Today 10.5, 10.6

L32

Today 10.5, 10.6

L32

Angular momentum

Today 10.5, 10.6

L32

Angular momentum

Conservation of angular momentum

Today 10.5, 10.6

Monday 11.1, 11.2

$\boxed{L32}$

Today 10.5, 10.6

Monday 11.1, 11.2

Conditions for equilibrium

L32

Today 10.5, 10.6

Monday 11.1, 11.2

$\underbrace{11.1}_{\text{Conditions for equilibrium}}$, $\underbrace{11.2}_{\text{Center of gravity}}$

L32

Today 10.5, 10.6

Monday 11.1, 11.2

L32

Important dates:

Today 10.5, 10.6 $\boxed{L32}$

Monday 11.1, 11.2

Important dates:

* Friday Nov. 27th No class

Today 10.5, 10.6

Monday 11.1, 11.2

Important dates:

* Friday Nov. 27th no class ☺

Today 10.5, 10.6

Monday 11.1, 11.2

L32

Important dates:

* Friday Nov. 27th No class ☺
* Monday Nov. 30th Exam 4

ASU

Today 10.5, 10.6

Monday 11.1, 11.2

L32

Important dates:

* Friday Nov. 27th No class ☺
* Monday Nov. 30th Exam 4
* Wednesday Dec 2nd Day of Reckoning

Today 10.5, 10.6

L32

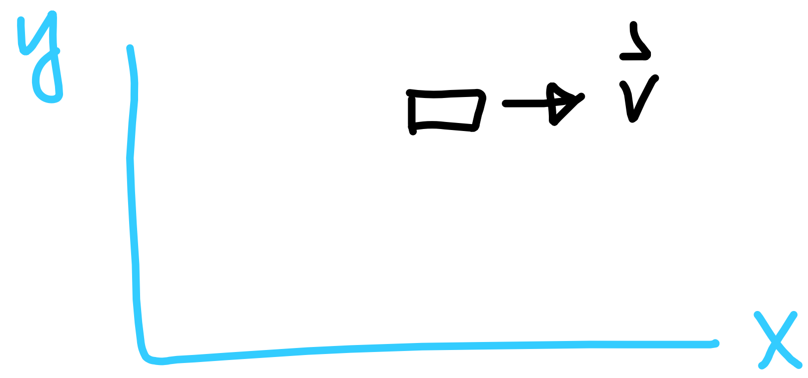
Monday 11.1, 11.2

Important dates:

* Friday Nov. 27th No class ☺
* Monday Nov. 30th Exam 4
* Wednesday Dec 2nd Day of Reckoning
* Friday Dec 4th Final exam

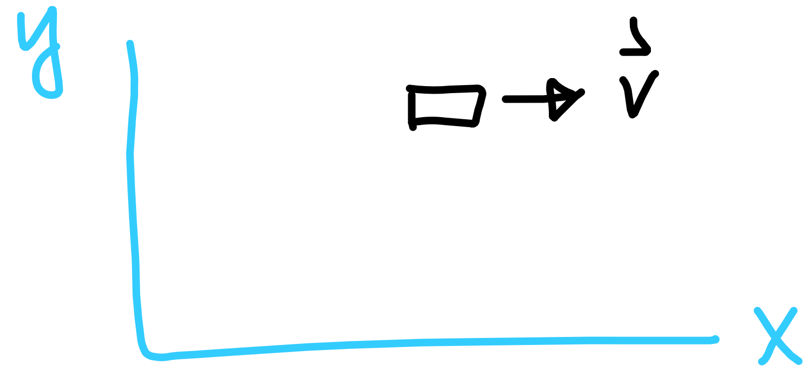
Angular momentum

Angular momentum



Angular momentum

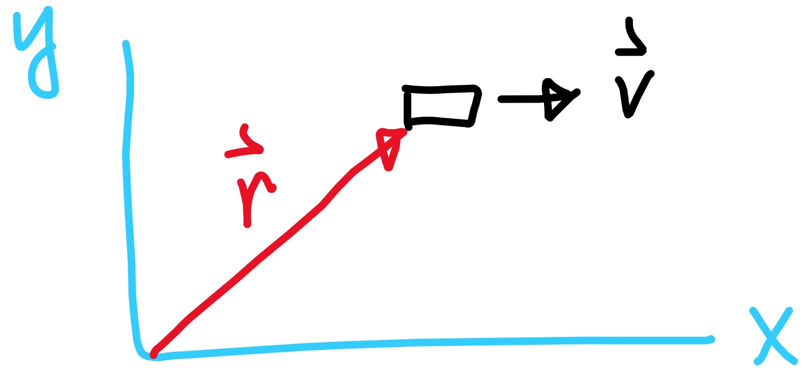
$\vec{p} = m\vec{v}$



Angular momentum

$\vec{p} = m\vec{v}$

so $\vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

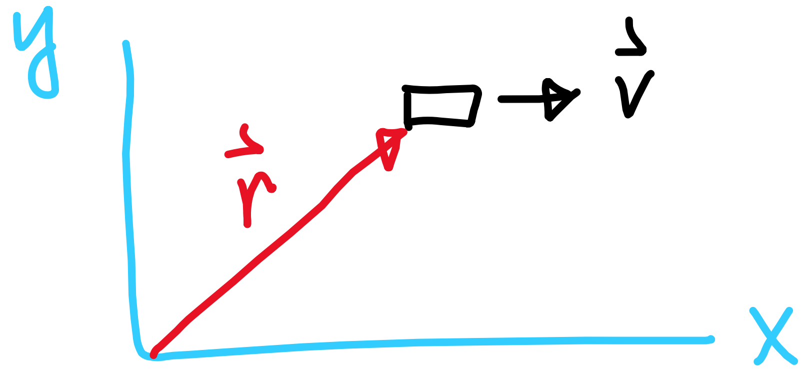


Angular momentum

$\vec{p} = m\vec{v}$

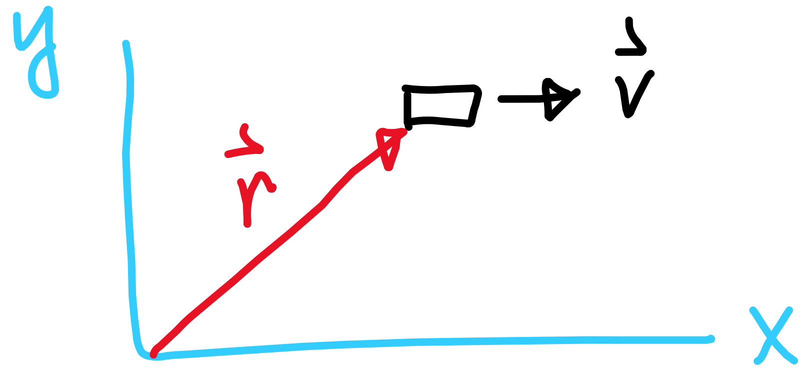
so $\vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

Let $\vec{L} \equiv \vec{r} \times \vec{p}$



Angular momentum

$\vec{p} = m\vec{v}$

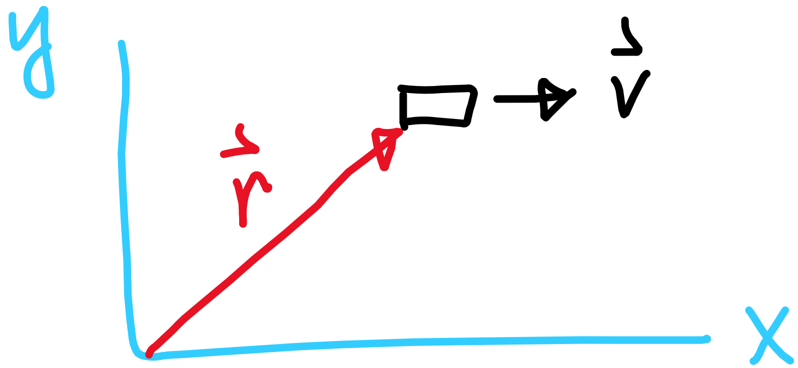


so $\vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

Let $\vec{L} \equiv \vec{r} \times \vec{p}$ & $L \equiv$ angular momentum

Angular momentum

$\vec{p} = m\vec{v}$



so $\vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

Let $\vec{L} \equiv \vec{r} \times \vec{p}$ & $L \equiv$ angular momentum

so $\vec{L} = \vec{r} \times m\vec{v}$

Torque & angular momentum

Torque & angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

Torque & angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \frac{d}{dt}\vec{L} = \frac{d}{dt}[\vec{r} \times \vec{p}]$$

Torque & angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \frac{d}{dt}\vec{L} = \frac{d}{dt}[\vec{r} \times \vec{p}]$$

$$\Rightarrow \frac{d}{dt}\vec{L} = \left(\frac{d\vec{r}}{dt}\right) \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

Torque & angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \frac{d}{dt}\vec{L} = \frac{d}{dt}[\vec{r} \times \vec{p}]$$

$$\Rightarrow \frac{d}{dt}\vec{L} = \left(\frac{d\vec{r}}{dt}\right) \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \left(\frac{d\vec{r}}{dt}\right) \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

Torque & angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \frac{d}{dt}\vec{L} = \frac{d}{dt}[\vec{r} \times \vec{p}]$$

$$\Rightarrow \frac{d}{dt}\vec{L} = \left(\frac{d\vec{r}}{dt}\right) \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \left(\frac{d\vec{r}}{dt}\right) \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$= \vec{v} \times m\vec{v} + \vec{r} \times m\vec{a}$$

ASU

Torque & angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \quad \Rightarrow \quad \frac{d}{dt}\vec{L} = \frac{d}{dt}\left[\vec{r} \times \vec{p}\right]$$

$$\Rightarrow \quad \frac{d}{dt}\vec{L} = \left(\frac{d\vec{r}}{dt}\right) \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \left(\frac{d\vec{r}}{dt}\right) \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$= \overset{0}{\cancel{\vec{v} \times m\vec{v}}} + \vec{r} \times m\vec{a}$$

Torque & angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \frac{d}{dt}\vec{L} = \frac{d}{dt}\left[\vec{r} \times \vec{p}\right]$$

$$\Rightarrow \frac{d}{dt}\vec{L} = \left(\frac{d\vec{r}}{dt}\right) \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \left(\frac{d\vec{r}}{dt}\right) \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$= \cancelto{0}{\vec{v} \times m\vec{v}} + \vec{r} \times m\vec{a}$$

Torque & angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \quad \Rightarrow \quad \frac{d}{dt}\vec{L} = \frac{d}{dt}\left[\vec{r} \times \vec{p}\right]$$

$$\Rightarrow \quad \frac{d}{dt}\vec{L} = \left(\frac{d\vec{r}}{dt}\right) \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \left(\frac{d\vec{r}}{dt}\right) \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$= \overset{0}{\cancel{\vec{v} \times m\vec{v}}} + \vec{r} \times m\vec{a} \qquad \text{But } \vec{F} = m\vec{a}$$

Torque & angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \frac{d}{dt}\vec{L} = \frac{d}{dt}[\vec{r} \times \vec{p}]$$

$$\Rightarrow \frac{d}{dt}\vec{L} = \left(\frac{d\vec{r}}{dt}\right) \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \left(\frac{d\vec{r}}{dt}\right) \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$= \cancel{\vec{v} \times m\vec{v}}^{\,0} + \vec{r} \times m\vec{a} \quad \text{But } \vec{F} = m\vec{a}$$

So $\quad \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}$

Torque & angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \frac{d}{dt}\vec{L} = \frac{d}{dt}[\vec{r} \times \vec{p}]$$

$$\Rightarrow \frac{d}{dt}\vec{L} = \left(\frac{d\vec{r}}{dt}\right) \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \left(\frac{d\vec{r}}{dt}\right) \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$= \cancel{\vec{v} \times m\vec{v}} \; 0 + \vec{r} \times m\vec{a} \quad \text{But } \vec{F} = m\vec{a}$$

So $\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}$ & since $\vec{\tau} = \vec{r} \times \vec{F}$

Torque & angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \frac{d}{dt}\vec{L} = \frac{d}{dt}[\vec{r} \times \vec{p}]$$

$$\Rightarrow \frac{d}{dt}\vec{L} = \left(\frac{d\vec{r}}{dt}\right) \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \left(\frac{d\vec{r}}{dt}\right) \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$= \cancel{\vec{v} \times m\vec{v}}^{\,0} + \vec{r} \times m\vec{a}$$ But $\vec{F} = m\vec{a}$

so $\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}$ & since $\vec{\tau} = \vec{r} \times \vec{F}$

then $\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$

Torque & angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \frac{d}{dt}\vec{L} = \frac{d}{dt}[\vec{r} \times \vec{p}]$$

$$\Rightarrow \frac{d}{dt}\vec{L} = \left(\frac{d\vec{r}}{dt}\right) \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= \left(\frac{d\vec{r}}{dt}\right) \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

