Today 10.3, 10.4

$\boxed{L31}$

Today 10.3, 10.4

$\boxed{L31}$

Rigid body rotation about a moving axis

Today 10.3, 10.4

L31

Rigid body rotation about a moving axis

Work & Power in rotational motion

Today 10.3, 10.4

$\boxed{L31}$

SI session today

* 5-6 pm

Today 10.3, 10.4

L31

SI session today

* 5-6 pm
* 10.1 [Torque]

Today 10.3, 10.4

L31

SI session today

* 5-6 pm
* 10.1 [Torque]
* 10.2 [Torque & angular acceleration for a rigid body]

Today 10.3, 10.4

L31

Wednesday Holiday

$\boxed{L31}$

Today 10.3, 10.4

Wednesday Holiday ☺

$\boxed{L31}$

Today 10.3, 10.4

Wednesday Holiday ☺

Friday 10.5, 10.6

Today 10.3, 10.4

L31

Wednesday Holiday ☺

Friday 10.5, 10.6

Angular momentum

L31

Today 10.3, 10.4

Wednesday Holiday ☺

Friday 10.5, 10.6

Angular momentum

Conservation of angular momentum

L31

Today 10.3, 10.4

Wednesday Holiday ☺

Friday 10.5, 10.6

Important dates:

L31

Today 10.3, 10.4

Wednesday Holiday ☺

Friday 10.5, 10.6

Important dates:

* Friday Nov. 27th No class

L31

Today 10.3, 10.4

Wednesday Holiday ☺

Friday 10.5, 10.6

Important dates:

* Friday Nov. 27th No class
* Monday Nov. 30th Exam 4

L31

Today 10.3, 10.4

Wednesday Holiday ☺

Friday 10.5, 10.6

Important dates:

* Friday Nov. 27th No class
* Monday Nov. 30th Exam 4
* Wednesday Dec 2nd Day of Reckoning

L31

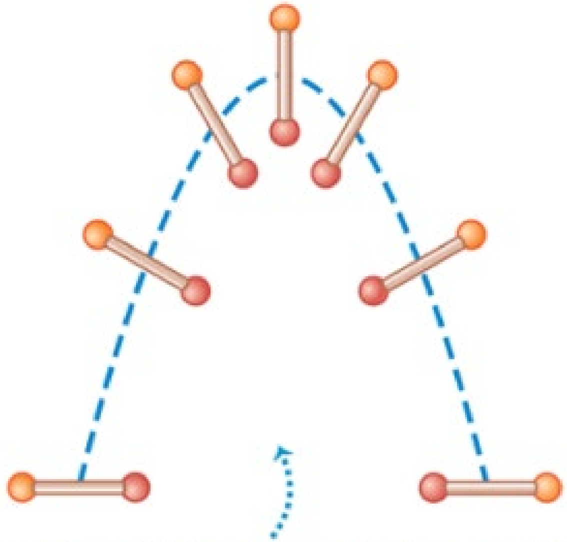
Today 10.3, 10.4

Wednesday Holiday ☺

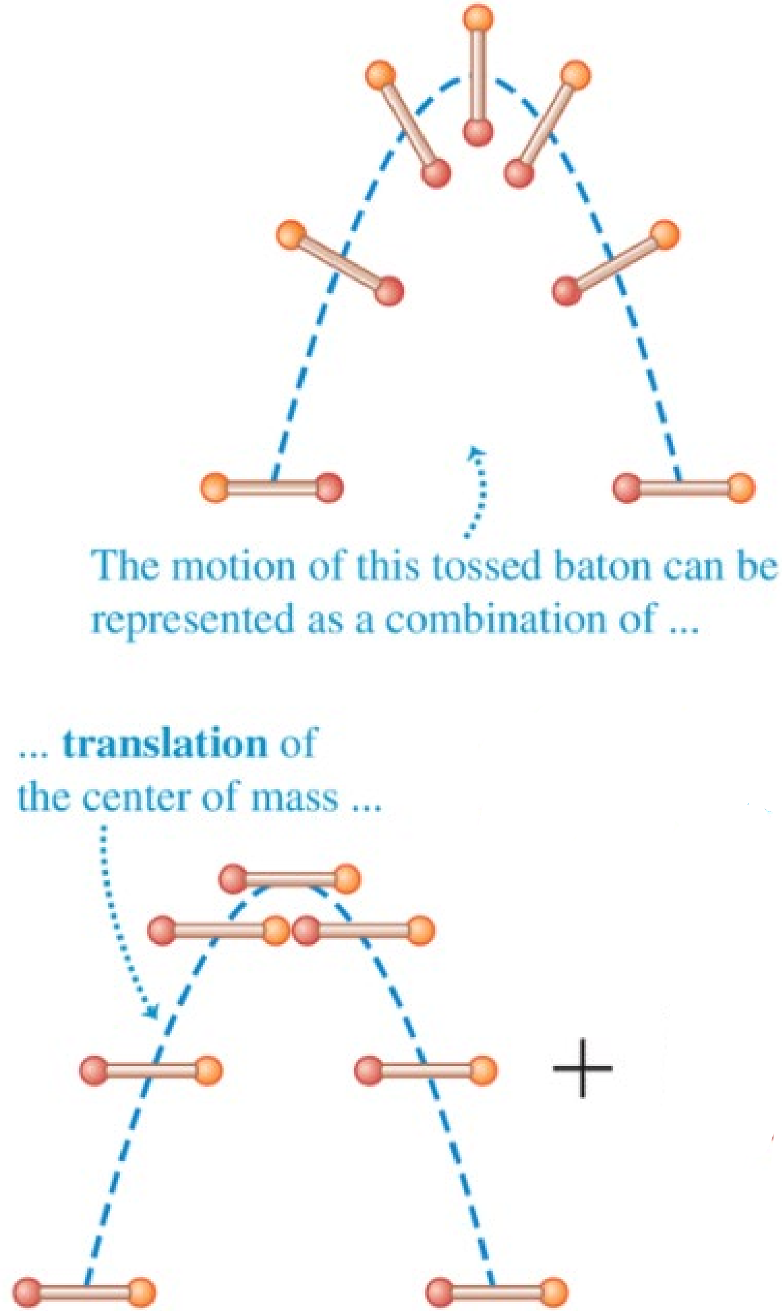
Friday 10.5, 10.6

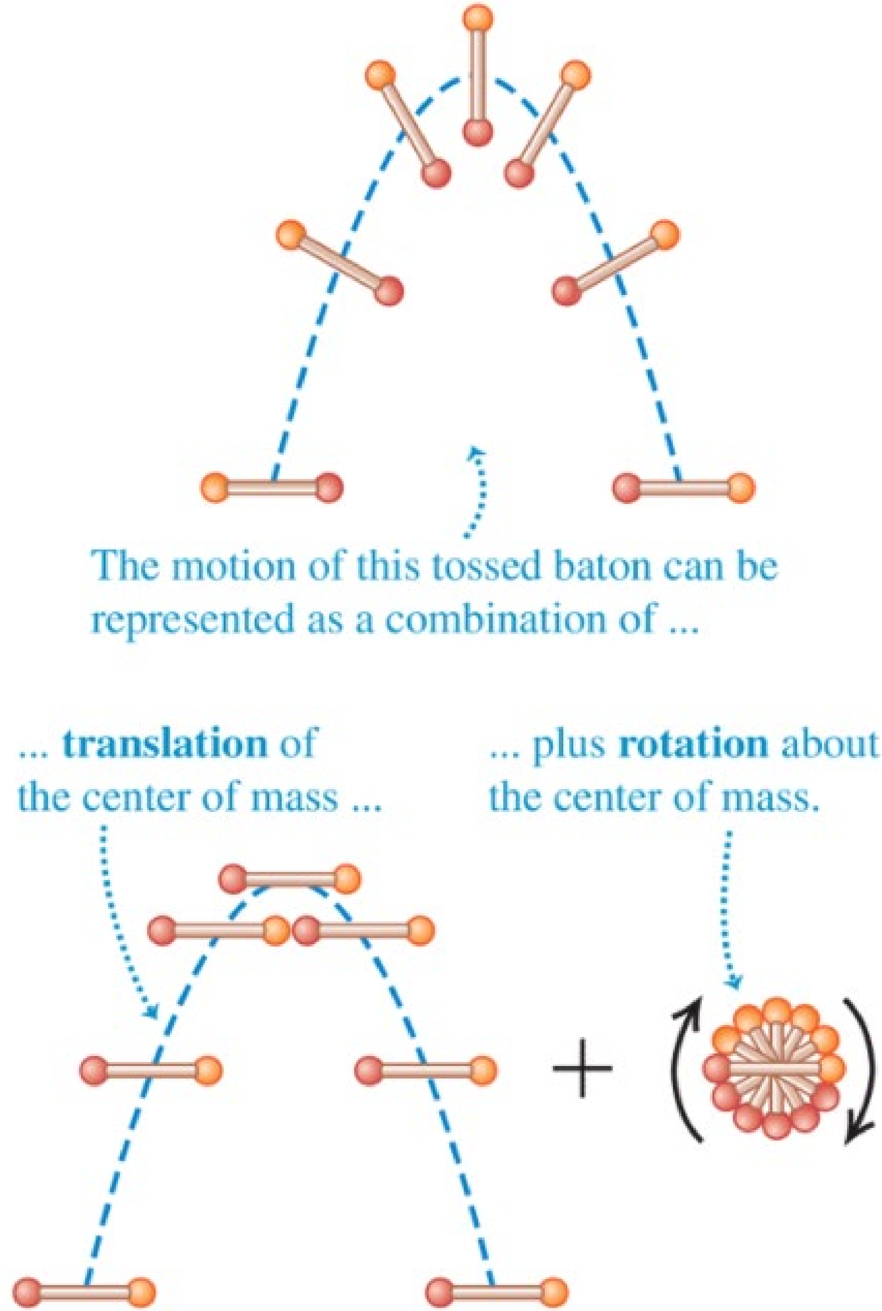
Important dates:

