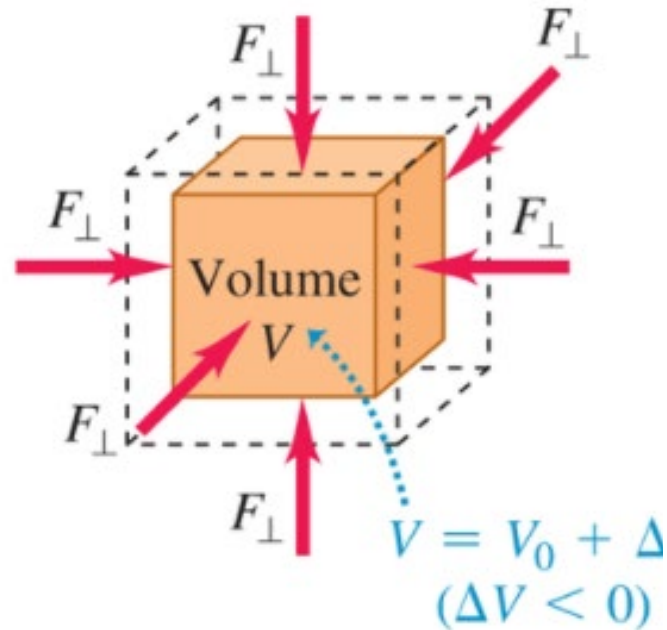
Initial state  
of the object

$$\text{Pressure} = p_0$$



Object under  
bulk stress

$$\text{Pressure} = p = p_0 + \Delta p$$



Bulk modulus  $\equiv B$

Bulk modulus  $\equiv B$  ‡

$$B \equiv \frac{\text{Bulk stress}}{\text{Bulk strain}}$$

Bulk modulus  $\equiv B$  ‡

$$B \equiv \frac{\text{Bulk stress}}{\text{Bulk strain}} = - \frac{\Delta P}{\Delta V/V_0}$$

Bulk modulus  $\equiv B$  ‡

$$B \equiv \frac{\text{Bulk stress}}{\text{Bulk strain}} = - \frac{\Delta P}{\Delta V/V_0}$$

Compressibility  $\equiv \kappa$

Bulk modulus  $\equiv B$   $\&$

$$B \equiv \frac{\text{Bulk stress}}{\text{Bulk strain}} = - \frac{\Delta P}{\Delta V/V_0}$$

Compressibility  $\equiv K$   $\&$

$$K \equiv \frac{1}{B}$$

Liquid	Compressibility, $k$	
	$\text{Pa}^{-1}$	$\text{atm}^{-1}$
Carbon disulfide	$93 \times 10^{-11}$	$94 \times 10^{-6}$
Ethyl alcohol	$110 \times 10^{-11}$	$111 \times 10^{-6}$
Glycerin	$21 \times 10^{-11}$	$21 \times 10^{-6}$
Mercury	$3.7 \times 10^{-11}$	$3.8 \times 10^{-6}$
Water	$45.8 \times 10^{-11}$	$46.4 \times 10^{-6}$



**Table 11.1 Approximate Elastic Moduli**

<b>Material</b>	<b>Young's Modulus, <math>Y</math> (Pa)</b>	<b>Bulk Modulus, <math>B</math> (Pa)</b>	<b>Shear Modulus, <math>S</math> (Pa)</b>
Aluminum	$7.0 \times 10^{10}$	$7.5 \times 10^{10}$	$2.5 \times 10^{10}$
Brass	$9.0 \times 10^{10}$	$6.0 \times 10^{10}$	$3.5 \times 10^{10}$
Copper	$11 \times 10^{10}$	$14 \times 10^{10}$	$4.4 \times 10^{10}$
Iron	$21 \times 10^{10}$	$16 \times 10^{10}$	$7.7 \times 10^{10}$
Lead	$1.6 \times 10^{10}$	$4.1 \times 10^{10}$	$0.6 \times 10^{10}$
Nickel	$21 \times 10^{10}$	$17 \times 10^{10}$	$7.8 \times 10^{10}$
Silicone rubber	$0.001 \times 10^{10}$	$0.2 \times 10^{10}$	$0.0002 \times 10^{10}$
Steel	$20 \times 10^{10}$	$16 \times 10^{10}$	$7.5 \times 10^{10}$
Tendon (typical)	$0.12 \times 10^{10}$	—	—

A hydraulic press contains  $0.25 \text{ m}^3$  (250 L) of oil. Find the decrease in the volume of the oil when it is subjected to a pressure increase  $\Delta p = 1.6 \times 10^7 \text{ Pa}$  (about 160 atm or 2300 psi). The bulk modulus of the oil is  $B = 5.0 \times 10^9 \text{ Pa}$  (about  $5.0 \times 10^4 \text{ atm}$ ), and its compressibility is  $k = 1/B = 20 \times 10^{-6} \text{ atm}^{-1}$ .

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$$B = 5 \times 10^9 \text{ Pa}$$

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$B = 5 \times 10^9 \text{ Pa}$       Find  $-\Delta V$  :

A hydraulic press contains  $0.25 \text{ m}^3$  (250 L) of oil. Find the decrease in the volume of the oil when it is subjected to a pressure increase  $\Delta p = 1.6 \times 10^7 \text{ Pa}$  (about 160 atm or 2300 psi).

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$B = 5 \times 10^9 \text{ Pa}$  Find  $-\Delta V$ :

$$B = -\frac{\Delta P}{(\Delta V/V_0)}$$

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$B = 5 \times 10^9 \text{ Pa}$  Find  $-\Delta V$ :

$$B = -\frac{\Delta P}{(\Delta V/V_0)} \Rightarrow -\Delta V = \frac{\Delta P}{B} V_0$$

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$B = 5 \times 10^9 \text{ Pa}$  Find  $-\Delta V$ :

$$B = -\frac{\Delta P}{(\Delta V/V_0)} \Rightarrow -\Delta V = \frac{\Delta P}{B} V_0$$

$$\Rightarrow -\Delta V = \frac{(1.6 \times 10^7)(0.25 \times 10^{-1})}{5 \times 10^9} \text{ m}^3$$

A hydraulic press contains  $0.25 \text{ m}^3$  (250 L) of oil. Find the decrease in the volume of the oil when it is subjected to a pressure increase  $\Delta p = 1.6 \times 10^7 \text{ Pa}$  (about 160 atm or 2300 psi).