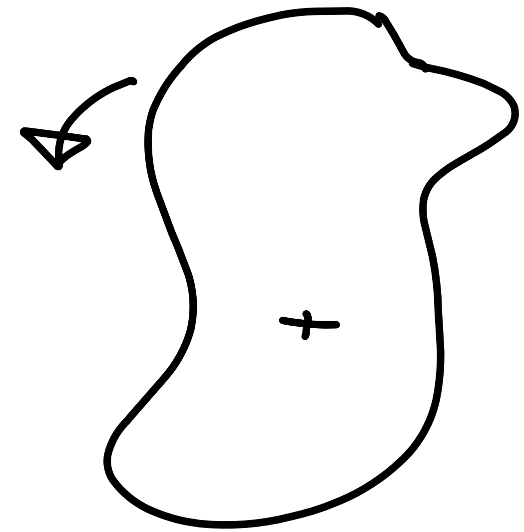
$$= \cancel{\vec{v} \times m\vec{v}}^{\,0} + \vec{r} \times m\vec{a} \qquad \text{But } \vec{F} = m\vec{a}$$

So $\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}$ & since $\vec{\tau} = \vec{r} \times \vec{F}$

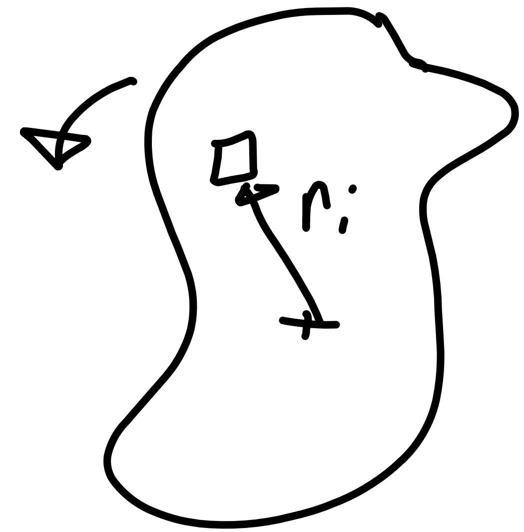
then $\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$ compare to

$$\vec{F} = \frac{d\vec{p}}{dt}$$

For a rigid body

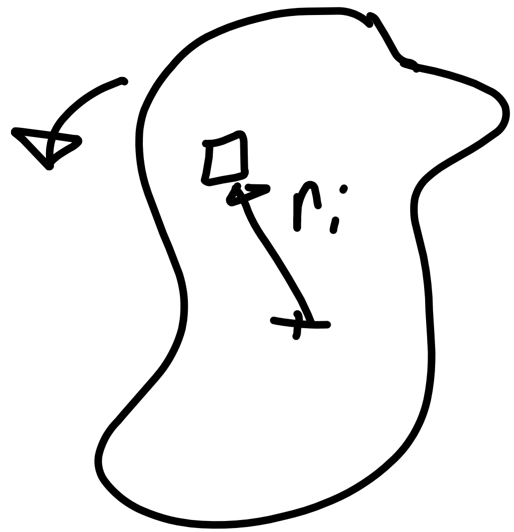


For a rigid body



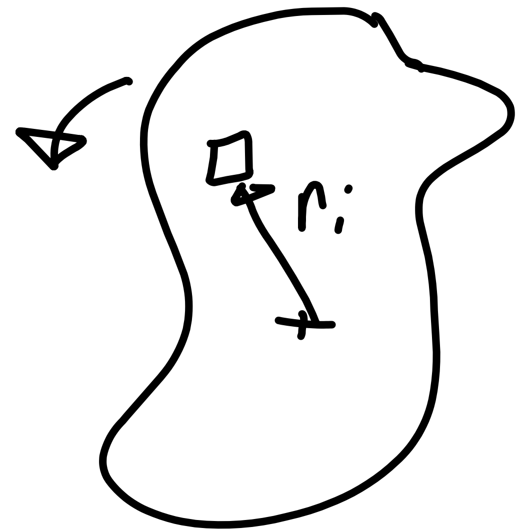
For a rigid body

$L = \Sigma L_i$



For a rigid body

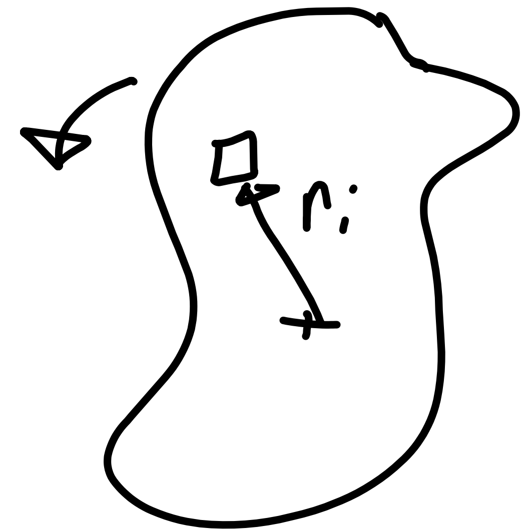
$$L = \sum L_i = \sum m_i r_i (r_i \omega)$$



For a rigid body

$$L = \sum L_i = \sum m_i r_i (r_i \omega)$$

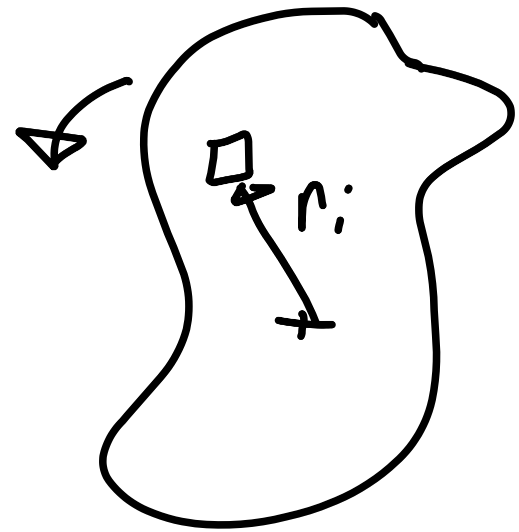
$$= \sum m_i r_i^2 \omega$$



For a rigid body

$$L = \Sigma L_i = \Sigma m_i r_i (r_i \omega)$$

$$= (\Sigma m_i r_i^2)\omega$$

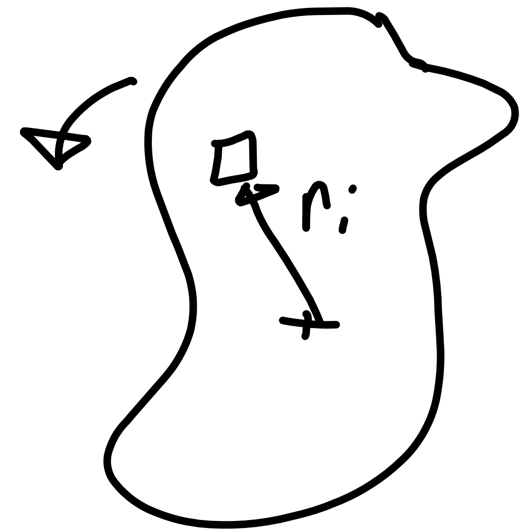


For a rigid body

$$L = \sum L_i = \sum m_i r_i (r_i \omega)$$

$$= \left(\sum m_i r_i^2\right) \omega$$

$$= I \omega$$



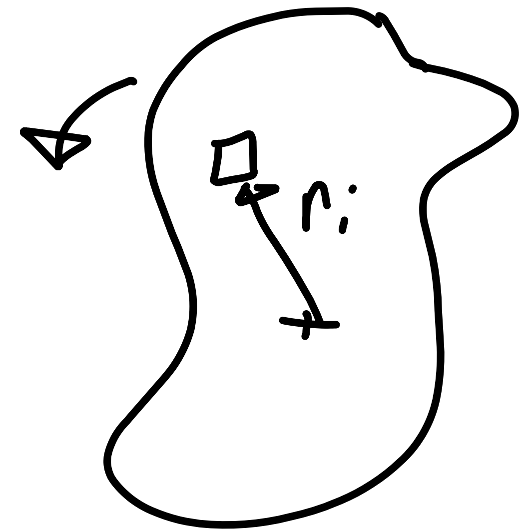
For a rigid body

$$L = \Sigma L_i = \Sigma m_i r_i (r_i \omega)$$

$$= \left(\Sigma m_i r_i^2\right) \omega$$

$$= I \omega$$

so $\quad \vec{L} = I \vec{\omega}$



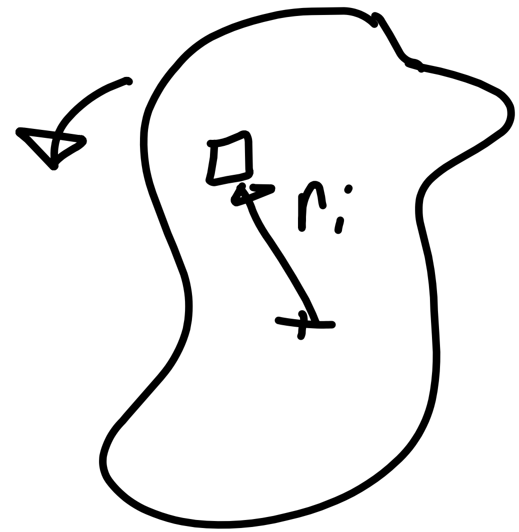
For a rigid body

$$L = \sum L_i = \sum m_i r_i (r_i \omega)$$

$$= (\sum m_i r_i^2) \omega$$

$$= I \omega$$

so $\vec{L} = I \vec{\omega}$



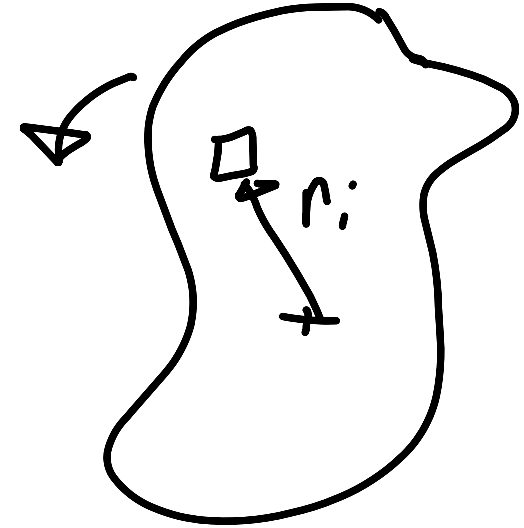
$$\Rightarrow \quad \frac{d\vec{L}}{dt} = I \frac{d\vec{\omega}}{dt}$$

For a rigid body

$$L = \sum L_i = \sum m_i r_i (r_i \omega)$$

$$= \left(\sum m_i r_i^2\right) \omega$$

$$= I \omega$$



so $\vec{L} = I \vec{\omega}$

$$\Rightarrow \quad \frac{d\vec{L}}{dt} = I \frac{d\vec{\omega}}{dt}$$

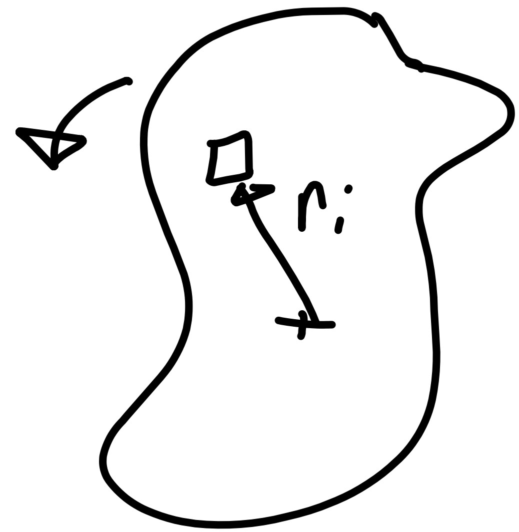
Assumed $\frac{dI}{dt} = 0$

For a rigid body

$$L = \sum L_i = \sum m_i r_i (r_i \omega)$$

$$= \left(\sum m_i r_i^2\right) \omega$$

$$= I \omega$$



so $\quad \vec{L} = I \vec{\omega}$

$$\Rightarrow \quad \frac{d\vec{L}}{dt} = I \frac{d\vec{\omega}}{dt} \qquad \text{But } \quad \vec{\tau} = \frac{d\vec{L}}{dt}$$