* Friday Nov. 27th No class
* Monday Nov. 30th Exam 4
* Wednesday Dec 2nd Day of Reckoning
* Friday Dec 4th Final exam



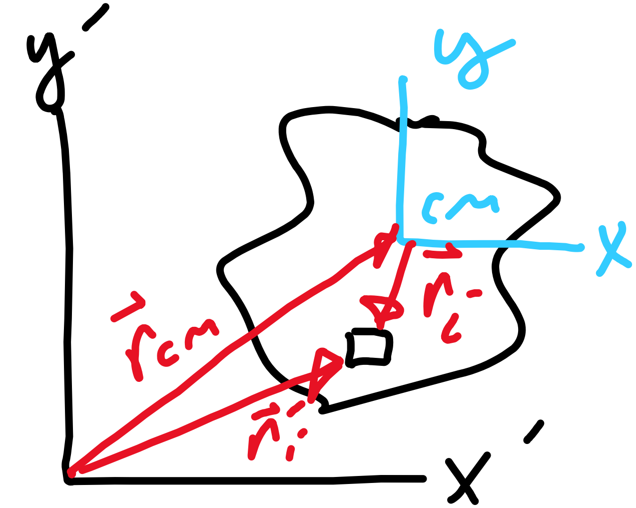
The motion of this tossed baton can be represented as a combination of ...


The motion of this tossed baton can be represented as a combination of ...
... translation of the center of mass ...
+


The motion of this tossed baton can be represented as a combination of ...
... **translation** of the center of mass ...
... plus **rotation** about the center of mass.
+

Kinetic energy

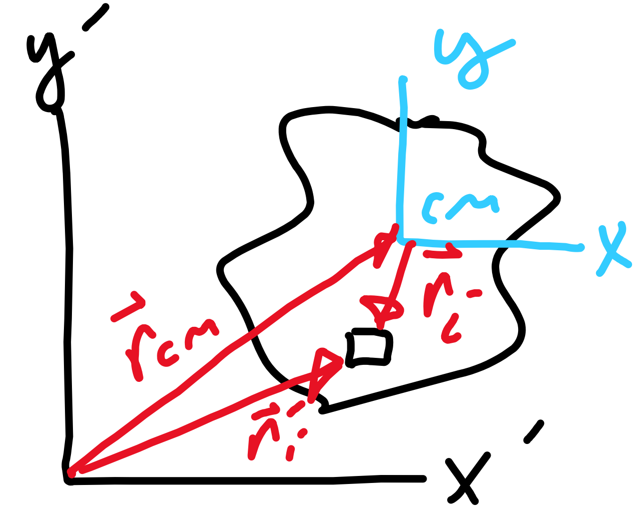
$$K = \frac{1}{2}\sum m_i (v_i')^2$$



Kinetic energy

$K = \frac{1}{2} \sum m_i (v_i')^2$ But

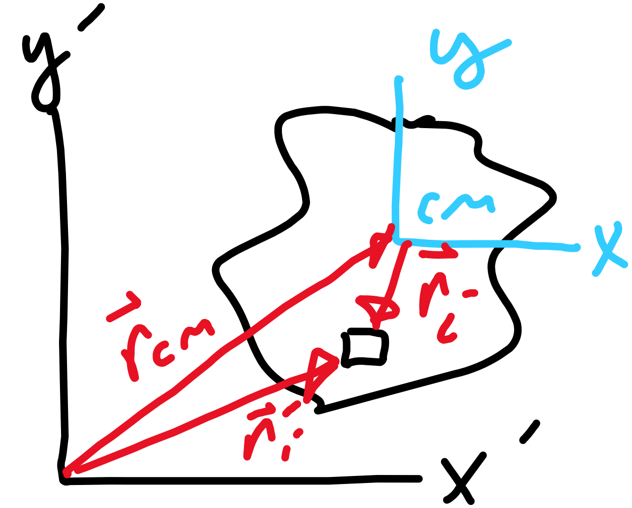
$(v_i')^2 = \vec{v}_i' \cdot \vec{v}_i'$



Kinetic energy

$K = \frac{1}{2}\sum m_i (v_i')^2$ But

$(v_i')^2 = \vec{v}_i' \cdot \vec{v}_i' = (\vec{v}_{cm} + \vec{v}_i)\cdot(\vec{v}_{cm} + \vec{v}_i)$



Kinetic energy

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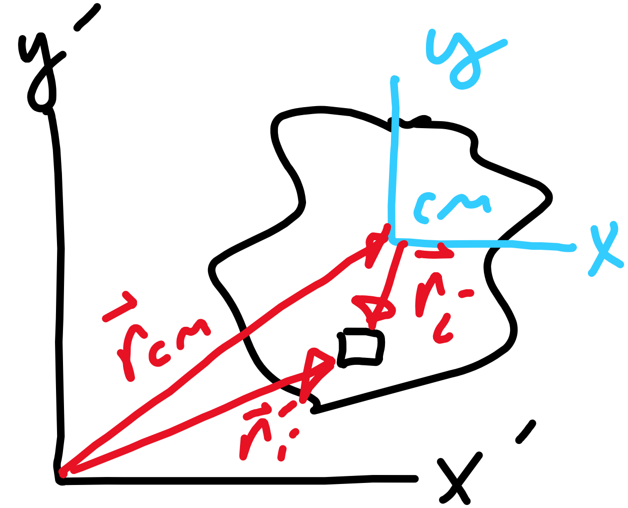
$(v_i')^2 = \vec{v}_i' \cdot \vec{v}_i' = (\vec{v}_{cm} + \vec{v}_i) \cdot (\vec{v}_{cm} + \vec{v}_i)$