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$B = 5 \times 10^9 \text{ Pa}$  Find  $-\Delta V$ :

$$B = -\frac{\Delta P}{(\Delta V/V_0)} \Rightarrow -\Delta V = \frac{\Delta P}{B} V_0$$

$$\Rightarrow -\Delta V = \frac{(1.6 \times 10^7)(2.5 \times 10^{-1})}{5 \times 10^9} \text{ m}^3 = \left(\frac{1.6 \times 2.5}{5}\right) \times 10^{-3} \text{ m}^3$$

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$B = 5 \times 10^9 \text{ Pa}$  Find  $-\Delta V$ :

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$$\Rightarrow -\Delta V = 0.8 \times 10^{-3} \text{ m}^3$$

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$$B = -\frac{\Delta P}{(\Delta V/V_0)} \Rightarrow -\Delta V = \frac{\Delta P}{B} V_0$$

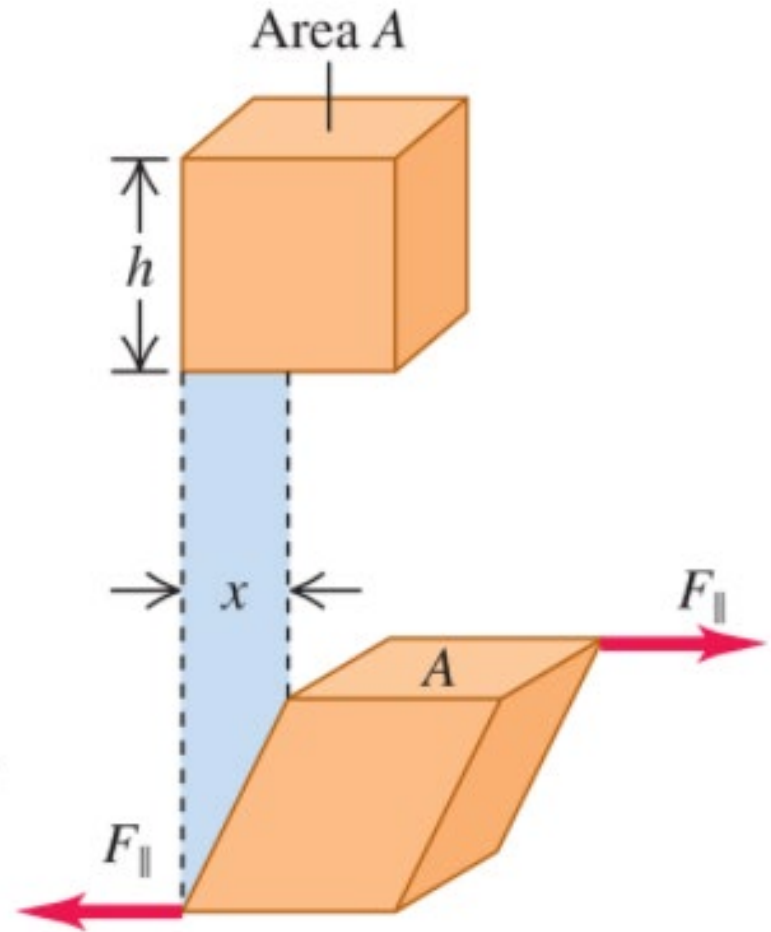
$$\Rightarrow -\Delta V = \frac{(1.6 \times 10^7)(2.5 \times 10^{-1})}{5 \times 10^9} \text{ m}^3 = \left(\frac{1.6 \times 2.5}{5}\right) \times 10^{-3} \text{ m}^3$$

$$\Rightarrow -\Delta V = 0.8 \times 10^{-3} \text{ m}^3 = 8 \times 10^{-4} \text{ m}^3$$