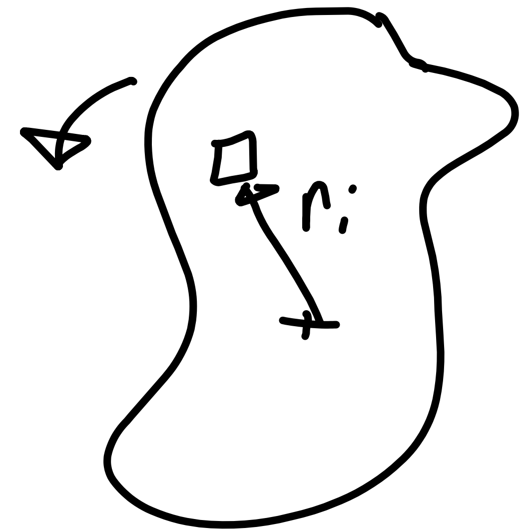
For a rigid body

$$L = \sum L_i = \sum m_i r_i (r_i \omega)$$

$$= \left(\sum m_i r_i^2\right)\omega$$

$$= I\omega$$

so $\vec{L} = I\vec{\omega}$



$$\Rightarrow \quad \frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt}$$

But $\vec{\tau} = \dfrac{d\vec{L}}{dt}$

& $\vec{\alpha} = \dfrac{d\vec{\omega}}{dt}$

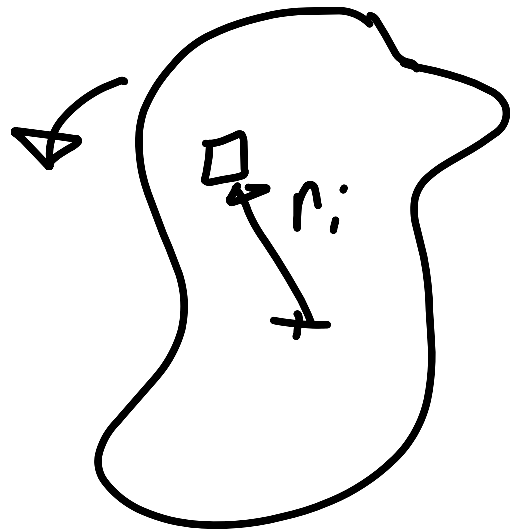
For a rigid body

$$L = \Sigma L_i = \Sigma m_i r_i (r_i \omega)$$

$$= (\Sigma m_i r_i^2)\omega$$

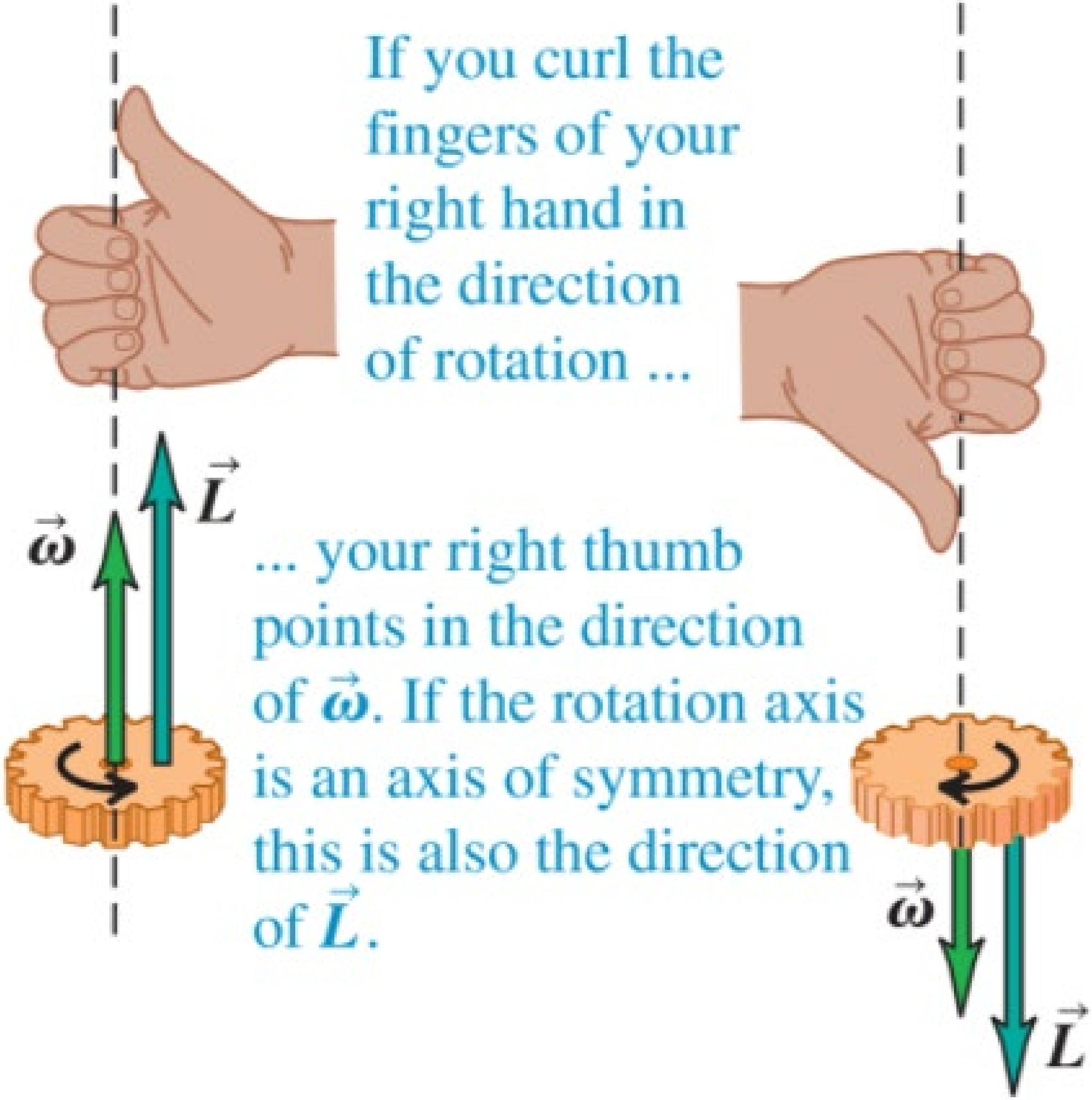
$$= I\omega$$

so $\vec{L} = I\vec{\omega}$



$\Rightarrow$ $\frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt}$ But $\vec{\tau} = \frac{d\vec{L}}{dt}$

& $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$ so $\vec{\tau} = I\vec{\alpha}$


If you curl the fingers of your right hand in the direction of rotation ...
$\vec{L}$
$\vec{\omega}$
... your right thumb points in the direction of $\vec{\omega}$. If the rotation axis is an axis of symmetry, this is also the direction of $\vec{L}$.
$\vec{\omega}$
$\vec{L}$

A turbine fan in a jet engine has a moment of inertia of $2.5\ \mathrm{kg \cdot m^2}$ about its axis of rotation. As the turbine starts up, its angular velocity is given by $\omega_z = (40\ \mathrm{rad/s^3})t^2$. (a) Find the fan's angular momentum as a function of time, and find its value at $t = 3.0\ \mathrm{s}$. (b) Find the net torque on the fan as a function of time, and find its value at $t = 3.0\ \mathrm{s}$.

A turbine fan in a jet engine has a moment of inertia of $2.5\ \mathrm{kg \cdot m^2}$ about its axis of rotation. As the turbine starts up, its angular velocity is given by $\omega_z = (40\ \mathrm{rad/s^3})t^2$. (a) Find the fan's angular momentum as a function of time, and find its value at $t = 3.0$ s. (b) Find the net torque on the fan as a function of time, and find its value at $t = 3.0$ s. $I = 2.5\ \mathrm{kg \cdot m^2}$

A turbine fan in a jet engine has a moment of inertia of $2.5\ \text{kg}\cdot\text{m}^2$ about its axis of rotation. As the turbine starts up, its angular velocity is given by $\omega_z = (40\ \text{rad/s}^3)t^2$. (a) Find the fan's angular momentum as a function of time, and find its value at $t = 3.0$ s. (b) Find the net torque on the fan as a function of time, and find its value at $t = 3.0$ s.

$I = 2.5\ \text{kg}\cdot\text{m}^2$

$\omega = 40 \frac{\text{rad}}{s^3} t^2$

A turbine fan in a jet engine has a moment of inertia of $2.5\ \text{kg}\cdot\text{m}^2$ about its axis of rotation. As the turbine starts up, its angular velocity is given by $\omega_z = (40\ \text{rad/s}^3)t^2$. (a) Find the fan's angular momentum as a function of time, and find its value at $t = 3.0$ s. (b) Find the net torque on the fan as a function of time, and find its value at $t = 3.0$ s.

$I = 2.5\ \text{kg}\cdot\text{m}^2$

$\omega = 40 \frac{\text{rad}}{s^3} t^2$.

<u>Find $L(t)$ & $L(t=3s)$</u>:

A turbine fan in a jet engine has a moment of inertia of 2.5 kg·m^2 about its axis of rotation. As the turbine starts up, its angular velocity is given by $\omega_z = (40\ \mathrm{rad/s^3})t^2$. (a) Find the fan's angular momentum as a function of time, and find its value at $t = 3.0$ s. (b) Find the net torque on the fan as a function of time, and find its value at $t = 3.0$ s.

$I = 2.5\ \mathrm{kg \cdot m^2}$

$\omega = 40 \frac{\mathrm{rad}}{\mathrm{s^3}} t^2$. Find $L(t)$ & $L(t=3\mathrm{s})$:

$L = I\omega$

A turbine fan in a jet engine has a moment of inertia of $2.5\ \mathrm{kg \cdot m^2}$ about its axis of rotation. As the turbine starts up, its angular velocity is given by $\omega_z = (40\ \mathrm{rad/s^3})t^2$. (a) Find the fan's angular momentum as a function of time, and find its value at $t = 3.0$ s. (b) Find the net torque on the fan as a function of time, and find its value at $t = 3.0$ s.

$I = 2.5\ \mathrm{kg \cdot m^2}$

$\omega = 40 \frac{\mathrm{rad}}{\mathrm{s^3}} t^2$. Find $L(t)$ & $L(t = 3s)$:

$L = I\omega \Rightarrow L = (2.5)(40)\left(\frac{\mathrm{kg \cdot m^2}}{\mathrm{s^3}}\right) t^2$

A turbine fan in a jet engine has a moment of inertia of $2.5\ \mathrm{kg \cdot m^2}$ about its axis of rotation. As the turbine starts up, its angular velocity is given by $\omega_z = (40\ \mathrm{rad/s^3})t^2$. (a) Find the fan's angular momentum as a function of time, and find its value at $t = 3.0\ \mathrm{s}$. (b) Find the net torque on the fan as a function of time, and find its value at $t = 3.0\ \mathrm{s}$.

$I = 2.5\ \mathrm{kg \cdot m^2}$

$\omega = 40 \frac{\mathrm{rad}}{\mathrm{s}^3} t^2$. Find $L(t)$ & $L(t = 3\mathrm{s})$:

$L = I\omega \Rightarrow L = (2.5)(40)\left(\frac{\mathrm{kg \cdot m^2}}{\mathrm{s}^3}\right) t^2 \Rightarrow$

$$\boxed{L = 100\left(\frac{\mathrm{kg \cdot m^2}}{\mathrm{s}^3}\right) t^2}$$

ASU

A turbine fan in a jet engine has a moment of inertia of $2.5\ \mathrm{kg\cdot m^2}$ about its axis of rotation. As the turbine starts up, its angular velocity is given by $\omega_z = (40\ \mathrm{rad/s^3})t^2$. (a) Find the fan's angular momentum as a function of time, and find its value at $t = 3.0$ s. (b) Find the net torque on the fan as a function of time, and find its value at $t = 3.0$ s.

$I = 2.5\ \mathrm{kg \cdot m^2}$

$\omega = 40 \frac{\mathrm{rad}}{\mathrm{s^3}} t^2$. Find $L(t)$ & $L(t = 3s)$:

$L = I\omega \Rightarrow L = (2.5)(40)\left(\frac{\mathrm{kg\cdot m^2}}{\mathrm{s^3}}\right) t^2 \Rightarrow$

$\boxed{L = 100\left(\frac{\mathrm{kg\cdot m^2}}{\mathrm{s^3}}\right) t^2}$ & $\boxed{L(t=3) = 900\ \frac{\mathrm{kg\, m^2}}{\mathrm{s}}}$

A turbine fan in a jet engine has a moment of inertia of $2.5\ \mathrm{kg \cdot m^2}$ about its axis of rotation. As the turbine starts up, its angular velocity is given by $\omega_z = (40\ \mathrm{rad/s^3})t^2$. (a) Find the fan's angular momentum as a function of time, and find its value at $t = 3.0$ s. (b) Find the net torque on the fan as a function of time, and find its value at $t = 3.0$ s.