$= v_{cm}^2 + v_i^2 + 2\vec{v}_{cm} \cdot \vec{v}_i$



Kinetic energy

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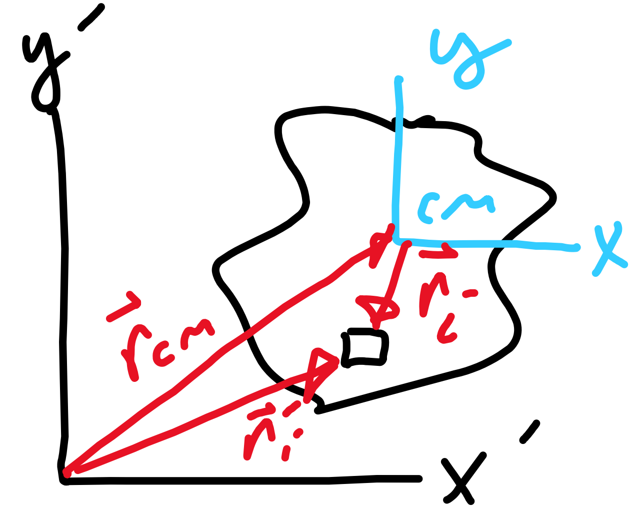
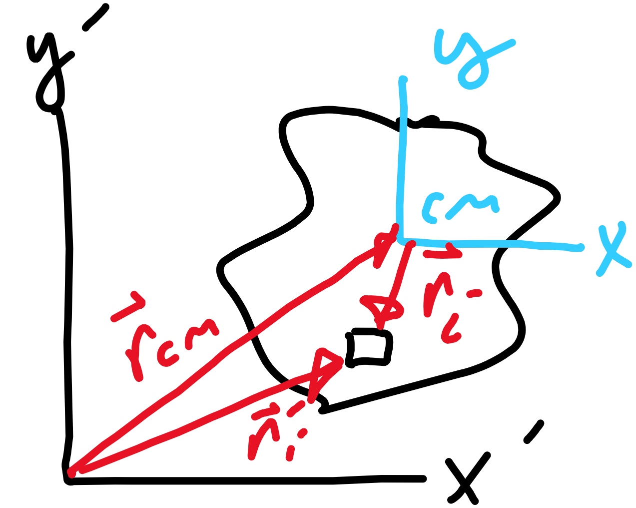


So $K = \frac{1}{2}\sum m_i v_{cm}^2 +$

Kinetic energy



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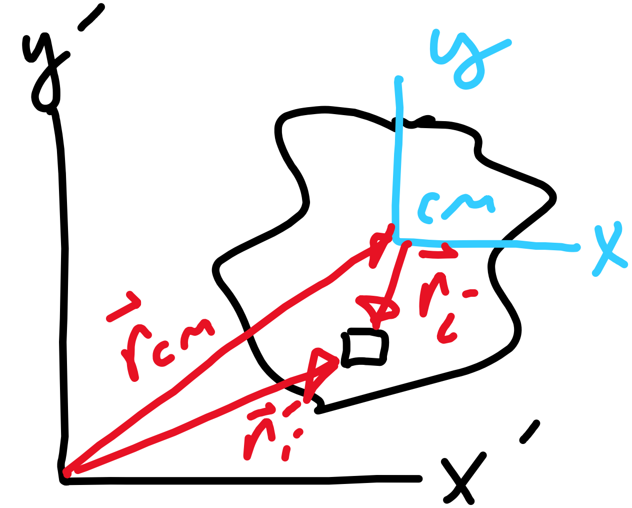
$= v_{cm}^2 + v_i^2 + 2\vec{v}_{cm} \cdot \vec{v}_i$

So $K = \frac{1}{2}\sum m_i v_{cm}^2 + \frac{1}{2}\sum m_i v_i^2 +$

Kinetic energy



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So $K = \frac{1}{2}\sum m_i v_{cm}^2 + \frac{1}{2}\sum m_i v_i^2 + \vec{v}_{cm}\cdot\sum m_i \vec{v}_i$

Kinetic energy

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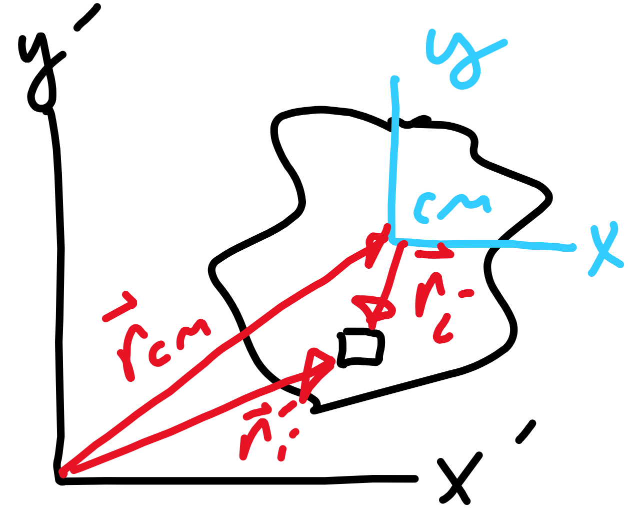

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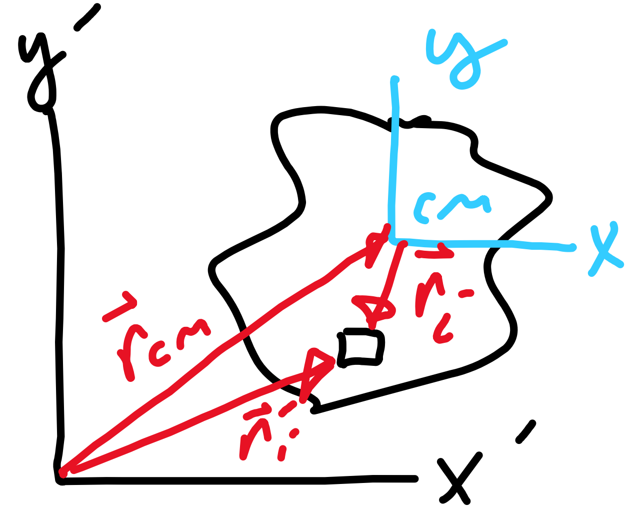
$= v_{cm}^2 + v_i^2 + 2\vec{v}_{cm} \cdot \vec{v}_i$

So $K = \frac{1}{2} \sum m_i v_{cm}^2 + \frac{1}{2} \sum m_i v_i^2 + \vec{v}_{cm} \cdot \sum m_i \vec{v}_i$

But $\frac{1}{2} \sum m_i v_{cm}^2 = \frac{1}{2} v_{cm}^2 \sum m_i$

Kinetic energy



$$K = \frac{1}{2}\sum m_i (v_i')^2 \quad \text{But}$$

$$(v_i')^2 = \vec{v}_i' \cdot \vec{v}_i' = (\vec{v}_{cm} + \vec{v}_i)\cdot(\vec{v}_{cm} + \vec{v}_i)$$

$$= v_{cm}^2 + v_i^2 + 2\vec{v}_{cm}\cdot\vec{v}_i$$

So $\quad K = \frac{1}{2}\sum m_i v_{cm}^2 + \frac{1}{2}\sum m_i v_i^2 + \vec{v}_{cm}\cdot\sum m_i \vec{v}_i$

But $\quad \frac{1}{2}\sum m_i v_{cm}^2 = \frac{1}{2} v_{cm}^2 \sum m_i = \frac{1}{2} M v_{cm}^2$

Kinetic energy

$K = \frac{1}{2}\sum m_i (v_i')^2$ But

$(v_i')^2 = \vec{v}_i' \cdot \vec{v}_i' = (\vec{v}_{cm} + \vec{v}_i)\cdot(\vec{v}_{cm} + \vec{v}_i)$

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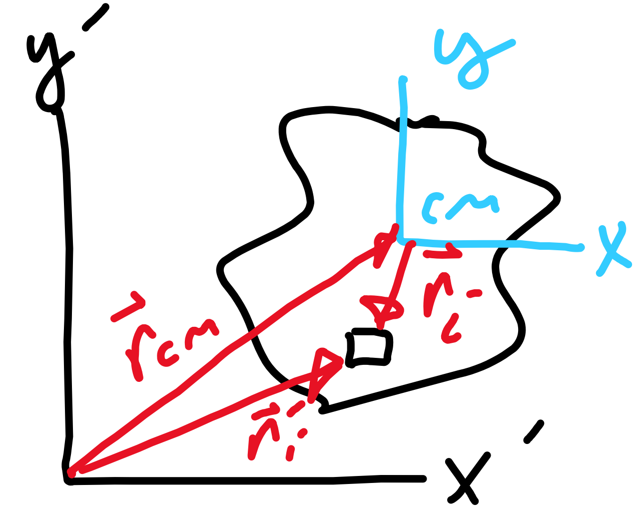