# Shear stress & strain

Initial state  
of the object

Object under  
shear stress

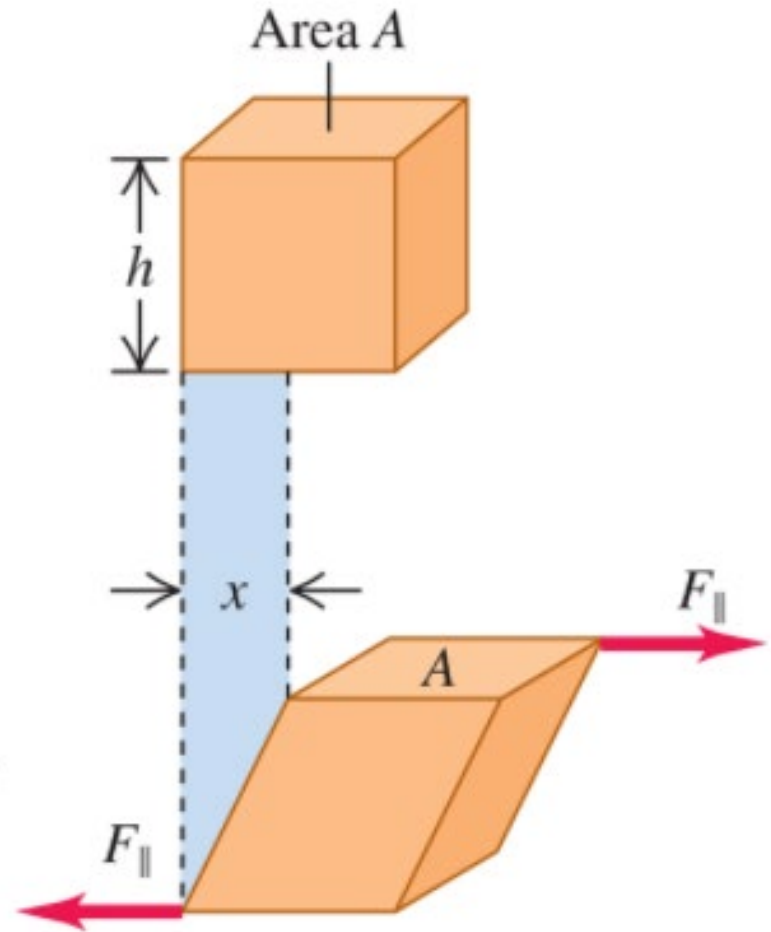


# Shear stress & strain

$$\text{Shear stress} = \frac{F_{\parallel}}{A}$$

Initial state  
of the object

Object under  
shear stress

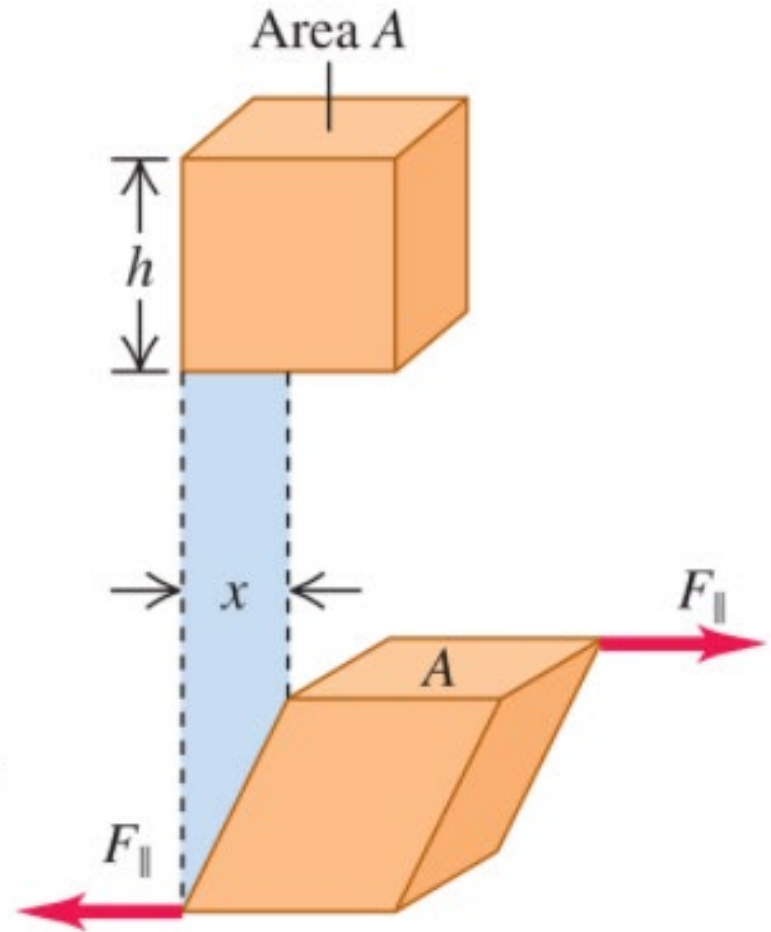


# Shear stress & strain

$$\text{Shear stress} = \frac{F_{\parallel}}{A} \quad \&$$

$$\text{Shear strain} = \frac{x}{h}$$

Initial state  
of the object



Shear modulus  $\equiv S$

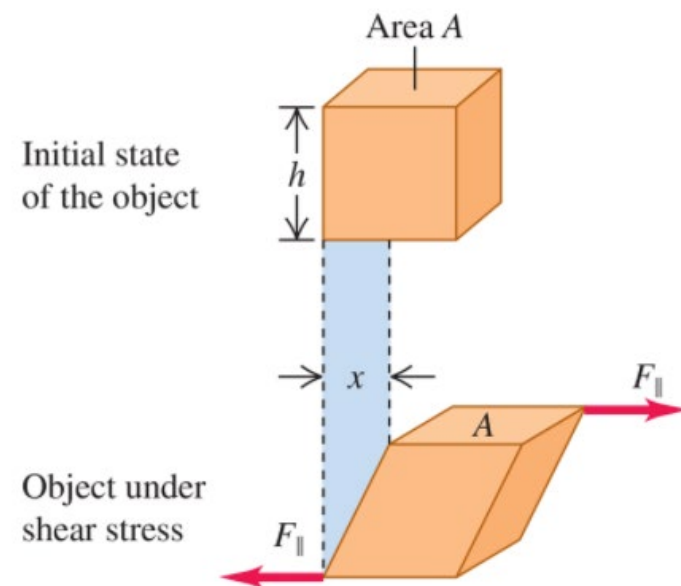
Shear modulus  $\equiv S$   $\$$

$$S \equiv \frac{\text{Shear stress}}{\text{Shear strain}}$$

Shear modulus  $\equiv S$   $\$$

$$S \equiv \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{(F_{||}/A)}{(x/h)}$$

Suppose the object in Fig. 11.18 is the brass base plate of an outdoor sculpture that experiences shear forces in an earthquake. The plate is 0.80 m square and 0.50 cm thick. What is the force exerted on each of its edges if the resulting displacement  $x$  is 0.16 mm?

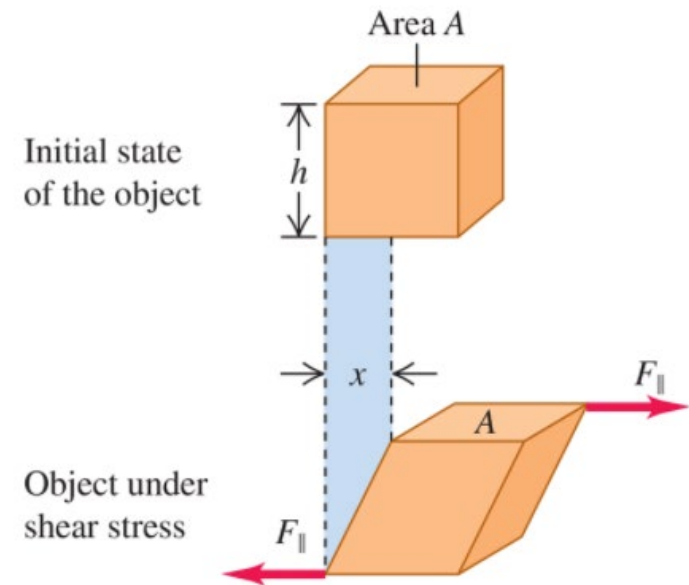


$$\text{Shear stress} = \frac{F_{\parallel}}{A} \quad \text{Shear strain} = \frac{x}{h}$$

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What is the force exerted on each of its edges if the resulting displacement  $x$  is 0.16 mm?