$I = 2.5\ \mathrm{Kg \cdot m^2}$

$\omega = 40 \frac{\mathrm{rad}}{\mathrm{s^3}} t^2$. Find $L(t)$ & $L(t = 3s)$:

$L = I\omega \Rightarrow L = (2.5)(40)\left(\frac{\mathrm{Kg \cdot m^2}}{\mathrm{s^3}}\right) t^2 \Rightarrow$

$\boxed{L = 100\left(\frac{\mathrm{Kg \cdot m^2}}{\mathrm{s^3}}\right) t^2}$ & $\boxed{L(t=3) = 900 \frac{\mathrm{Kg\,m^2}}{\mathrm{s}}}$

A turbine fan in a jet engine has a moment of inertia of $2.5\ \mathrm{kg\cdot m^2}$ about its axis of rotation. As the turbine starts up, its angular velocity is given by $\omega_z = (40\ \mathrm{rad/s^3})t^2$. (a) Find the fan's angular momentum as a function of time, and find its value at $t = 3.0\ \mathrm{s}$. (b) Find the net torque on the fan as a function of time, and find its value at $t = 3.0\ \mathrm{s}$.

$I = 2.5\ \mathrm{kg\cdot m^2}$

$\omega = 40\ \frac{\mathrm{rad}}{\mathrm{s^3}} t^2$. Find $L(t)$ & $L(t=3\mathrm{s})$:

$L = I\omega \Rightarrow L = (2.5)(40)\left(\frac{\mathrm{kg\cdot m^2}}{\mathrm{s^3}}\right)t^2 \Rightarrow$

$\boxed{L = 100\left(\frac{\mathrm{kg\cdot m^2}}{\mathrm{s^3}}\right)t^2}$ & $\boxed{L(t=3) = 900\ \frac{\mathrm{kg\,m^2}}{\mathrm{s}}}$

$\tau = \frac{dL}{dt}$

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ASU

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$\boxed{L = 100\left(\frac{\text{kg}\cdot\text{m}^2}{\text{s}^3}\right) t^2}$ & $\boxed{L(t=3) = 900 \frac{\text{kg}\,\text{m}^2}{\text{s}}}$

$\boxed{\tau = \frac{dL}{dt} = \left(200 \frac{\text{kg}\,\text{m}^2}{\text{s}^3}\right) t} \Rightarrow$

$\boxed{\tau(t=3) = 600 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}}$

ASU

Conservation of angular momentum

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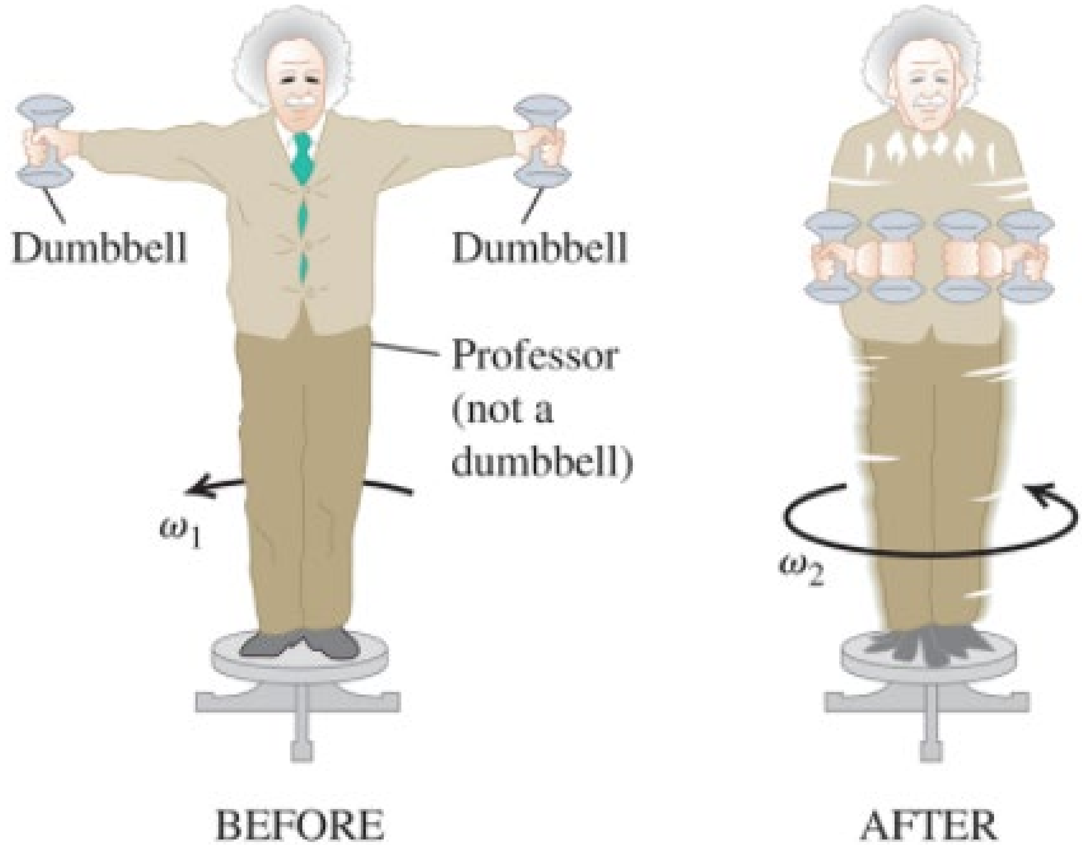
Since $\vec{\tau} = \frac{d\vec{L}}{dt}$ then

$\vec{\tau}_{EXT} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow$

$\vec{L} = \text{constant}$ for the system

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is $3.0\ \mathrm{kg \cdot m^2}$ with arms outstretched and $2.2\ \mathrm{kg \cdot m^2}$ with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.



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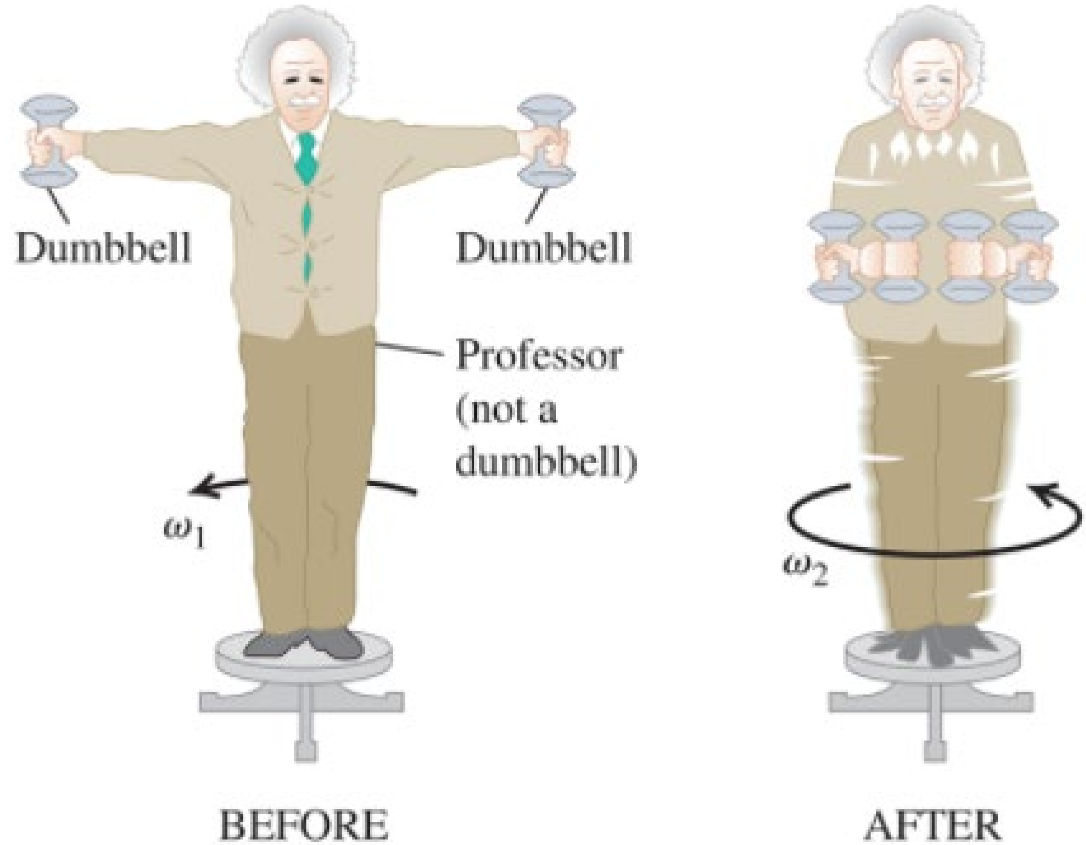
Let subscript
$p \equiv$ professor



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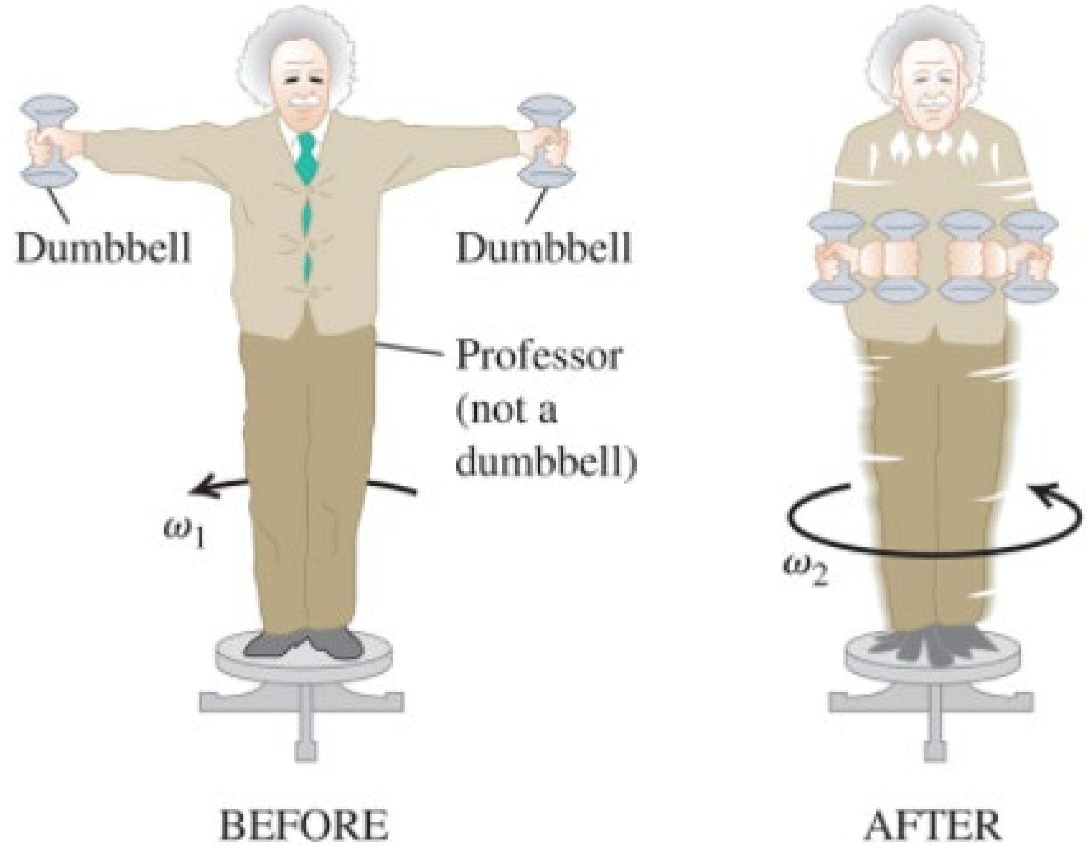
Let subscript
$p \equiv$ professor &
$d \equiv$ dumbbells