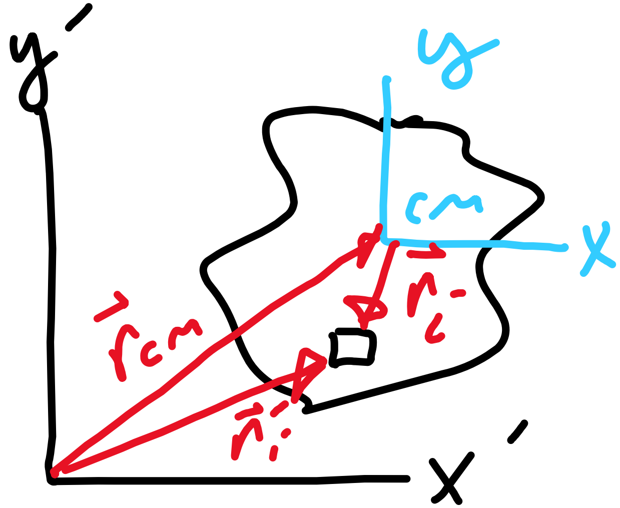
So $K = \frac{1}{2}\sum m_i v_{cm}^2 + \frac{1}{2}\sum m_i v_i^2 + \vec{v}_{cm}\cdot\sum m_i \vec{v}_i$

But $\frac{1}{2}\sum m_i v_{cm}^2 = \frac{1}{2}v_{cm}^2 \sum m_i = \frac{1}{2}M v_{cm}^2$ &

$\frac{1}{2}\sum m_i v_i^2 = \frac{1}{2}\sum m_i r_i^2 \omega^2$

Kinetic energy

$K = \frac{1}{2}\sum m_i (v_i')^2$ But



$(v_i')^2 = \vec{v}_i' \cdot \vec{v}_i' = (\vec{V}_{cm} + \vec{v}_i)\cdot(\vec{V}_{cm} + \vec{v}_i)$

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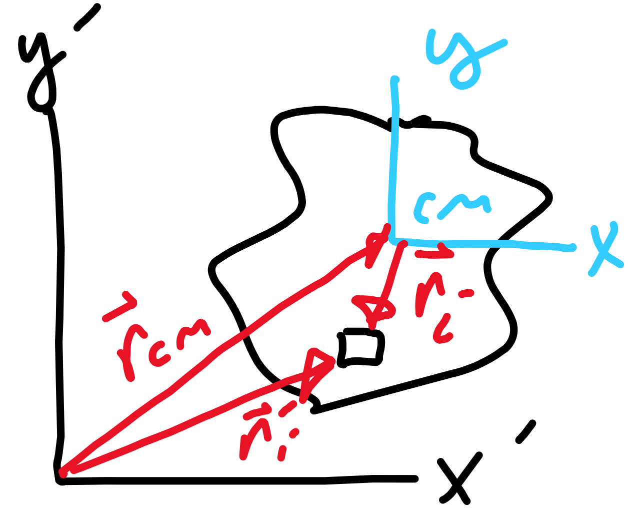
So $K = \frac{1}{2}\sum m_i V_{cm}^2 + \frac{1}{2}\sum m_i v_i^2 + \vec{V}_{cm}\cdot\sum m_i \vec{v}_i$

But $\frac{1}{2}\sum m_i V_{cm}^2 = \frac{1}{2}V_{cm}^2 \sum m_i = \frac{1}{2} M V_{cm}^2$ &

$\frac{1}{2}\sum m_i v_i^2 = \frac{1}{2}\sum m_i r_i^2 \omega^2 = \frac{\omega^2}{2}\sum m_i r_i^2$

Kinetic energy



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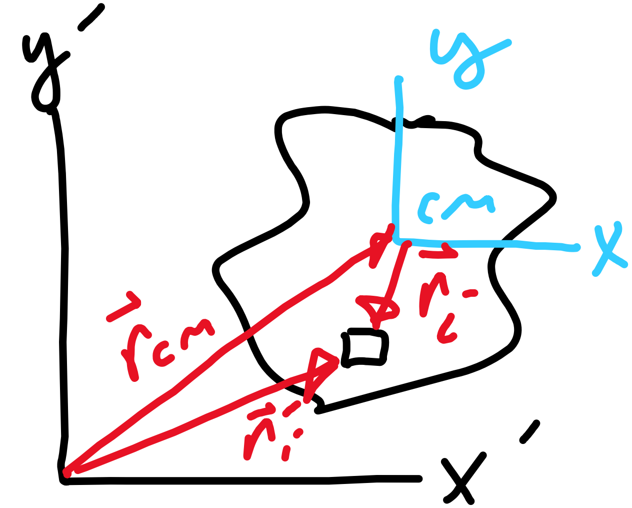
$= \frac{1}{2} I_{cm}\omega^2$ &

$\vec{V}_{cm}\cdot\sum m_i \vec{v}_i = \vec{V}_{cm}\cdot\frac{d}{dt}\sum m_i \vec{r}_i$

Kinetic energy

$K = \frac{1}{2}\sum m_i (v_i')^2$ But



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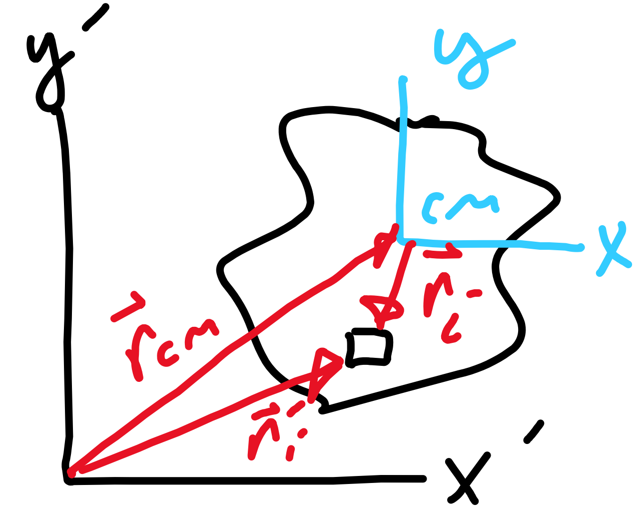
$\vec{V}_{cm}\cdot\sum m_i \vec{v}_i = \vec{V}_{cm}\cdot\frac{d}{dt}\cancel{\sum m_i \vec{r}_i}$

Kinetic energy



$$K = \frac{1}{2}\sum m_i (v_i')^2 \quad \text{But}$$

$$(v_i')^2 = \vec{v}_i' \cdot \vec{v}_i' = (\vec{V}_{cm} + \vec{v}_i)\cdot(\vec{V}_{cm} + \vec{v}_i)$$

$$= V_{cm}^2 + v_i^2 + 2\vec{V}_{cm}\cdot\vec{v}_i$$

So $K = \frac{1}{2}\sum m_i V_{cm}^2 + \frac{1}{2}\sum m_i v_i^2 + \vec{V}_{cm}\cdot\sum m_i \vec{v}_i$

But $\frac{1}{2}\sum m_i V_{cm}^2 = \frac{1}{2}V_{cm}^2 \sum m_i = \frac{1}{2}MV_{cm}^2$ &

$$\frac{1}{2}\sum m_i v_i^2 = \frac{1}{2}\sum m_i r_i^2 \omega^2 = \frac{\omega^2}{2}\sum m_i r_i^2$$

$$= \frac{1}{2}I_{cm}\omega^2 \quad \&$$

$\vec{V}_{cm}\cdot\sum m_i \vec{v}_i = \vec{V}_{cm}\cdot\frac{d}{dt}\cancel{\sum m_i \vec{r}_i}$ So

$$K = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

Kinetic energy

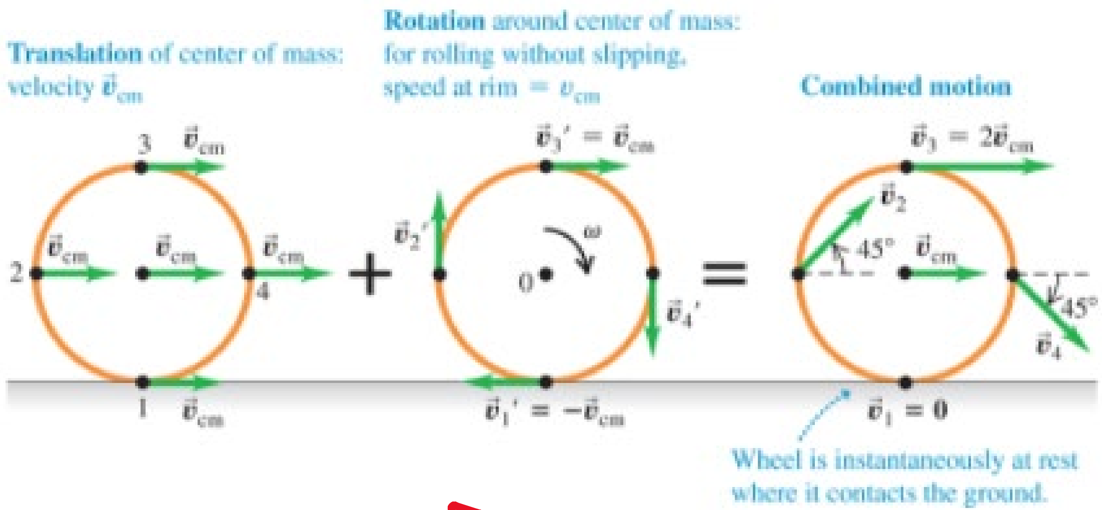
$$K = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