$$A = (0.8\text{ m} \times 0.005\text{ m}) = 0.004\text{ m}^2$$

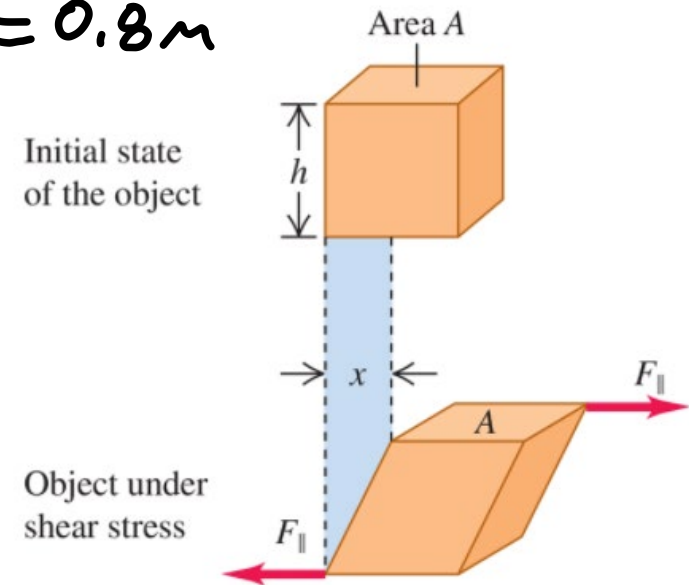


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$$A = (0.8\text{ m} \times 0.005\text{ m}) = 0.004\text{ m}^2, \quad h = 0.8\text{ m}$$



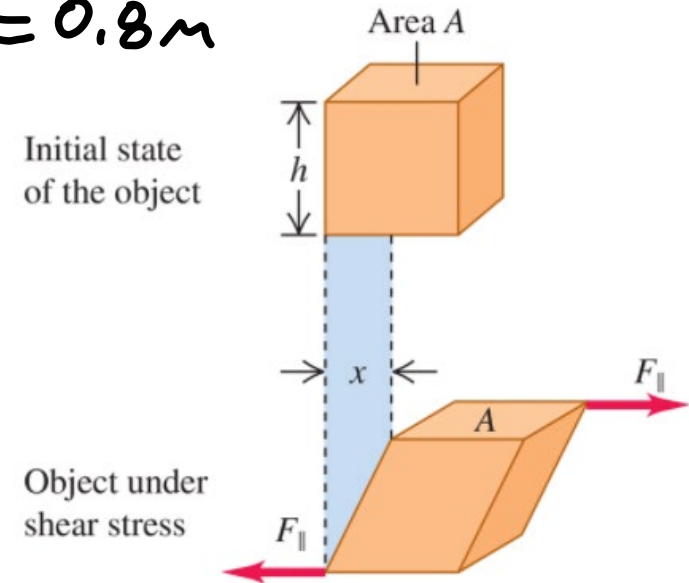
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$$A = (0.8\text{ m} \times 0.005\text{ m}) = 0.004\text{ m}^2, \quad h = 0.8\text{ m}$$

$$x = 0.16\text{ mm} = 0.00016\text{ m}$$



$$\text{Shear stress} = \frac{F_{\parallel}}{A} \quad \text{Shear strain} = \frac{x}{h}$$

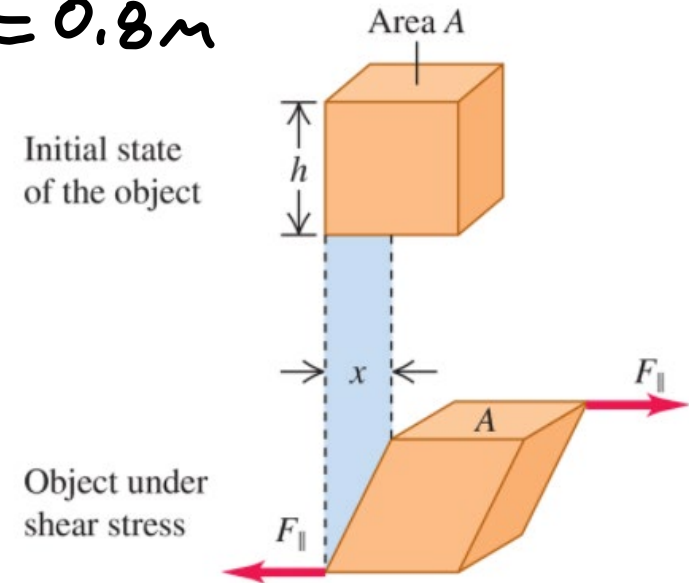
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$$x = 0.16\text{ mm} = 0.00016\text{ m}$$

$$S = \frac{(F_{\parallel}/A)}{(x/h)}$$



$$\text{Shear stress} = \frac{F_{\parallel}}{A} \quad \text{Shear strain} = \frac{x}{h}$$

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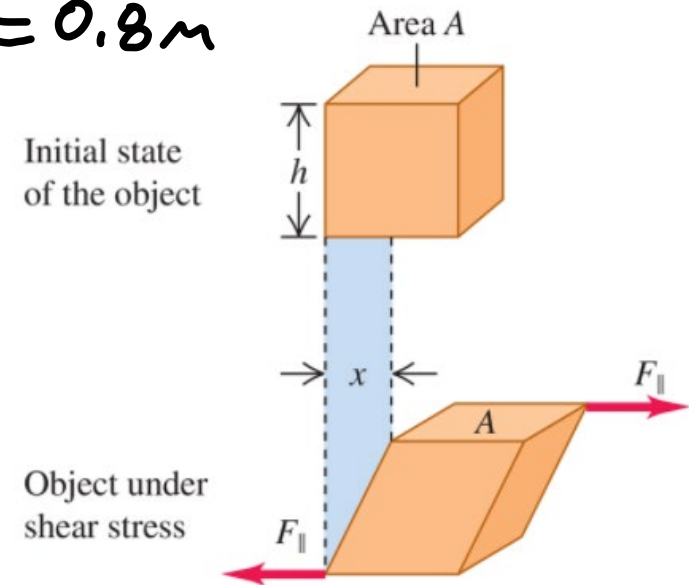
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$$x = 0.16\text{ mm} = 0.00016\text{ m}$$

$$S = \frac{(F_{\parallel}/A)}{(x/h)} \Rightarrow$$

$$F_{\parallel} = S \left(\frac{x}{h}\right) A$$



$$\text{Shear stress} = \frac{F_{\parallel}}{A} \quad \text{Shear strain} = \frac{x}{h}$$