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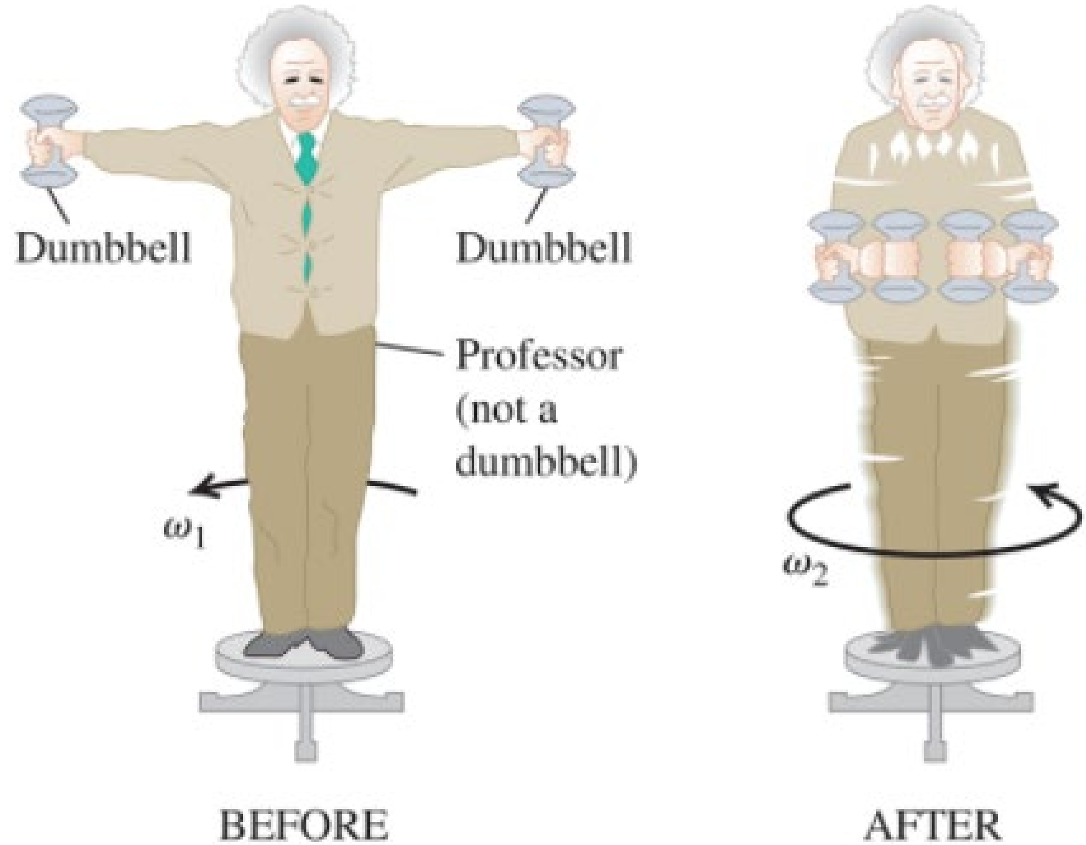
Let subscript
$p \equiv$ professor &
$d \equiv$ dumbbells &
$1 \equiv$ initial time



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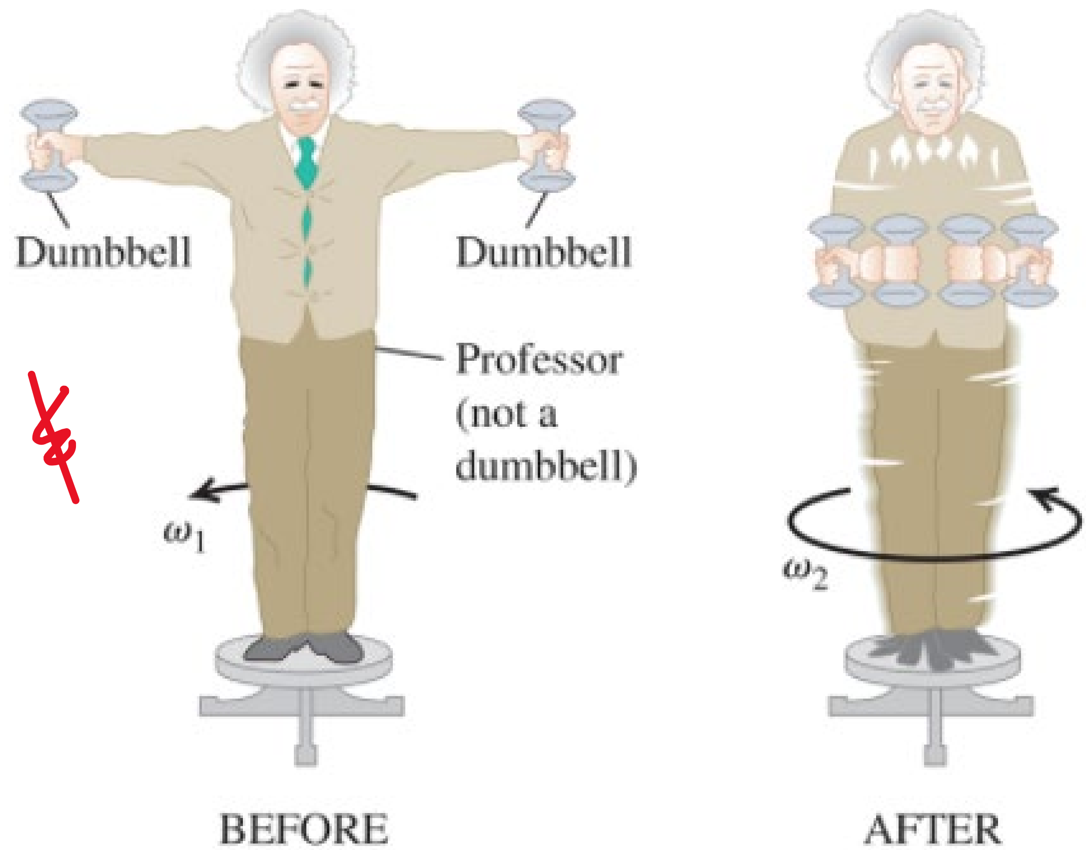
Let subscript
$p \equiv$ professor &
$d \equiv$ dumbbells &
$1 \equiv$ initial time &
$2 \equiv$ final time



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$I_{1p} = 3 kg \cdot m$

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is 3.0 kg·m^2 with arms outstretched and 2.2 kg·m^2 with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

$I_{1p} = 3 \text{kg} \cdot \text{m}, I_{2p} = 2.2 \text{kg} \cdot \text{m}^2$

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$I_{1p} = 3 kg \cdot m, \quad I_{2p} = 2.2 kg \cdot m^2$

$I_{1Q} = Mr_{1Q}^2 + Mr_{1Q}^2$

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$I_{1p} = 3 kg\cdot m, I_{2p} = 2.2 kg\cdot m^2$

$I_{1d} = Mr_{1d}^2 + Mr_{1d}^2 = 2 * 5 kg * 1 m^2$

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$I_{1p} = 3 kg \cdot m$, $I_{2p} = 2.2 kg \cdot m^2$

$I_{1Q} = Mr_{1Q}^2 + Mr_{1Q}^2 = 2 * 5 kg * 1 m^2 = 10 kg \cdot m^2$

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is 3.0 kg·m^2 with arms outstretched and 2.2 kg·m^2 with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

$I_{1p} = 3 kg \cdot m, I_{2p} = 2.2 kg \cdot m^2$

$I_{1d} = Mr_{1d}^2 + Mr_{1d}^2 = 2 * 5 kg * 1 m^2 = 10 kg \cdot m^2$

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A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is 3.0 kg·m^2 with arms outstretched and 2.2 kg·m^2 with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

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A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is 3.0 kg·m² with arms outstretched and 2.2 kg·m² with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

$I_{1p} = 3 kg \cdot m, I_{2p} = 2.2 kg \cdot m^2$

$I_{1d} = Mr_{1d}^2 + Mr_{1d}^2 = 2 * 5 kg * 1 m^2 = 10 kg \cdot m^2$

$I_{2d} = Mr_{2d}^2 + Mr_{2d}^2 = 2 * 5 kg * 0.04 m^2 = 0.4 kg \cdot m^2$

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is 3.0 kg·m^2 with arms outstretched and 2.2 kg·m^2 with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

$I_{1p} = 3 kg \cdot m, \; I_{2p} = 2.2 kg \cdot m^2$

$I_{1d} = M r_{1d}^2 + M r_{1d}^2 = 2 * 5 kg * 1 m^2 = 10 kg \cdot m^2$

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$L_1 = L_2$

ASU

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is 3.0 kg·m^2 with arms outstretched and 2.2 kg·m^2 with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

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$L_1 = L_2$, where $L = (I_d + I_p)\omega$

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$L_1 = L_2$, where $L = (I_d + I_p)\omega \Rightarrow$

$(I_{1d} + I_{1p})\omega_1 = (I_{2d} + I_{2p})\omega_2$

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is 3.0 kg·m^2 with arms outstretched and 2.2 kg·m^2 with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

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$(I_{1d} + I_{1p})\omega_1 = (I_{2d} + I_{2p})\omega_2 \Rightarrow$

$\omega_2 = \left[\frac{I_{1d} + I_{1p}}{I_{2d} + I_{2p}}\right]\omega_1$

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is 3.0 kg·m^2 with arms outstretched and 2.2 kg·m^2 with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

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$I_{1d} = Mr_{1d}^2 + Mr_{1d}^2 = 2 * 5\,\mathrm{kg} * 1\,\mathrm{m}^2 = 10\,\mathrm{kg\cdot m^2}$

$I_{2d} = Mr_{2d}^2 + Mr_{2d}^2 = 2 * 5\,\mathrm{kg} * 0.04\,\mathrm{m}^2 = 0.4\,\mathrm{kg\cdot m^2}$

$L_1 = L_2$, where $L = (I_d + I_p)\omega \Rightarrow$

$(I_{1d} + I_{1p})\omega_1 = (I_{2d} + I_{2p})\omega_2 \Rightarrow$

$\omega_2 = \left[\dfrac{I_{1d} + I_{1p}}{I_{2d} + I_{2p}}\right]\omega_1 = \dfrac{13}{}$

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is $3.0\ \mathrm{kg \cdot m^2}$ with arms outstretched and $2.2\ \mathrm{kg \cdot m^2}$ with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

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$L_1 = L_2$, where $L = (I_d + I_p)\omega \Rightarrow$

$(I_{1d} + I_{1p})\omega_1 = (I_{2d} + I_{2p})\omega_2 \Rightarrow$

$\omega_2 = \left[\dfrac{I_{1d} + I_{1p}}{I_{2d} + I_{2p}}\right]\omega_1 = \left(\dfrac{13}{2.6}\right)$

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is 3.0 $kg \cdot m^2$ with arms outstretched and 2.2 $kg \cdot m^2$ with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

$$I_{1p} = 3 kg \cdot m, \quad I_{2p} = 2.2 kg \cdot m^2$$

$$I_{1d} = Mr_{1d}^2 + Mr_{1d}^2 = 2 * 5 kg * 1 m^2 = 10 kg \cdot m^2$$

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$$L_1 = L_2, \text{ where } L = (I_d + I_p)\omega \Rightarrow$$

$$(I_{1d} + I_{1p})\omega_1 = (I_{2d} + I_{2p})\omega_2 \Rightarrow$$

$$\omega_2 = \left[\frac{I_{1d} + I_{1p}}{I_{2d} + I_{2p}}\right]\omega_1 = \left(\frac{13}{2.6}\right)\left(\frac{rev}{2 s}\right)$$

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is 3.0 kg·m^2 with arms outstretched and 2.2 kg·m^2 with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

$I_{1p} = 3 kg \cdot m, \; I_{2p} = 2.2 kg \cdot m^2$

$I_{1d} = M r_{1d}^2 + M r_{1d}^2 = 2 * 5 kg * 1 m^2 = 10 kg \cdot m^2$

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$L_1 = L_2$, where $L = (I_d + I_p)\omega \Rightarrow$

$(I_{1d} + I_{1p})\omega_1 = (I_{2d} + I_{2p})\omega_2 \Rightarrow$

$\omega_2 = \left[\frac{I_{1d} + I_{1p}}{I_{2d} + I_{2p}}\right]\omega_1 = \left(\frac{13}{2.6}\right)\left(\frac{rev}{2 s}\right) = \left(\frac{5}{2}\right)\frac{rev}{s}$