Kinetic energy

$$K = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

<u>If</u> rolling without slipping

$$v_{cm} = R\omega$$



Translation of center of mass: velocity $\vec{v}_{cm}$
Rotation around center of mass: for rolling without slipping, speed at rim = v_{cm}
Combined motion
3
2
1
4
0
ω
$\vec{v}_{cm}$
$\vec{v}_3{}' = \vec{v}_{cm}$
$\vec{v}_2{}'$
$\vec{v}_4{}'$
$\vec{v}_1{}' = -\vec{v}_{cm}$
$\vec{v}_3 = 2\vec{v}_{cm}$
$\vec{v}_2$
45°
$\vec{v}_4$
$\vec{v}_1 = 0$
Wheel is instantaneously at rest where it contacts the ground.

Roll
no
slip

Roll with Slipping ?

A primitive yo-yo has a massless string wrapped around a solid cylinder with mass M and radius R (Fig. 10.15). You hold the free end of the string stationary and release the cylinder from rest. The string unwinds but does not slip or stretch as the cylinder descends and rotates. Using energy considerations, find the speed v_{cm} of the cylinder's center of mass after it has descended a distance h.

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ASU

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$$K_1 + U_1 = K_2 + U_2$$

ASU

A primitive yo-yo has a massless string wrapped around a solid cylinder with mass M and radius R (Fig. 10.15). You hold the free end of the string stationary and release the cylinder from rest. The string unwinds but does not slip or stretch as the cylinder descends and rotates. Using energy considerations, find the speed v_{cm} of the cylinder's center of mass after it has descended a distance h.

$$K_1 + U_1 = K_2 + U_2 \Rightarrow$$

$$U_1 - U_2 = K_2 - K_1$$

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$$U_1 - U_2 = K_2 - \cancelto{0}{K_1} \quad , \text{ where } U_1 - U_2 = mgh$$

$$\& \quad K_2 = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

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$$I_{cm} = \frac{1}{2} M R^2$$

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$$I_{cm} = \tfrac{1}{2} M R^2 \Rightarrow K_2 = \tfrac{1}{2} M V_{cm}^2 + \tfrac{1}{2}\left(\tfrac{M}{2} R^2\right)\omega^2$$

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$$\& \text{ since } \quad R\omega = V_{cm}$$

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$$U_1 - U_2 = K_2 - \cancelto{0}{K_1}, \quad \text{where } U_1 - U_2 = mgh$$

$$\& \quad K_2 = \tfrac{1}{2} M V_{cm}^2 + \tfrac{1}{2} I_{cm} \omega^2 \quad \text{But}$$

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$$\& \text{ since } \quad R\omega = V_{cm} \quad K_2 = \left(\tfrac{1}{2} + \tfrac{1}{4}\right) M V_{cm}^2$$

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$$U_1 - U_2 = K_2 - \cancel{K_1}^{\,0}, \text{ where } U_1 - U_2 = mgh$$

$$\& \quad K_2 = \tfrac{1}{2} M V_{cm}^2 + \tfrac{1}{2} I_{cm} \omega^2 \quad \text{But}$$

$$I_{cm} = \tfrac{1}{2} M R^2 \Rightarrow K_2 = \tfrac{1}{2} M V_{cm}^2 + \tfrac{1}{2}\left(\tfrac{M}{2} R^2\right)\omega^2$$

$$\& \text{ since } \quad R\omega = V_{cm} \quad K_2 = \left(\tfrac{1}{2} + \tfrac{1}{4}\right) M V_{cm}^2$$

$$\Rightarrow \quad K_2 = \tfrac{3}{4} M V_{cm}^2$$

A primitive yo-yo has a massless string wrapped around a solid cylinder with mass M and radius R (Fig. 10.15). You hold the free end of the string stationary and release the cylinder from rest. The string unwinds but does not slip or stretch as the cylinder descends and rotates. Using energy considerations, find the speed v_{cm} of the cylinder's center of mass after it has descended a distance h.

$$K_1 + U_1 = K_2 + U_2 \Rightarrow$$

$$U_1 - U_2 = K_2 - \cancelto{0}{K_1}, \quad \text{where } U_1 - U_2 = mgh$$

$$\& \quad K_2 = \tfrac{1}{2} M V_{cm}^2 + \tfrac{1}{2} I_{cm} \omega^2 \quad \text{But}$$

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$$\Rightarrow \quad K_2 = \tfrac{3}{4} M V_{cm}^2 \quad \text{Now} \quad mgh = \tfrac{3}{4} M V_{cm}^2$$

$$\Rightarrow \quad \boxed{V_{cm} = \sqrt{\frac{4gh}{3}}}$$

ASU

$$\sum \vec{F}_{EXT} = M\vec{a}_{cm}$$

$$\sum \vec{F}_{EXT} = M\vec{a}_{cm}$$

$$\Updownarrow$$

$$\sum \vec{\tau} = I_{cm}\vec{\alpha}$$

For the primitive yo-yo in Example 10.4 (Fig. 10.18a), find the downward acceleration of the cylinder and the tension in the string.

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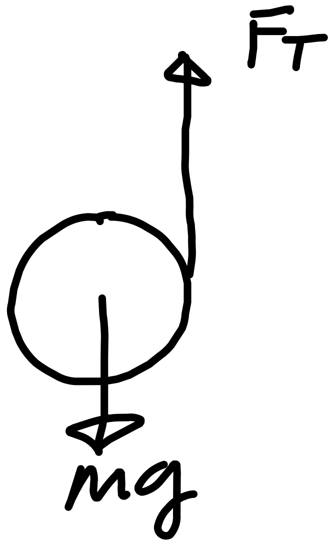
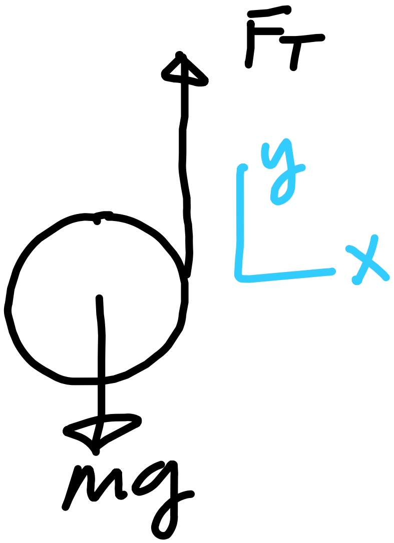
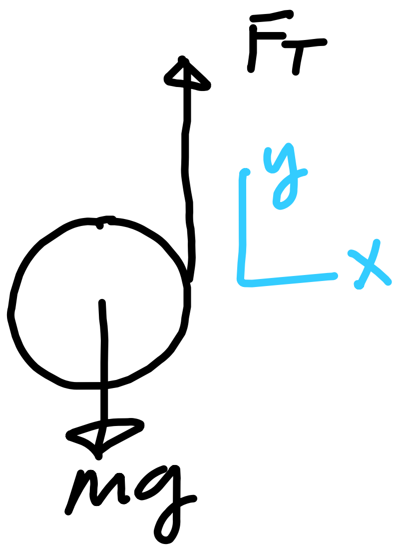


For the primitive yo-yo in Example 10.4 (Fig. 10.18a), find the downward acceleration of the cylinder and the tension in the string.



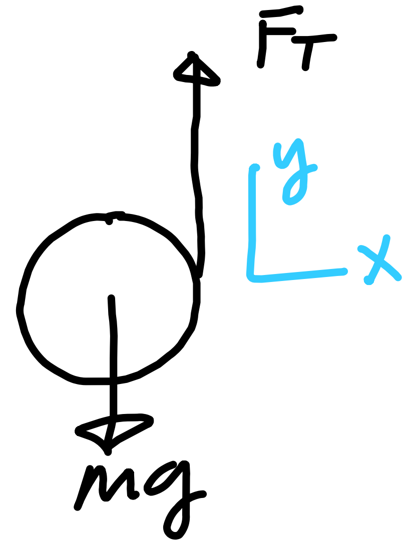
For the primitive yo-yo in Example 10.4 (Fig. 10.18a), find the downward acceleration of the cylinder and the tension in the string.

$$\Sigma F_y = m a_y$$



For the primitive yo-yo in Example 10.4 (Fig. 10.18a), find the downward acceleration of the cylinder and the tension in the string.