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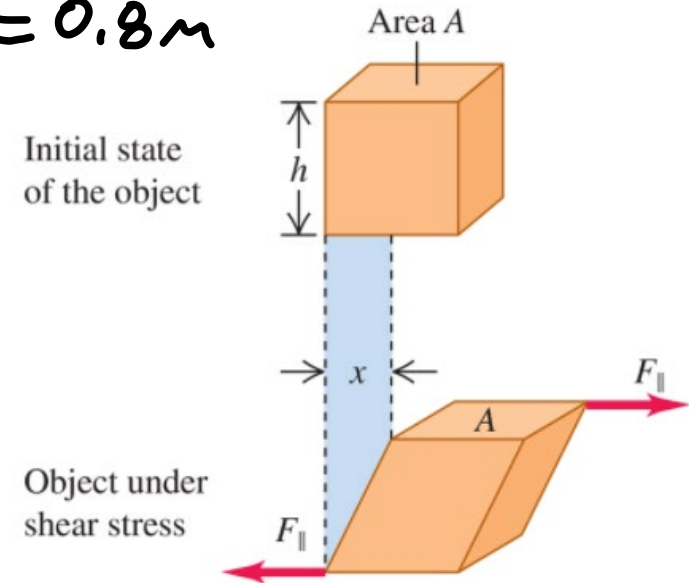
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$$S = \frac{(F_{\parallel}/A)}{(x/h)} \Rightarrow$$

$$F_{\parallel} = S \left(\frac{x}{h}\right) A, \text{ where}$$

$$S = 3.5 \times 10^{10} \text{ Pa}$$



$$\text{Shear stress} = \frac{F_{\parallel}}{A} \quad \text{Shear strain} = \frac{x}{h}$$

Material	Young's Modulus, $Y$ (Pa)	Bulk Modulus, $B$ (Pa)	Shear Modulus, $S$ (Pa)
Aluminum	$7.0 \times 10^{10}$	$7.5 \times 10^{10}$	$2.5 \times 10^{10}$
Brass	$9.0 \times 10^{10}$	$6.0 \times 10^{10}$	$3.5 \times 10^{10}$

Suppose the object in Fig. 11.18 is the brass base plate of an outdoor sculpture that experiences shear forces in an earthquake. The plate is 0.80 m square and 0.50 cm thick.

What is the force exerted on each of its edges if the resulting displacement  $x$  is 0.16 mm?

$$A = (0.8\text{m} \times 0.005\text{m}) = 0.004\text{m}^2, \quad h = 0.8\text{m}$$

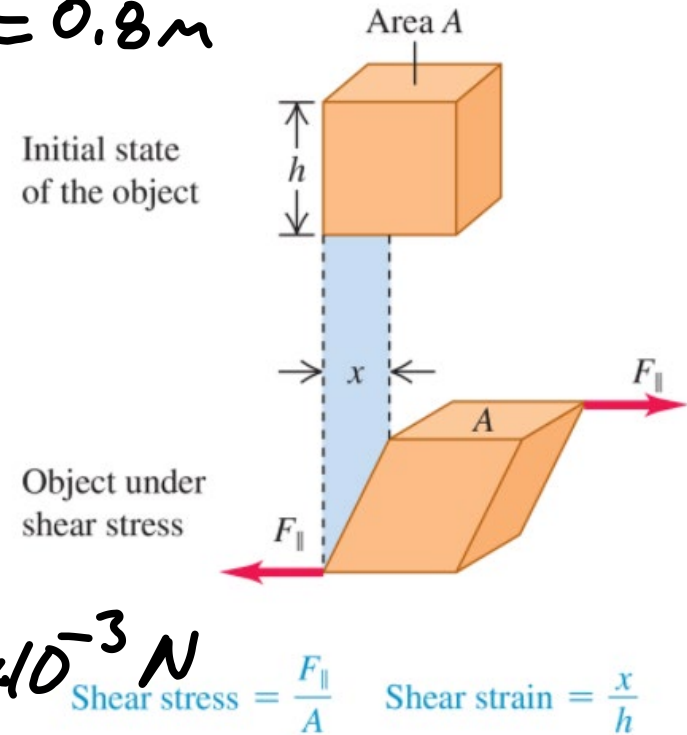
$$x = 0.16\text{mm} = 0.00016\text{m}$$

$$S = \frac{(F_{\parallel}/A)}{(x/h)} \Rightarrow$$

$$F_{\parallel} = S \left(\frac{x}{h}\right) A, \text{ where}$$

$$S = 3.5 \times 10^{10} \text{ Pa} \quad \text{No}$$

$$F_{\parallel} = (3.5 \times 10^{10}) \left(\frac{1.6 \times 10^{-4}}{0.8}\right) \times 4 \times 10^{-3} \text{ N}$$



Suppose the object in Fig. 11.18 is the brass base plate of an outdoor sculpture that experiences shear forces in an earthquake. The plate is 0.80 m square and 0.50 cm thick.

What is the force exerted on each of its edges if the resulting displacement  $x$  is 0.16 mm?

$$A = (0.8\text{m} \times 0.005\text{m}) = 0.004\text{m}^2, \quad h = 0.8\text{m}$$

$$x = 0.16\text{mm} = 0.00016\text{m}$$

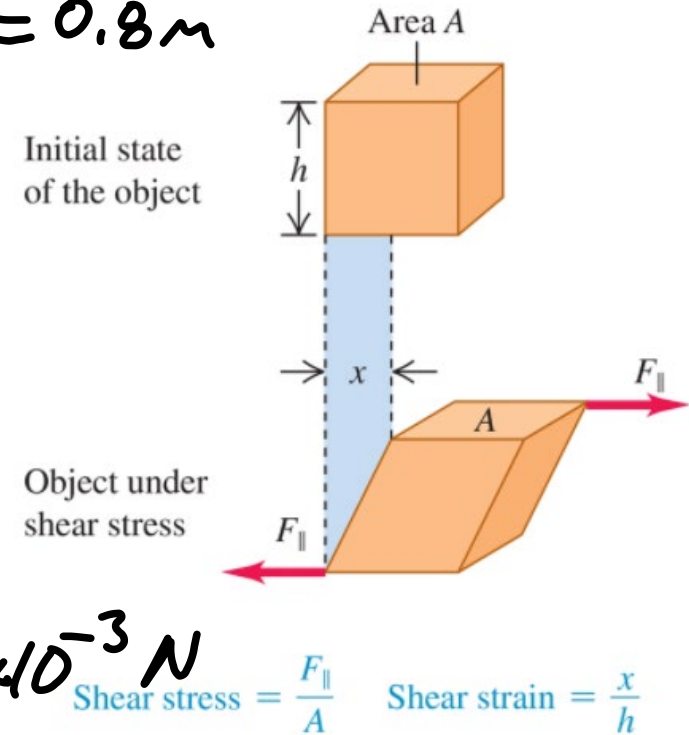
$$S = \frac{(F_{\parallel}/A)}{(x/h)} \Rightarrow$$

$$F_{\parallel} = S \left(\frac{x}{h}\right) A, \text{ where}$$

$$S = 3.5 \times 10^{10} \text{ Pa} \quad \text{No}$$

$$F_{\parallel} = (3.5 \times 10^{10}) \left(\frac{1.6 \times 10^{-4}}{0.8}\right) \times 4 \times 10^{-3} \text{ N}$$

$$\Rightarrow F_{\parallel} = 28 \times 10^3 \text{ N}$$



Suppose the object in Fig. 11.18 is the brass base plate of an outdoor sculpture that experiences shear forces in an earthquake. The plate is 0.80 m square and 0.50 cm thick.