ASU

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0 kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells inward to his stomach. His moment of inertia (without the dumbbells) is 3.0 kg·m^2 with arms outstretched and 2.2 kg·m^2 with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

$$I_{1p} = 3\,\mathrm{kg\cdot m},\quad I_{2p} = 2.2\,\mathrm{kg\cdot m^2}$$

$$I_{1d} = Mr_{1d}^2 + Mr_{1d}^2 = 2 * 5\,\mathrm{kg} * 1\,\mathrm{m}^2 = 10\,\mathrm{kg\cdot m^2}$$

$$I_{2d} = Mr_{2d}^2 + Mr_{2d}^2 = 2 * 5\,\mathrm{kg} * 0.04\,\mathrm{m}^2 = 0.4\,\mathrm{kg\cdot m^2}$$

$$L_1 = L_2,\ \text{where}\quad L = (I_d + I_p)\omega \Rightarrow$$

$$(I_{1d} + I_{1p})\omega_1 = (I_{2d} + I_{2p})\omega_2 \Rightarrow$$

$$\omega_2 = \left[\frac{I_{1d} + I_{1p}}{I_{2d} + I_{2p}}\right]\omega_1 = \left(\frac{13}{2.6}\right)\left(\frac{\mathrm{rev}}{2\,\mathrm{s}}\right) = \left(\frac{5}{2}\right)\frac{\mathrm{rev}}{\mathrm{s}}$$

$$\Rightarrow \boxed{\omega_2 = 2.5\,\frac{\mathrm{rev}}{\mathrm{s}}}$$

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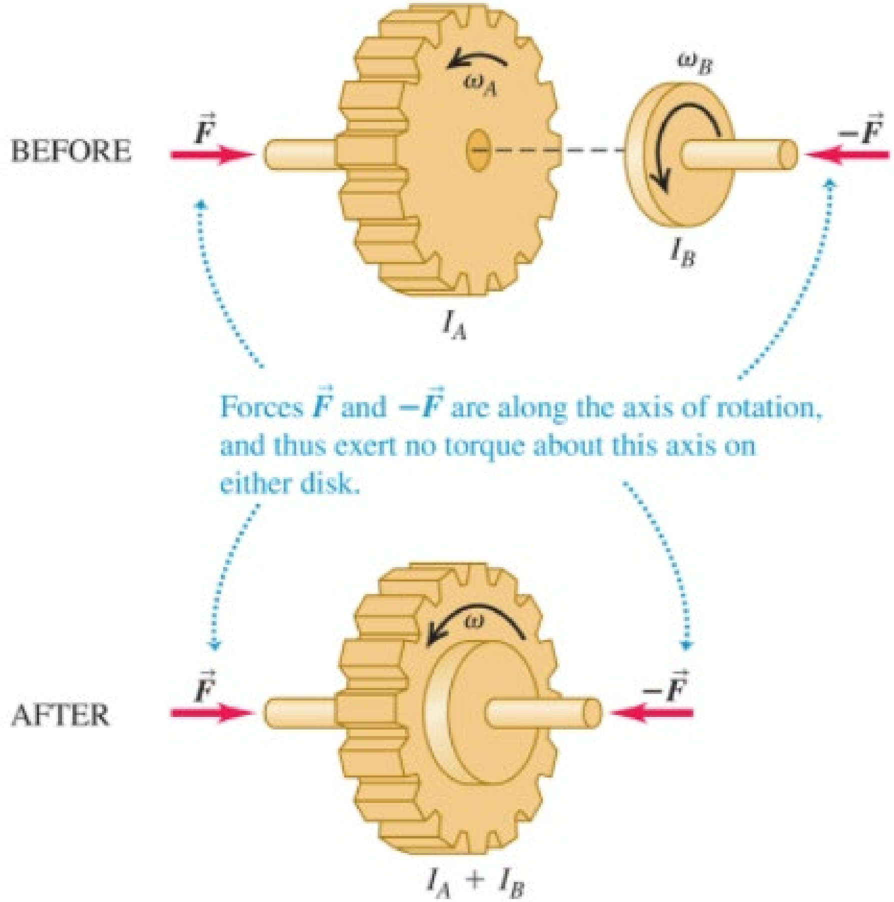
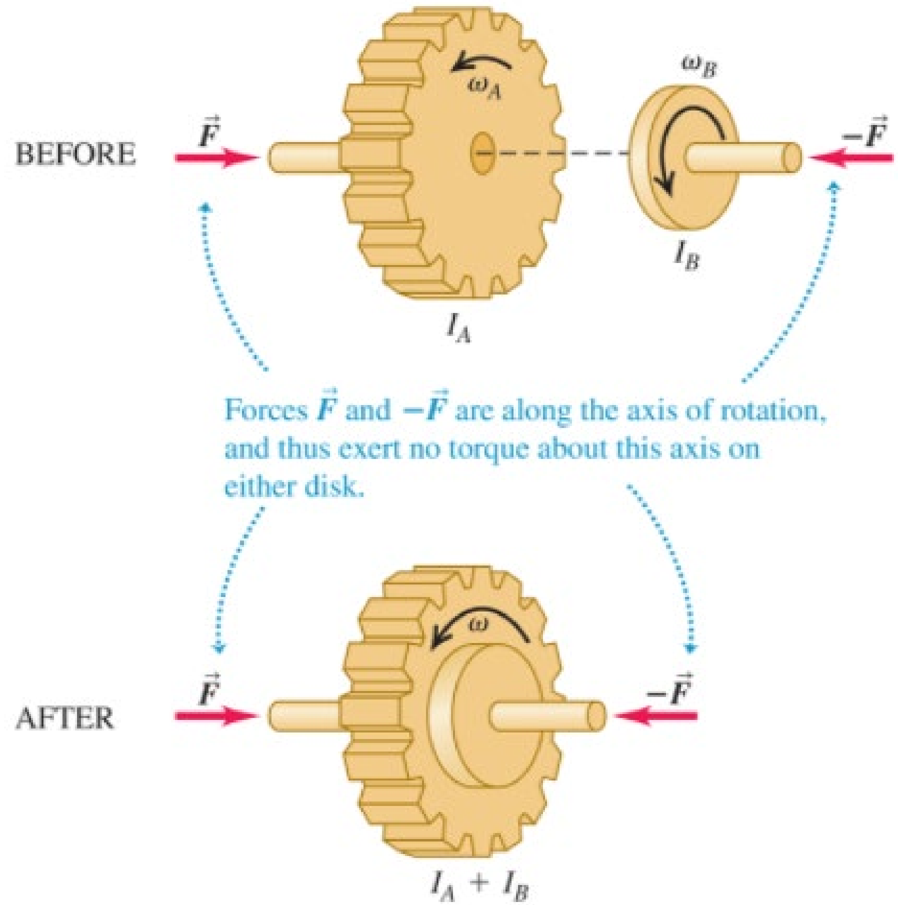
Figure 10.30 shows two disks: an engine flywheel (A) and a clutch plate (B) attached to a transmission shaft. Their moments of inertia are I_A and I_B; initially, they are rotating in the same direction with constant angular speeds ω_A and ω_B, respectively. We push the disks together with forces acting along the axis, so as not to apply any torque on either disk. The disks rub against each other and eventually reach a common angular speed ω. Derive an expression for ω.



ASU

Figure 10.30 shows two disks: an engine flywheel (A) and a clutch plate (B) attached to a transmission shaft. Their moments of inertia are I_A and I_B; initially, they are rotating in the same direction with constant angular speeds ω_A and ω_B, respectively. We push the disks together with forces acting along the axis, so as not to apply any torque on either disk. The disks rub against each other and eventually reach a common angular speed ω. Derive an expression for ω.

$L_1 = L_2$



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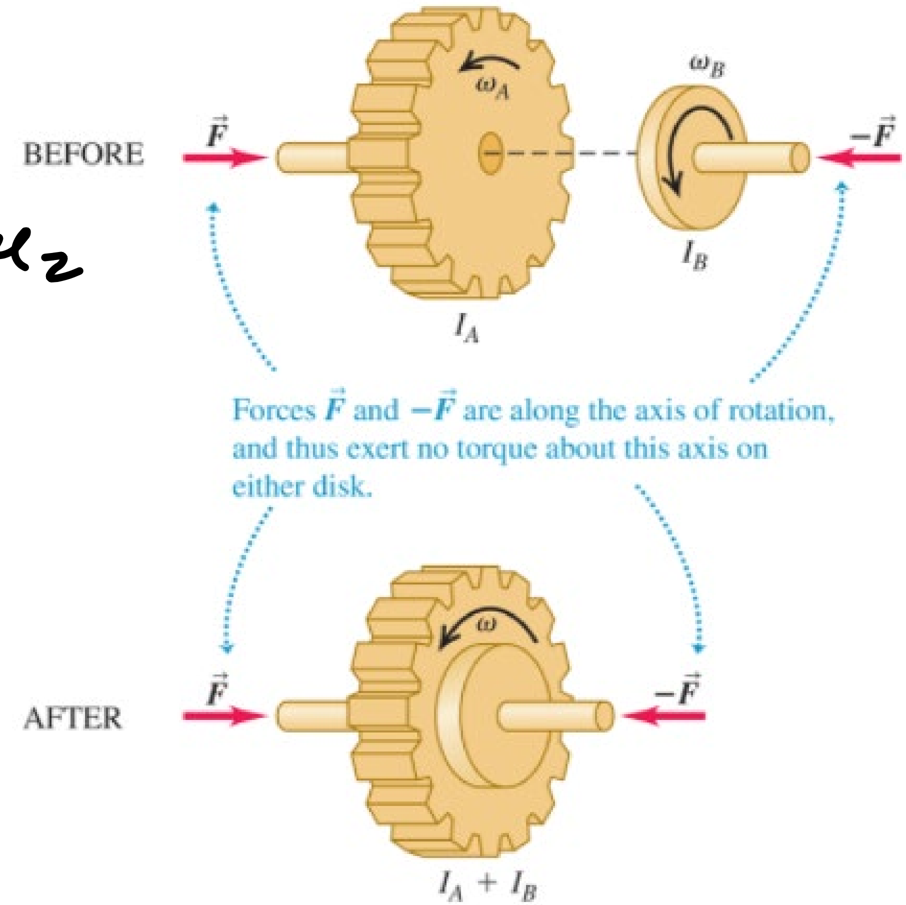
$L_1 = L_2 \Rightarrow$

$I_A \omega_A + I_B \omega_B = (I_A + I_B)\omega_2$



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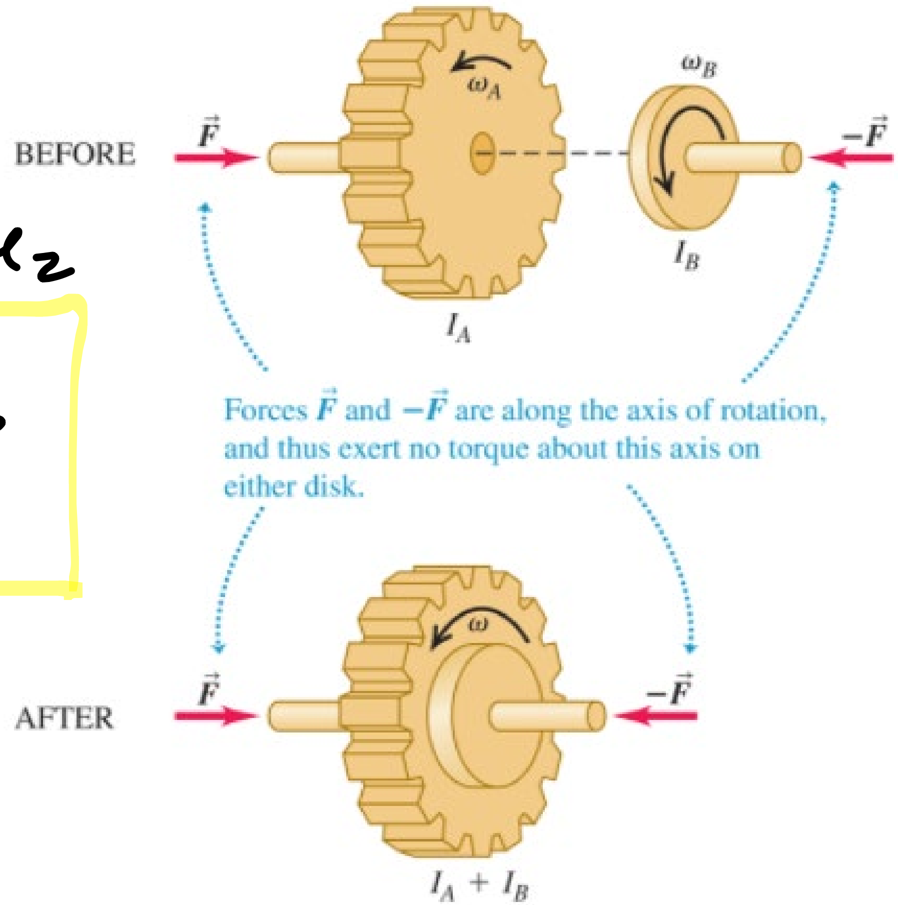
$\Rightarrow \boxed{\omega_2 = \dfrac{I_A \omega_A + I_B \omega_B}{(I_A + I_B)}}$



ASU

A door 1.00 m wide, of mass 15 kg, can rotate freely about a vertical axis through its hinges. A bullet with a mass of 10 g and a speed of 400 m/s strikes the center of the door, in a direction perpendicular to the plane of the door, and embeds itself there. Find the door's angular speed. Is kinetic energy conserved?