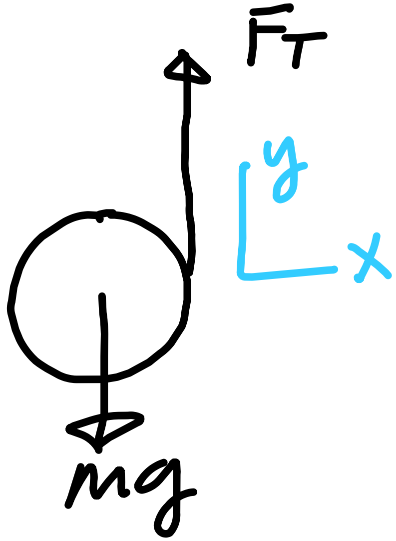
$$\sum F_y = ma_y \Rightarrow mg - F_T = ma_y$$



For the primitive yo-yo in Example 10.4 (Fig. 10.18a), find the downward acceleration of the cylinder and the tension in the string.

$$\sum F_y = ma_y \Rightarrow mg - F_T = ma_y$$

Also $\sum \tau_z = I_{cm}\alpha_z$



For the primitive yo-yo in Example 10.4 (Fig. 10.18a), find the downward acceleration of the cylinder and the tension in the string.

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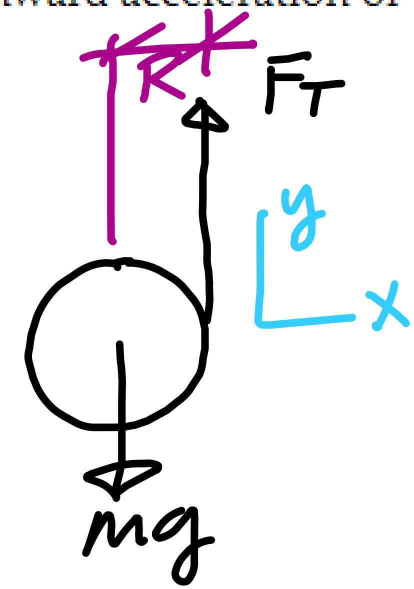

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$$\Sigma F_y = ma_y \Rightarrow mg - F_T = ma_y$$

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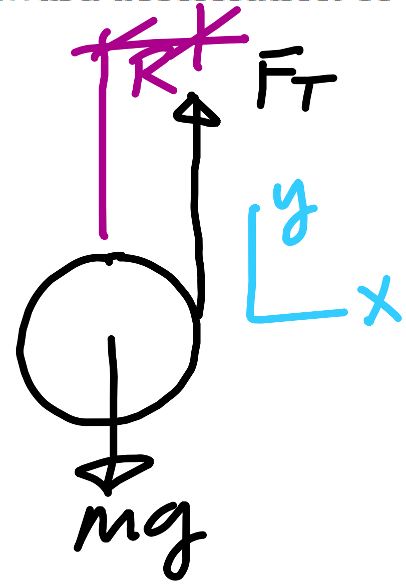
But $a_y = \alpha R$



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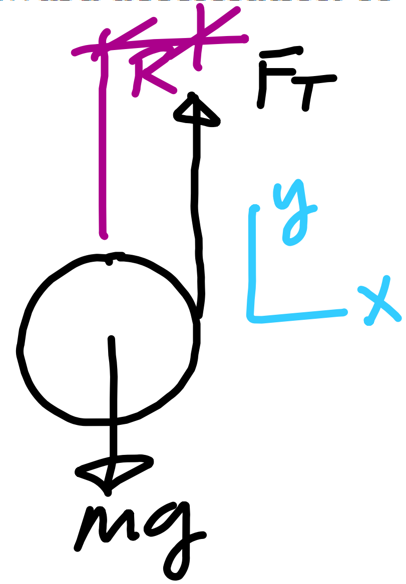
Also $\sum \tau_z = I_{cm}\alpha_z \Rightarrow F_T R = I_{cm}\alpha$

But $a_y = \alpha R$ & $I_{cm} = \frac{1}{2}MR^2$



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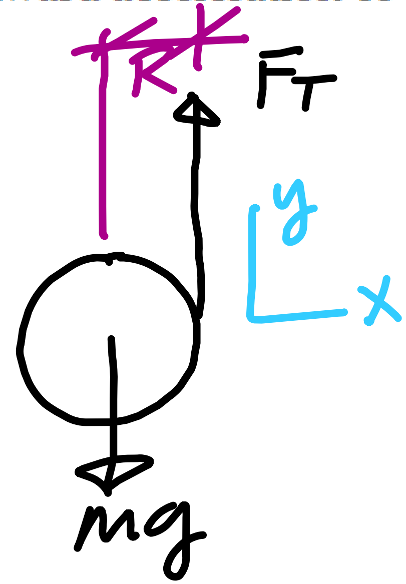

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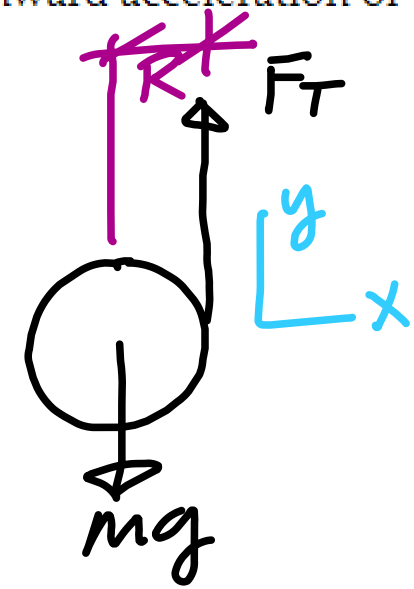
$$\Sigma F_y = m a_y \Rightarrow mg - F_T = m a_y$$

Also $\Sigma \tau_z = I_{cm} \alpha_z \Rightarrow F_T R = I_{cm} \alpha$

But $a_y = \alpha R$ & $I_{cm} = \frac{1}{2} M R^2$

So $F_T R = \frac{1}{2} M R^2 \alpha = \frac{M}{2} R a_y$



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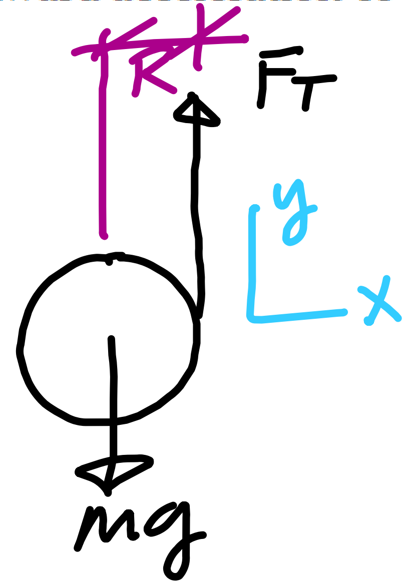

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So $F_T R = \frac{1}{2}MR^2\alpha = \frac{M}{2} R a_y$

$$\Rightarrow F_T = \frac{M}{2} a_y$$

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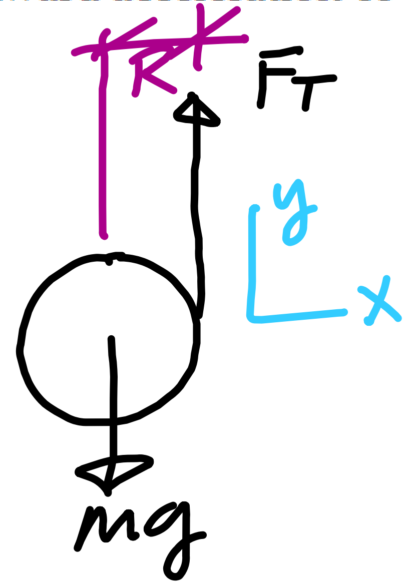
So $F_T R = \frac{1}{2}MR^2\alpha = \frac{M}{2}R a_y$