What is the force exerted on each of its edges if the resulting displacement  $x$  is 0.16 mm?

$$A = (0.8\text{ m} \times 0.005\text{ m}) = 0.004\text{ m}^2, \quad h = 0.8\text{ m}$$

$$x = 0.16\text{ mm} = 0.00016\text{ m}$$

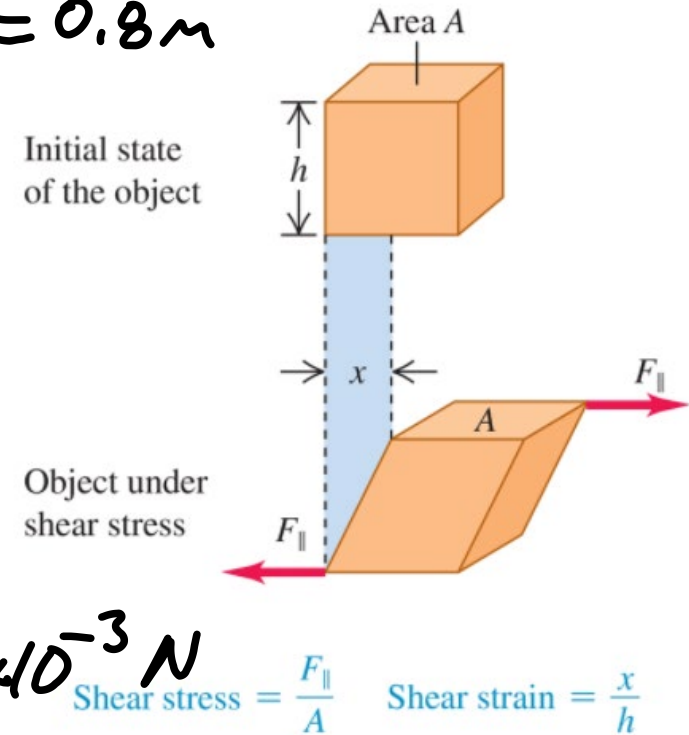
$$S = \frac{(F_{\parallel}/A)}{(x/h)} \Rightarrow$$

$$F_{\parallel} = S \left(\frac{x}{h}\right) A, \text{ where}$$

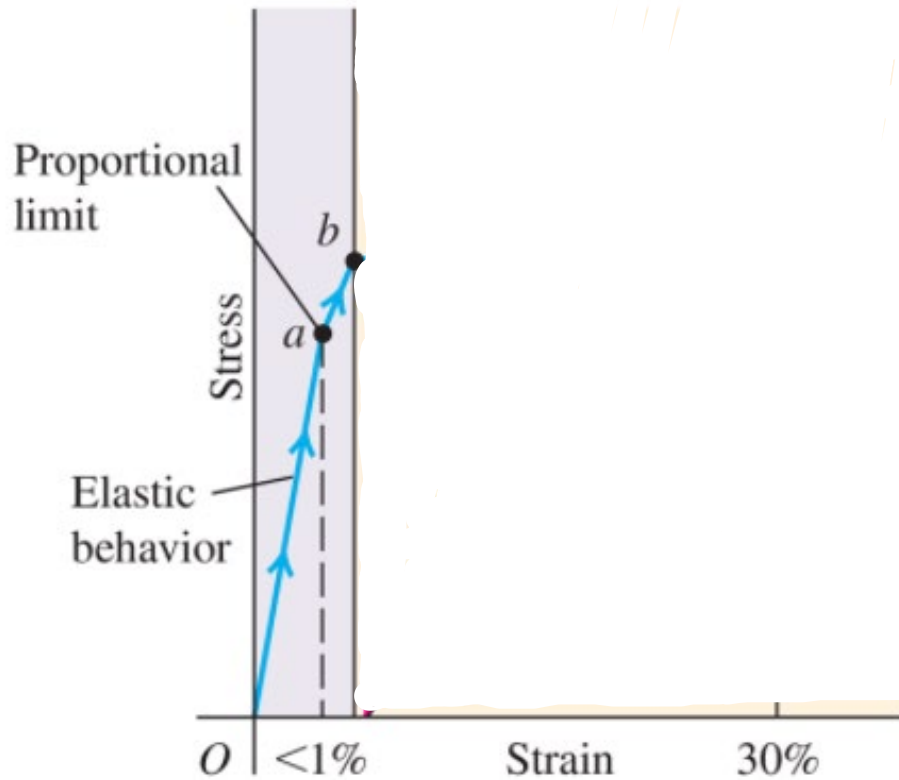
$$S = 3.5 \times 10^{10} \text{ Pa} \quad \text{No}$$

$$F_{\parallel} = (3.5 \times 10^{10}) \left(\frac{1.6 \times 10^{-4}}{0.8}\right) \times 4 \times 10^{-3} \text{ N}$$