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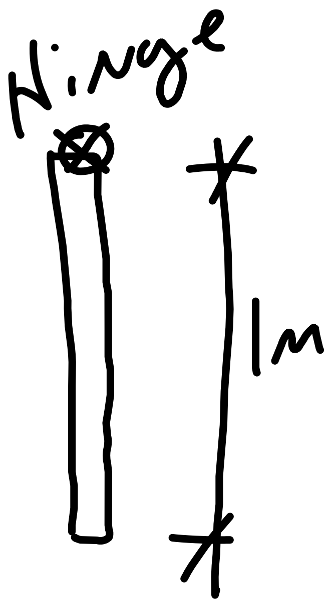

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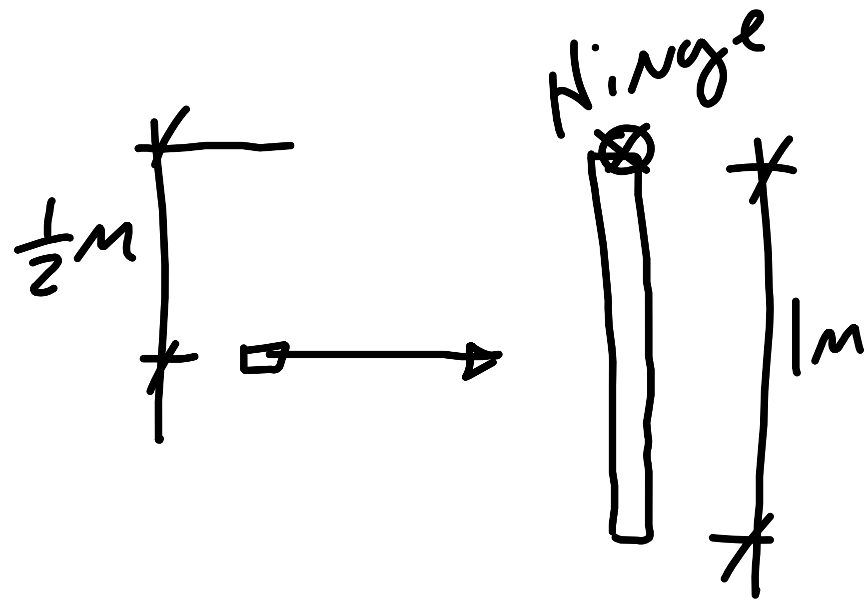

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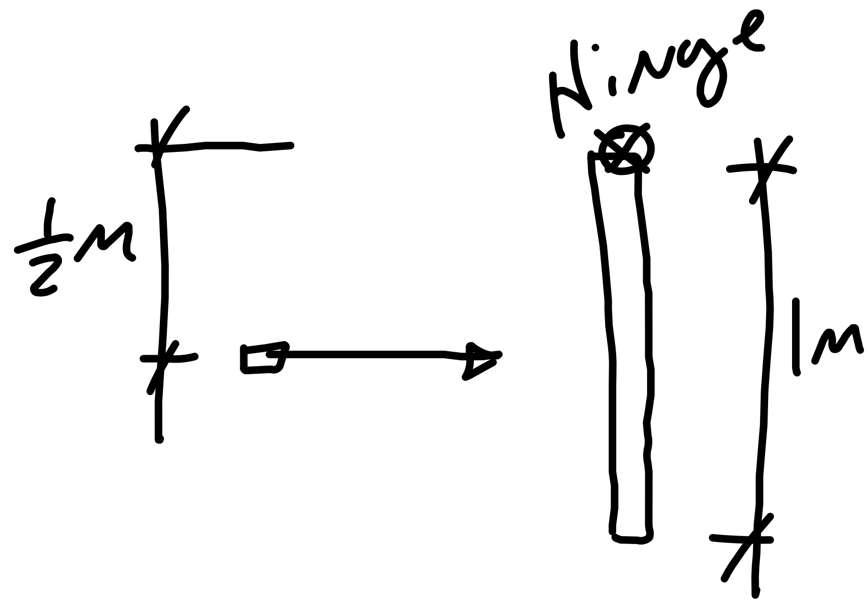
$M_d = 15\,kg$



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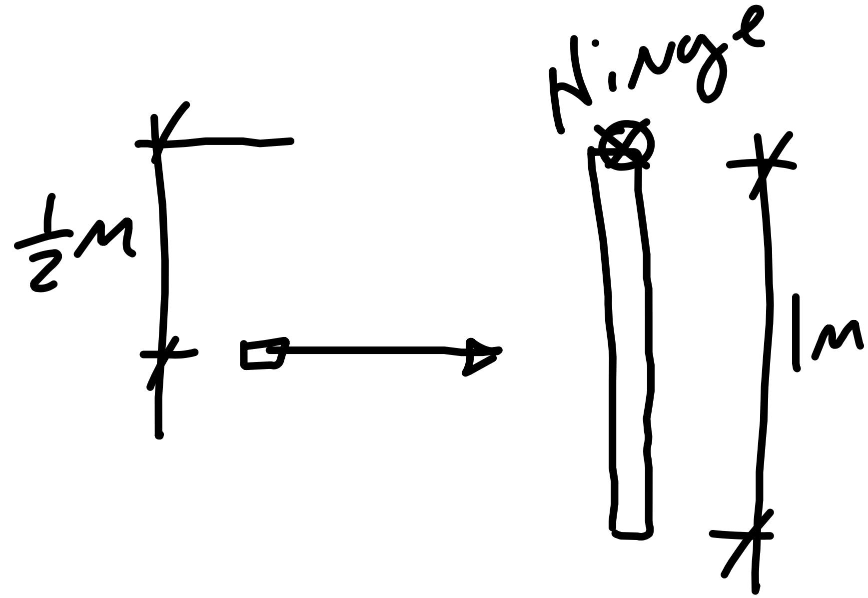
$M_d = 15\,kg,\ m_b = 0.01\,kg$



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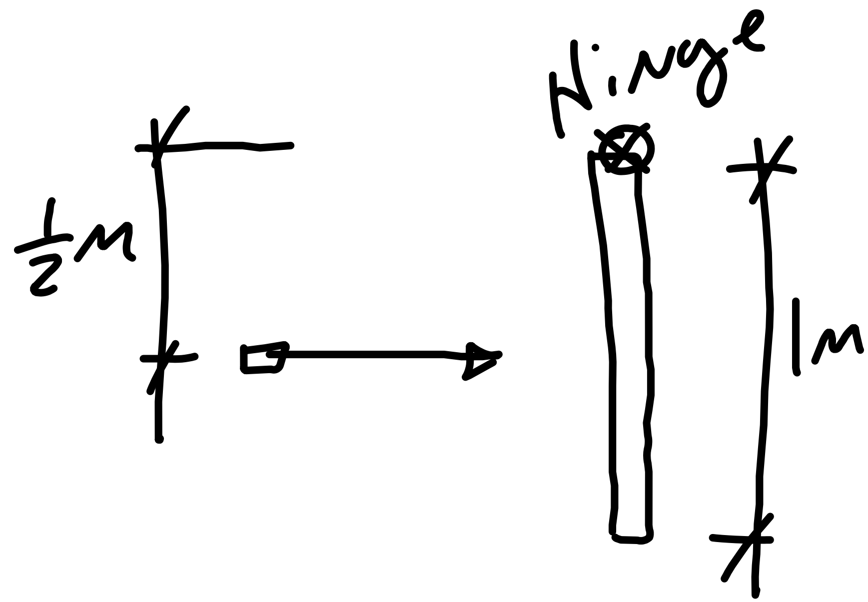
$m_d = 15\,kg$, $m_b = 0.01\,kg$

$V_b = 400\,m/s$



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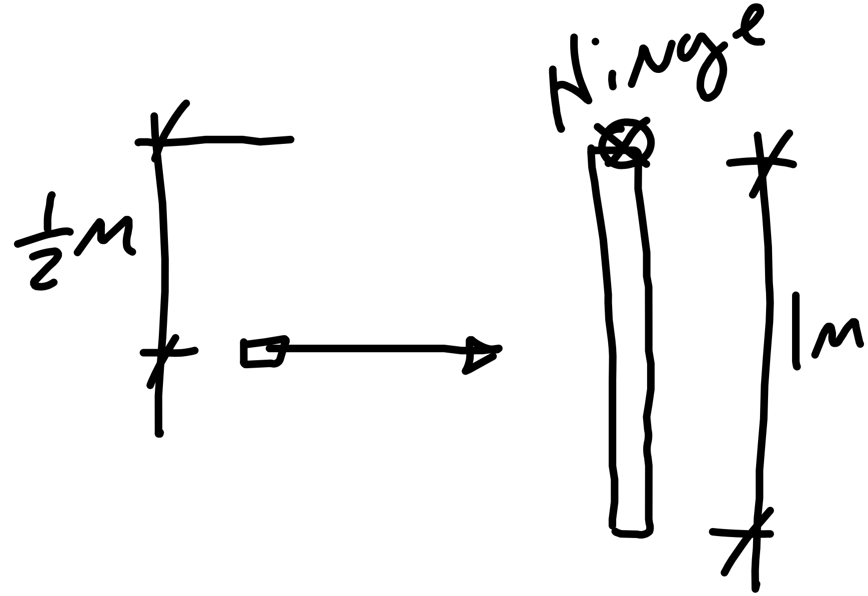
$m_d = 15\,kg$, $m_b = 0.01\,kg$

$V_b = 400\,m/s$

$I_d = \frac{m_d L}{3}$



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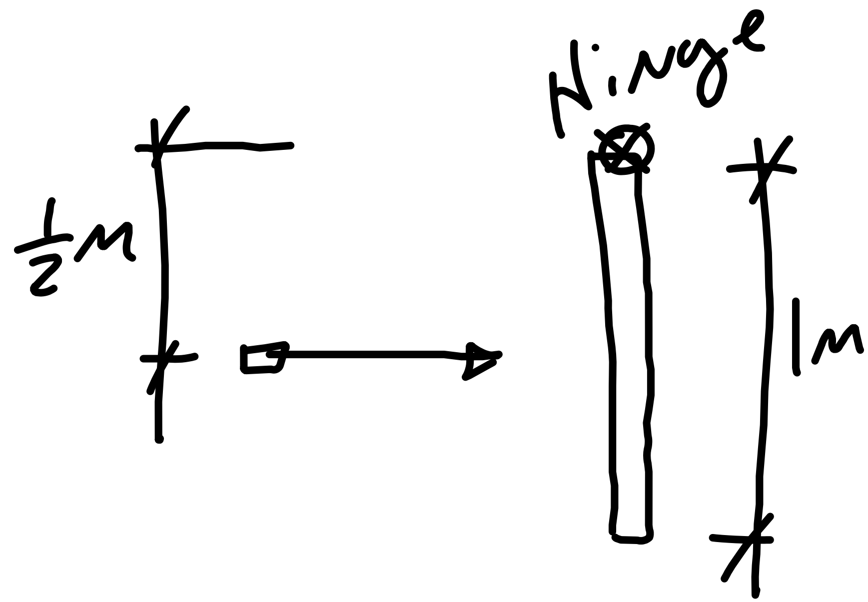
$$L_1 = L_2$$



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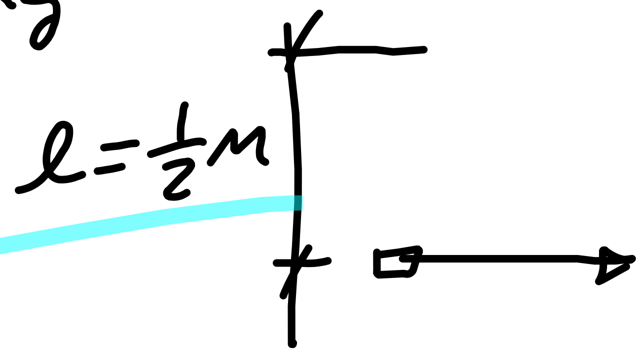

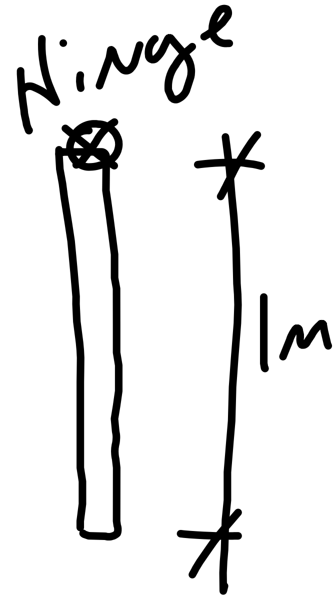

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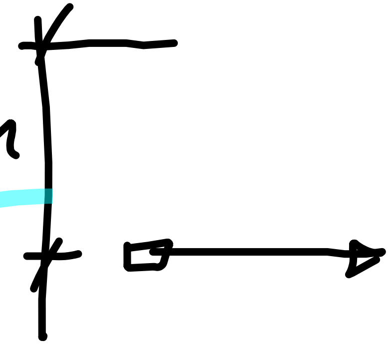
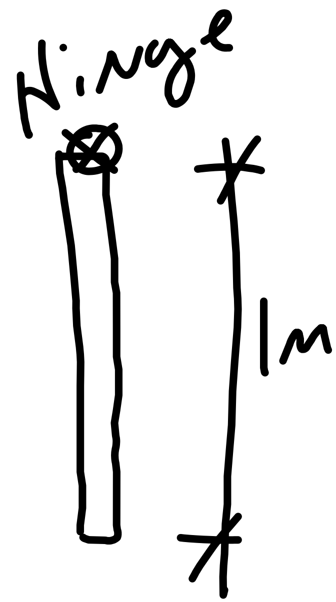
$M_d = 15\,kg$, $M_b = 0.01\,kg$

$V_b = 400\,m/s$

$l = \frac{1}{2}m$





$I_d = \frac{M_d L}{3}$

$L_1 = L_2 \Rightarrow M_b V_b l = M_b (V_{bF}) l + I_d \omega$

A door 1.00 m wide, of mass 15 kg, can rotate freely about a vertical axis through its hinges. A bullet with a mass of 10 g and a speed of 400 m/s strikes the center of the door, in a direction perpendicular to the plane of the door, and embeds itself there. Find the door's angular speed. Is kinetic energy conserved?