$\Rightarrow F_T = \frac{M}{2}a_y$ Sub into



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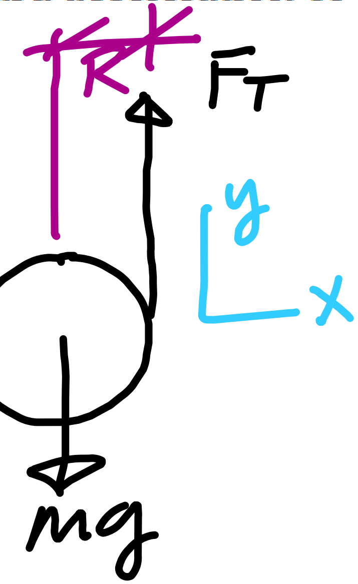

$\Sigma F_y = ma_y \Rightarrow \boxed{mg - F_T = ma_y}$

Also $\Sigma \tau_z = I_{cm}\alpha_z \Rightarrow F_T R = I_{cm}\alpha$

But $a_y = \alpha R$ & $I_{cm} = \frac{1}{2}MR^2$

So $F_T R = \frac{1}{2}MR^2\alpha = \frac{M}{2}R a_y$

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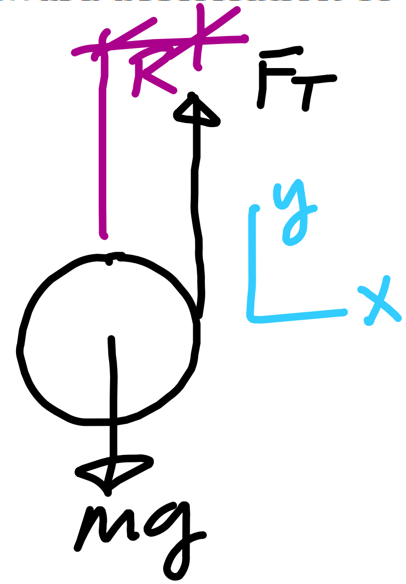
get $mg - \frac{M}{2}a_y = ma_y$

For the primitive yo-yo in Example 10.4 (Fig. 10.18a), find the downward acceleration of the cylinder and the tension in the string.

$$\Sigma F_y = m a_y \Rightarrow mg - F_T = m a_y$$



Also $\Sigma \tau_z = I_{cm} \alpha_z \Rightarrow F_T R = I_{cm} \alpha$

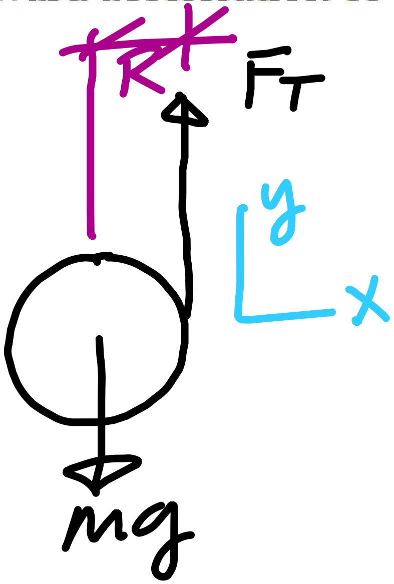
But $a_y = \alpha R$ & $I_{cm} = \frac{1}{2} M R^2$

So $F_T R = \frac{1}{2} M R^2 \alpha = \frac{M}{2} R a_y$

$\Rightarrow F_T = \frac{M}{2} a_y$ Sub into to

get $\cancel{m} g - \cancel{\frac{M}{2}} a_y = \cancel{m} a_y$

For the primitive yo-yo in Example 10.4 (Fig. 10.18a), find the downward acceleration of the cylinder and the tension in the string.



$$\Sigma F_y = m a_y \Rightarrow \underline{mg - F_T = m a_y}$$

Also $\Sigma \tau_z = I_{cm} \alpha_z \Rightarrow F_T R = I_{cm} \alpha$

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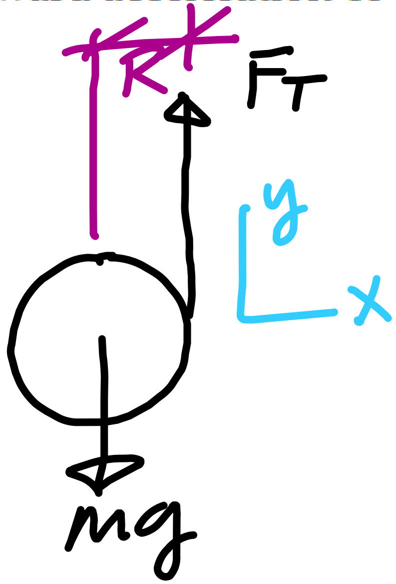

$$\Sigma F_y = ma_y \Rightarrow \boxed{mg - F_T = ma_y}$$

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$\Rightarrow F_T = \frac{M}{2}a_y$ Sub into to

get $\cancel{m}g - \frac{\cancel{m}}{2}a_y = \cancel{m}a_y \Rightarrow g - \frac{1}{2}a_y = a_y$

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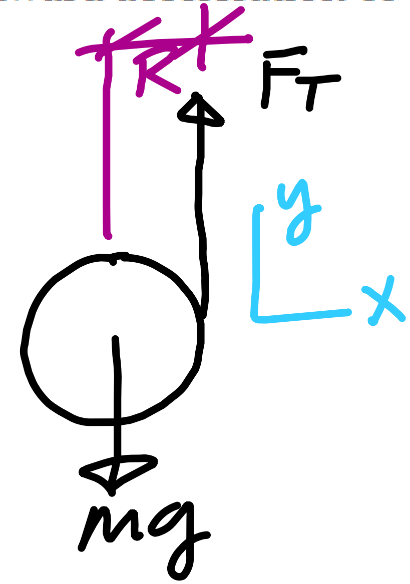

$\sum F_y = ma_y \Rightarrow mg - F_T = ma_y$

Also $\sum \tau_z = I_{cm}\alpha_z \Rightarrow F_T R = I_{cm}\alpha$

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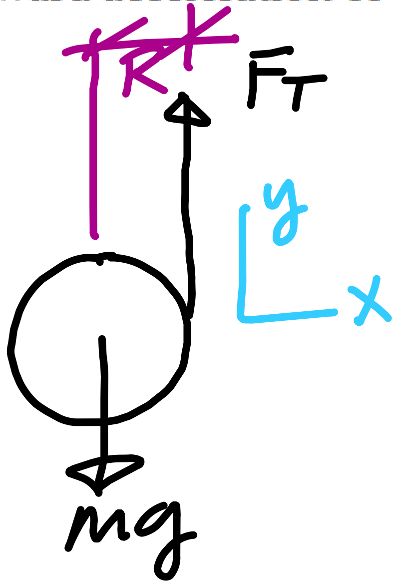
$\Rightarrow F_T = \frac{M}{2} a_y$ Sub into to

get $\cancel{m}g - \frac{\cancel{M}}{2}a_y = \cancel{m}a_y \Rightarrow g - \frac{1}{2}a_y = a_y$

$\Rightarrow g = \frac{3}{2}a_y \Rightarrow \boxed{a_y = \frac{2}{3}g}$

For the primitive yo-yo in Example 10.4 (Fig. 10.18a), find the downward acceleration of the cylinder and the tension in the string.