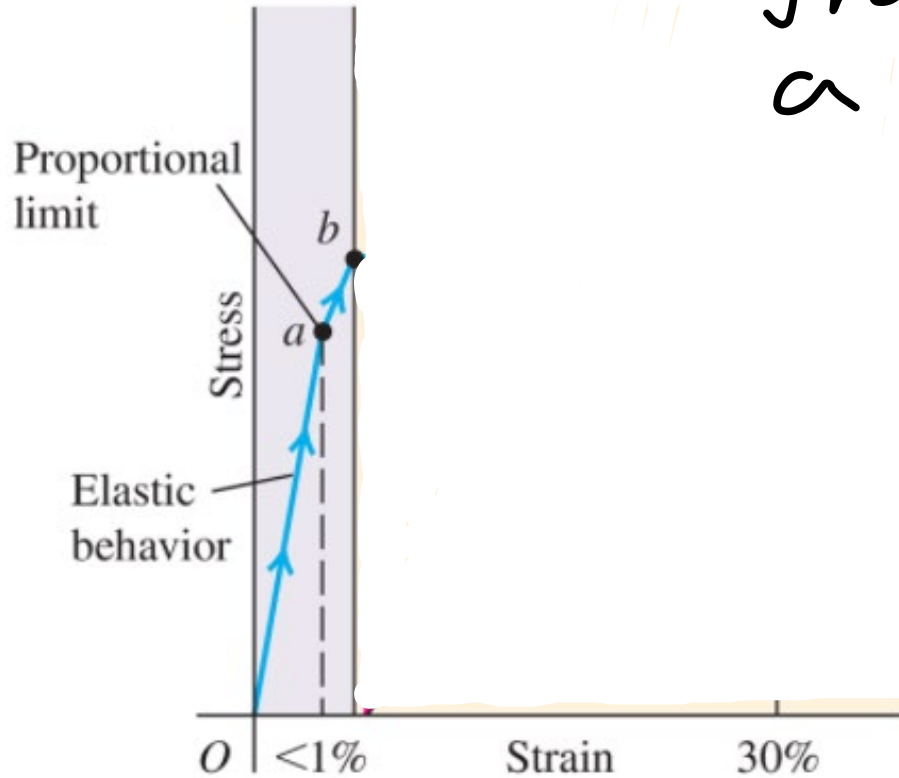
$$\Rightarrow F_{\parallel} = 28 \times 10^3 \text{ N} = 2.8 \times 10^4 \text{ N}$$



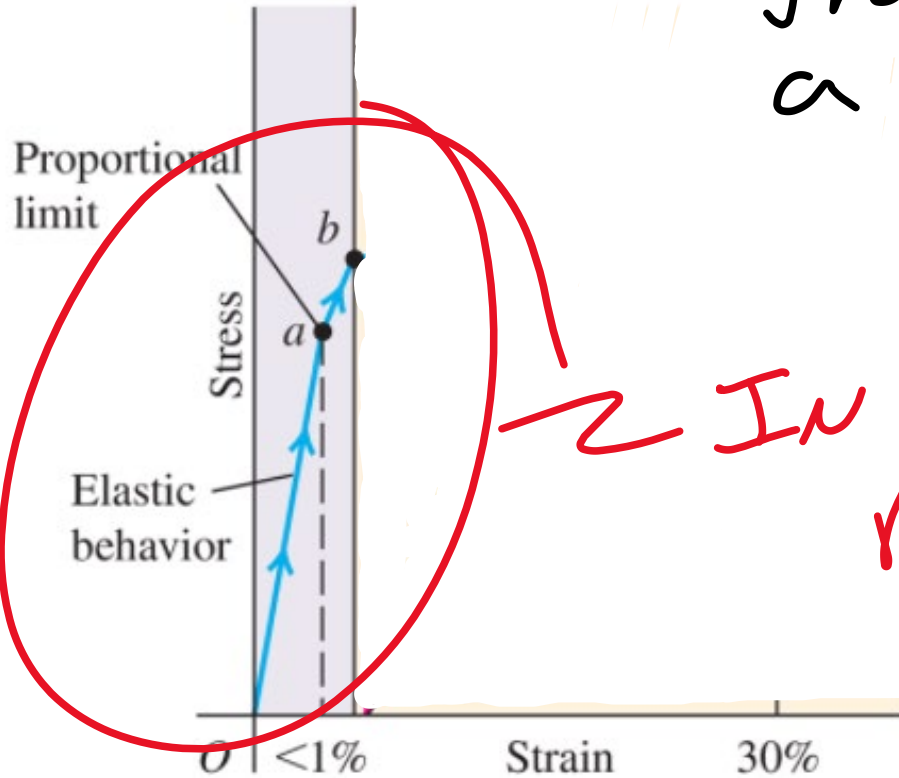
Blue line  $\equiv$  elastic



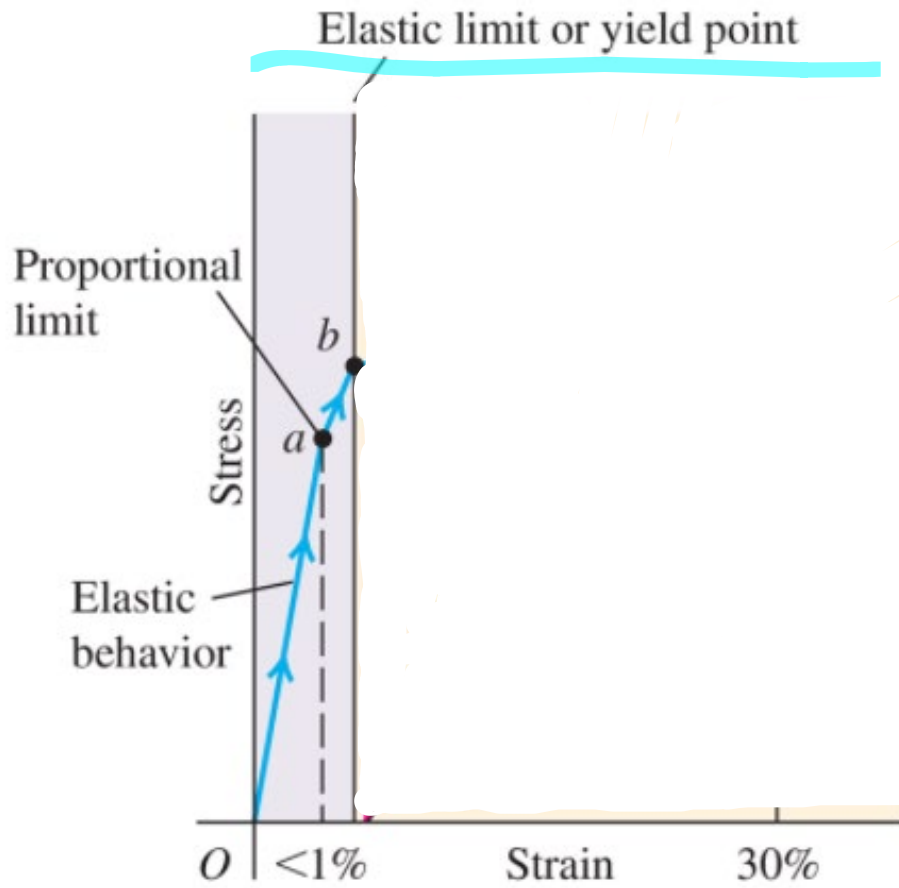
Blue line  $\equiv$  elastic  
From  $\theta$  to point  
a : proportional