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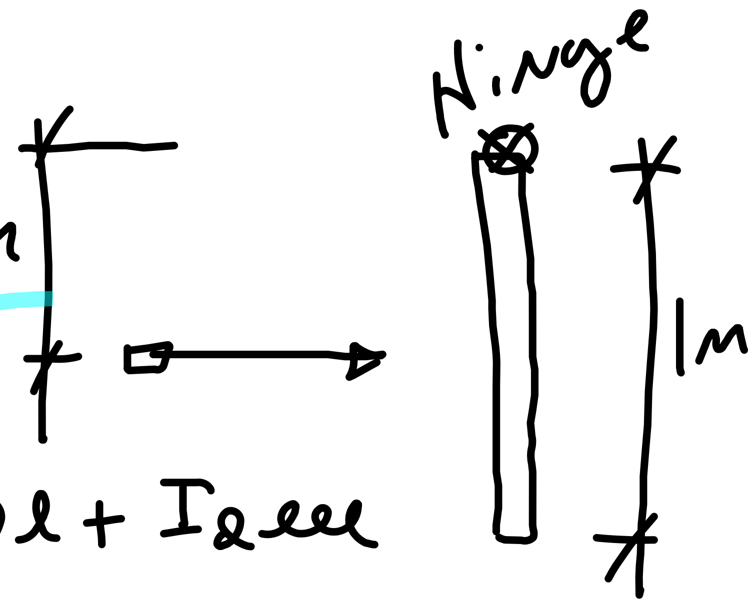

$L_1 = L_2 \Rightarrow m_b V_b l = m_b (V_{bF}) l + I_d \omega$

But $V_{bF} = l \omega$

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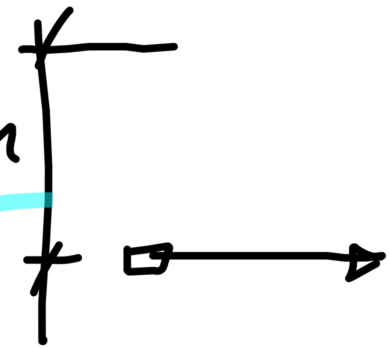

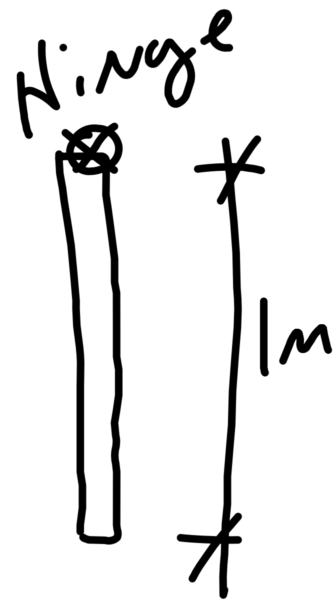

$I_d = \frac{M_d L}{3}$

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But $V_{bF} = l\omega$ so $M_b V_b l = M_b l^2 \omega + I_d \omega$

A door 1.00 m wide, of mass 15 kg, can rotate freely about a vertical axis through its hinges. A bullet with a mass of 10 g and a speed of 400 m/s strikes the center of the door, in a direction perpendicular to the plane of the door, and embeds itself there. Find the door's angular speed. Is kinetic energy conserved?

$M_d = 15\,kg$, $M_b = 0.01\,kg$

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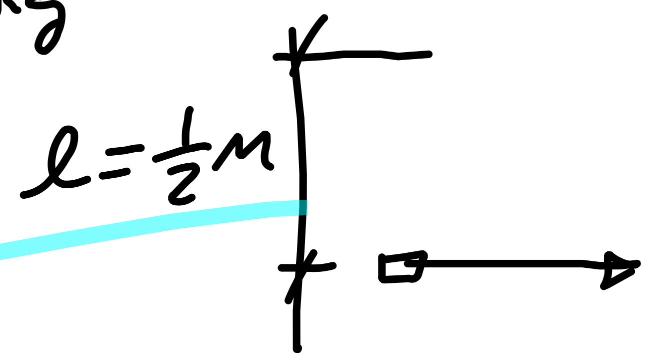

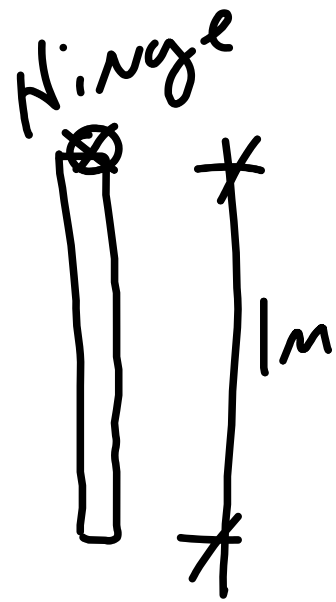

$$I_d = \frac{M_d L}{3}$$

$$L_1 = L_2 \Rightarrow M_b V_b l = M_b (V_{bF}) l + I_d \omega$$

But $V_{bF} = l\omega$ so $M_b V_b l = M_b l^2 \omega + I_d \omega$

$$\Rightarrow \omega = \frac{M_b V_b l}{M_b l^2 + I_d}$$

A door 1.00 m wide, of mass 15 kg, can rotate freely about a vertical axis through its hinges. A bullet with a mass of 10 g and a speed of 400 m/s strikes the center of the door, in a direction perpendicular to the plane of the door, and embeds itself there. Find the door's angular speed. Is kinetic energy conserved?

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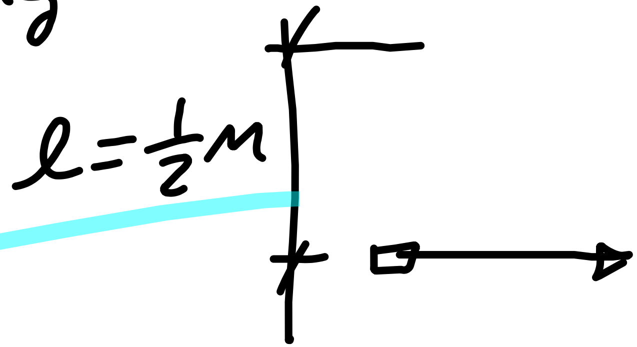

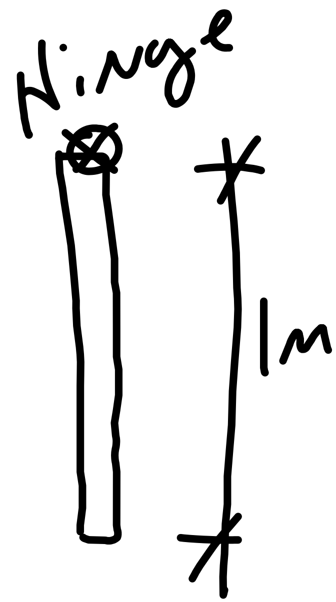

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$$\Rightarrow \omega = \frac{M_b V_b l}{M_b l^2 + I_d} = \frac{M_b V_b l}{M_b l^2 + M_d L/3}$$

A door 1.00 m wide, of mass 15 kg, can rotate freely about a vertical axis through its hinges. A bullet with a mass of 10 g and a speed of 400 m/s strikes the center of the door, in a direction perpendicular to the plane of the door, and embeds itself there. Find the door's angular speed. Is kinetic energy conserved?

$M_d = 15\text{ kg}$, $M_b = 0.01\text{ kg}$

$V_b = 400\text{ m/s}$

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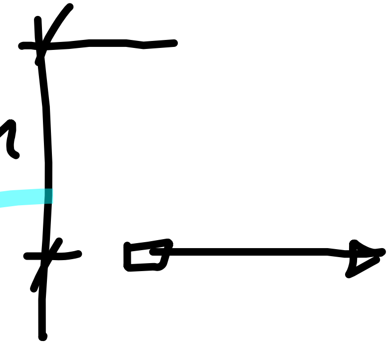

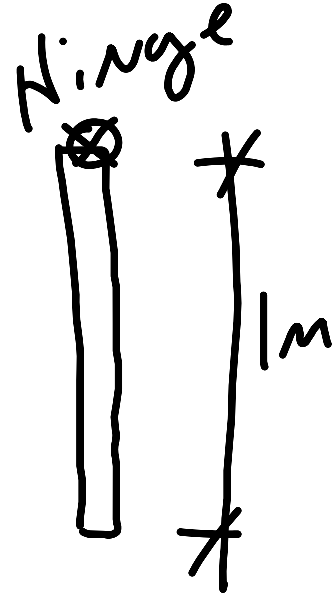

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But $V_{bF} = l\omega$ so $M_b V_b l = M_b l^2 \omega + I_d \omega$

$$\Rightarrow \omega = \frac{M_b V_b l}{M_b l^2 + I_d} = \frac{M_b V_b l}{M_b l^2 + M_d L/3} \Rightarrow$$

$$\omega = \left[\frac{(0.01)(400)(\frac{1}{2})}{(0.01)\frac{1}{4} + 15 * 1/3}\right]\frac{\text{rad}}{\text{s}} = 0.4\text{ rad/s}$$

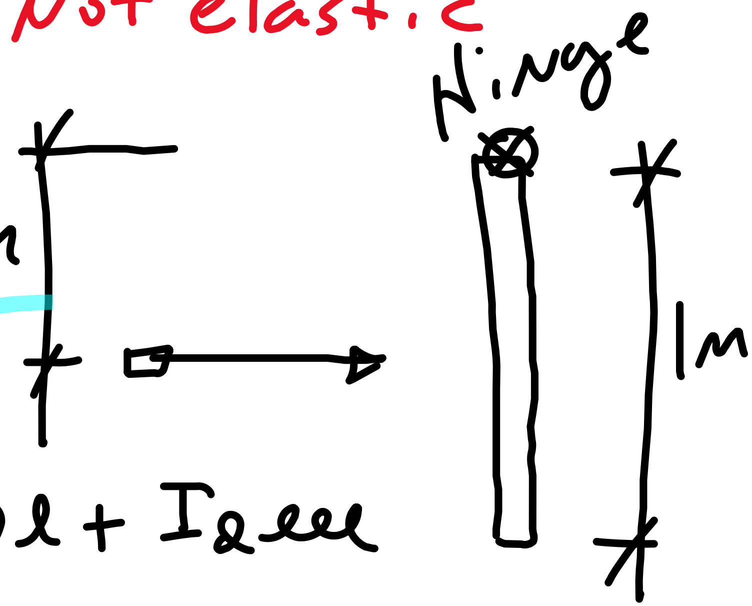
A door 1.00 m wide, of mass 15 kg, can rotate freely about a vertical axis through its hinges. A bullet with a mass of 10 g and a speed of 400 m/s strikes the center of the door, in a direction perpendicular to the plane of the door, and embeds itself there. Find the door's angular speed. Is kinetic energy conserved? → Collision is Not elastic

$M_d = 15\,kg,\ M_b = 0.01\,kg$

$V_b = 400\,m/s$

$I_d = \frac{M_d L}{3}$

$l = \frac{1}{2} m$



$L_1 = L_2 \Rightarrow M_b V_b l = M_b (V_{bF}) l + I_d \omega$

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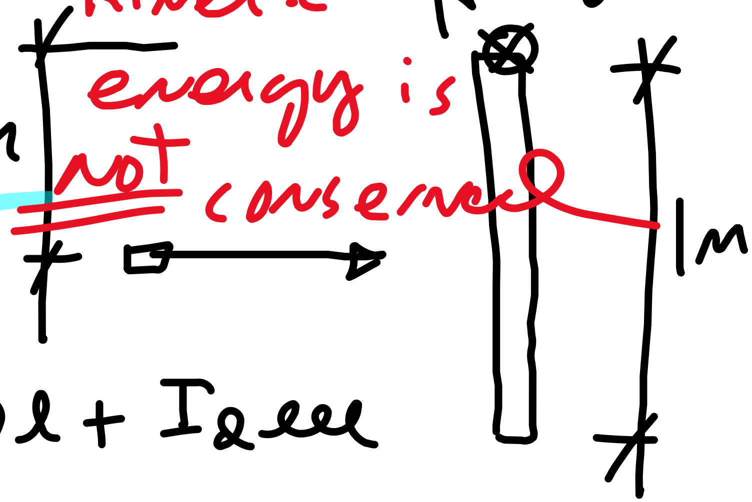
→ Collision is Not elastic so Kinetic energy is not conserved

$M_d = 15\,kg$, $M_b = 0.01\,kg$

$V_b = 400\,m/s$

$l = \frac{1}{2}\,m$

$I_d = \frac{M_d L}{3}$



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