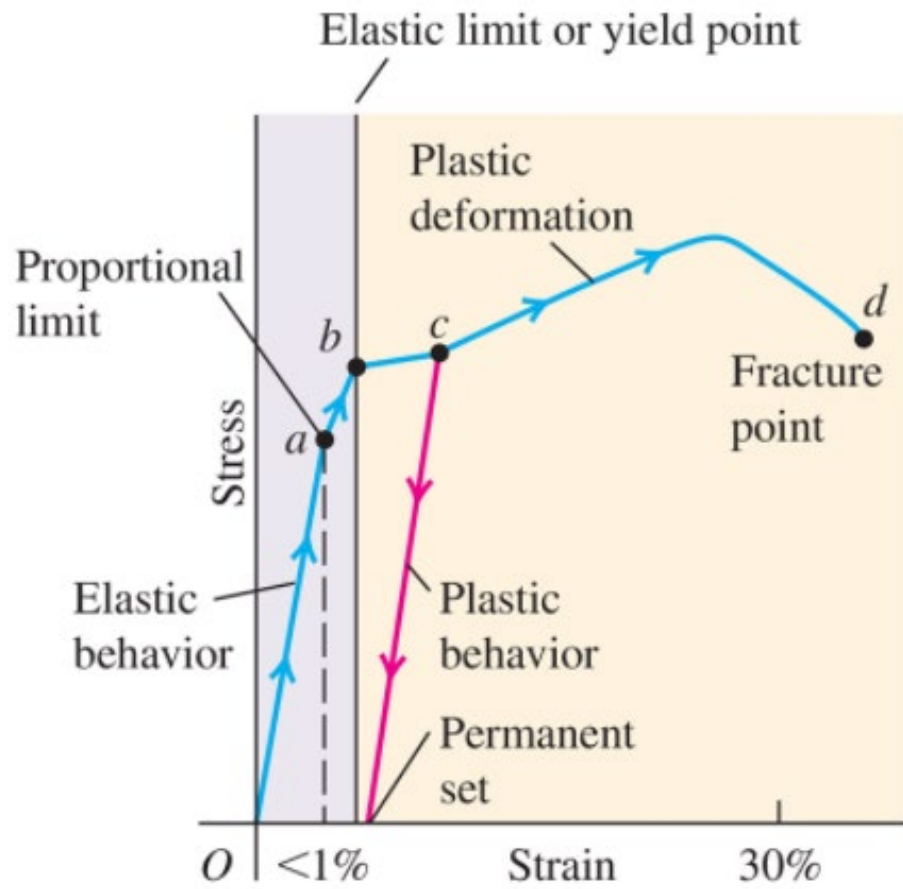


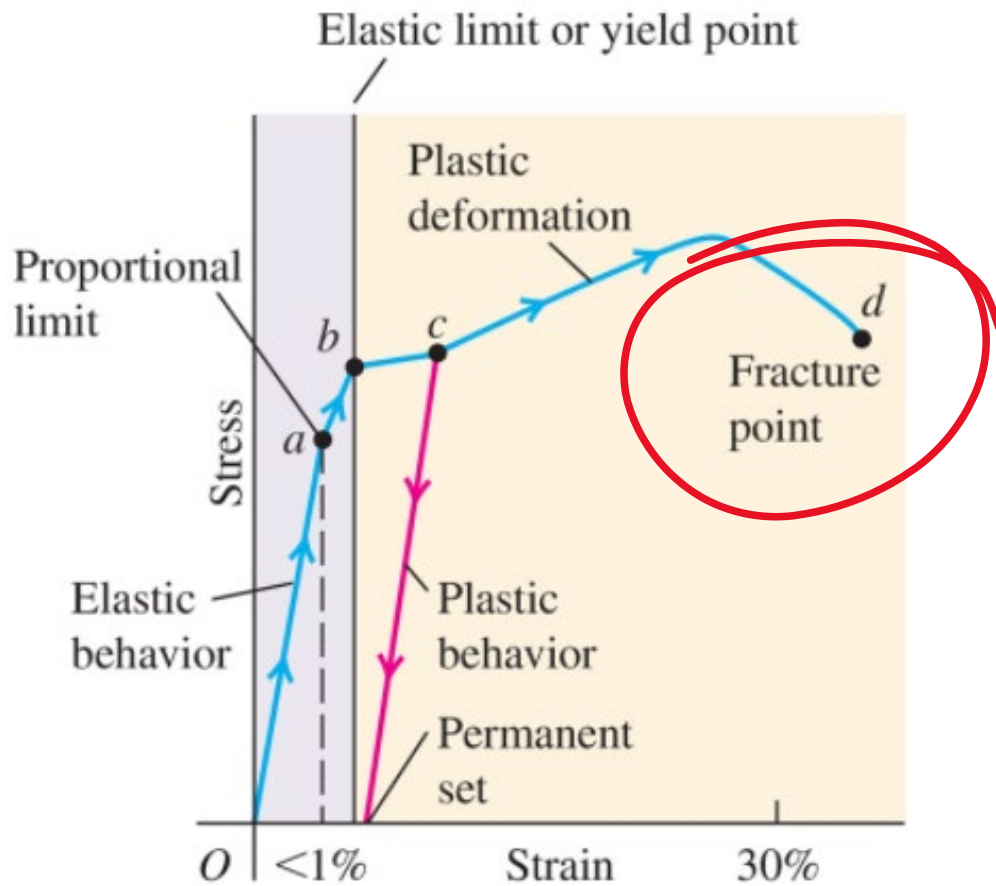
Blue line  $\equiv$  elastic  
From  $\theta$  to point  
a : proportional



In elastic region, item can go back to original shape







Material	Breaking Stress (Pa or N/m <sup>2</sup> )
Aluminum	$2.2 \times 10^8$
Brass	$4.7 \times 10^8$
Glass	$10 \times 10^8$
Iron	$3.0 \times 10^8$
Steel	$5-20 \times 10^8$
Tendon (typical)	$1 \times 10^8$