$\Sigma F_y = m a_y \Rightarrow \underbrace{mg - F_T = m a_y}$

Also $\Sigma \tau_z = I_{cm} \alpha_z \Rightarrow F_T R = I_{cm} \alpha$

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$\Rightarrow \underbrace{F_T = \frac{M}{2} a_y}$ Sub into to

get $\cancel{m} g - \frac{\cancel{M}}{2} a_y = \cancel{m} a_y \Rightarrow g - \frac{1}{2} a_y = a_y$

$\Rightarrow g = \frac{3}{2} a_y \Rightarrow \boxed{a_y = \frac{2}{3} g}$

Sub into

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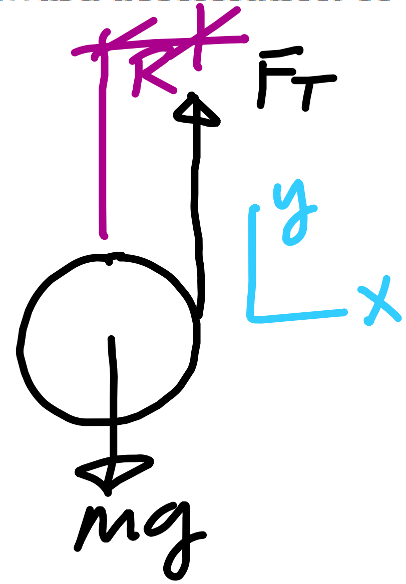

$\Sigma F_y = ma_y \Rightarrow mg - F_T = ma_y$

Also $\Sigma \tau_z = I_{cm}\alpha_z \Rightarrow F_T R = I_{cm}\alpha$

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So $F_T R = \frac{1}{2}MR^2\alpha = \frac{M}{2}R a_y$

$\Rightarrow F_T = \frac{M}{2}a_y$ Sub into to

get $\cancel{m}g - \frac{\cancel{M}}{2}a_y = \cancel{m}a_y \Rightarrow g - \frac{1}{2}a_y = a_y$

$\Rightarrow g = \frac{3}{2}a_y \Rightarrow \boxed{a_y = \frac{2}{3}g}$

Sub into to get $\boxed{F_T = \frac{mg}{3}}$

A bowling ball of mass M rolls without slipping down a ramp that is inclined at an angle β to the horizontal (Fig. 10.19a). What are the ball's acceleration and the magnitude of the friction force on the ball? Treat the ball as a uniform solid sphere, ignoring the finger holes.

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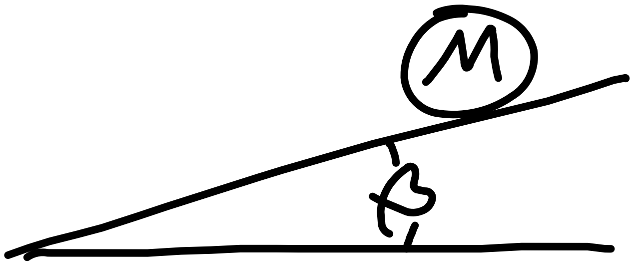

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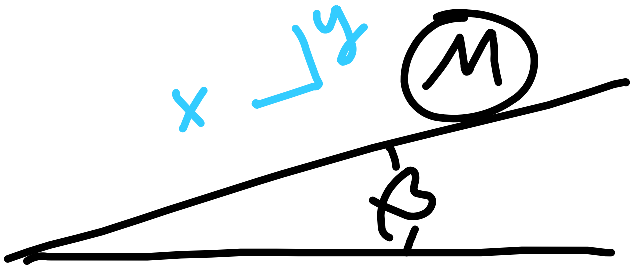

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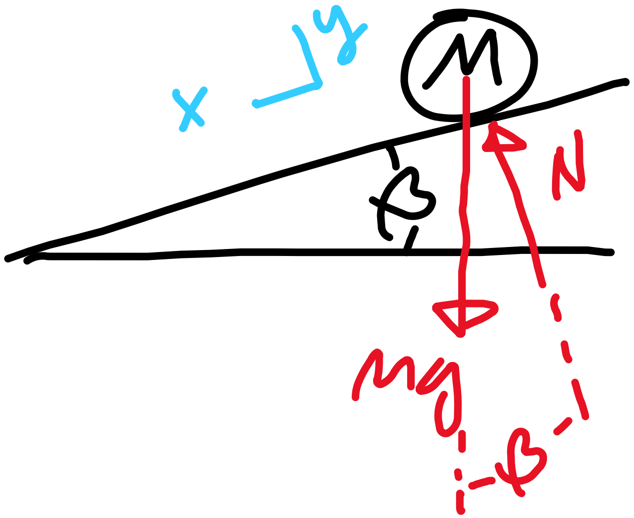

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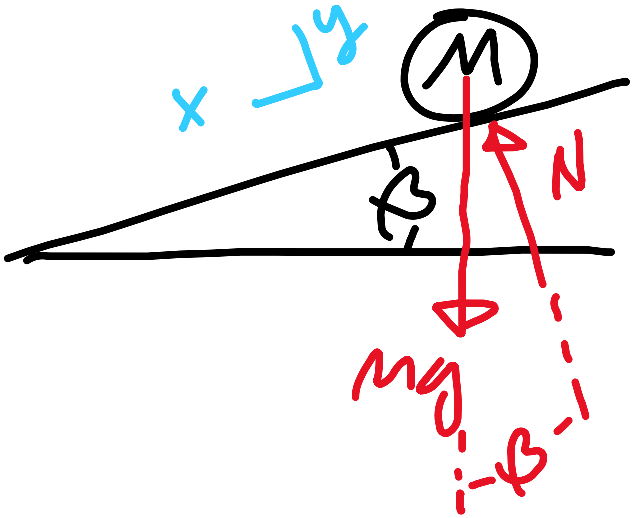

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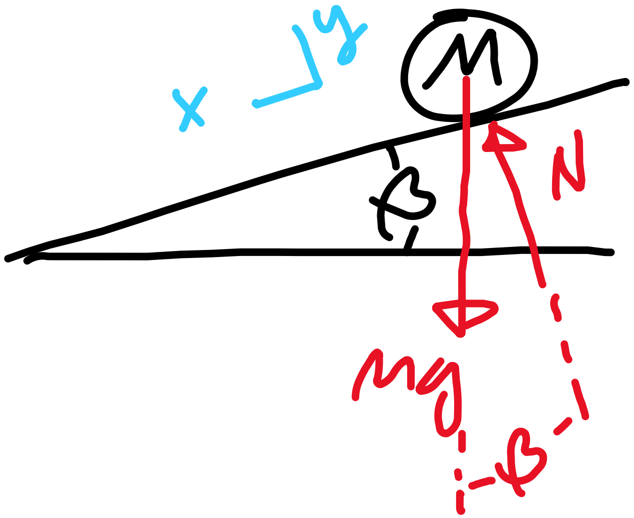

$$F_F = N\mu_s$$

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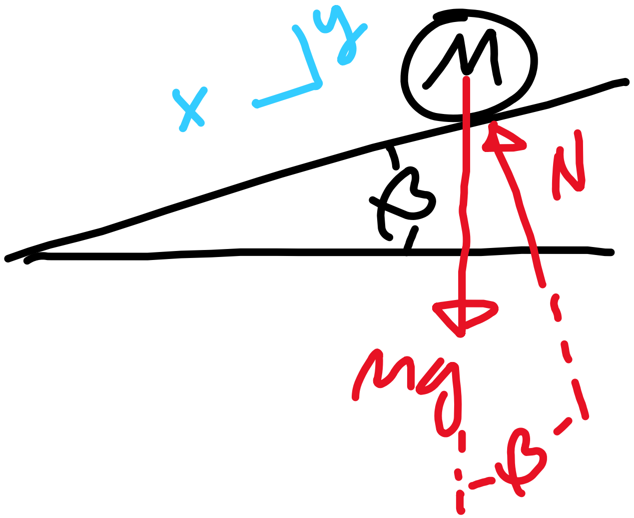
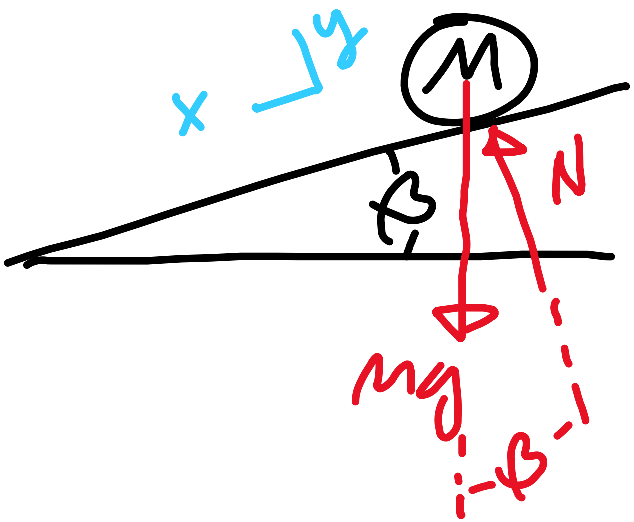


$F_F = N\mu_s$

$\sum F_y = \cancel{0}$

A bowling ball of mass M rolls without slipping down a ramp that is inclined at an angle β to the horizontal (Fig. 10.19a). What are the ball's acceleration and the magnitude of the friction force on the ball? Treat the ball as a uniform solid sphere, ignoring the finger holes.

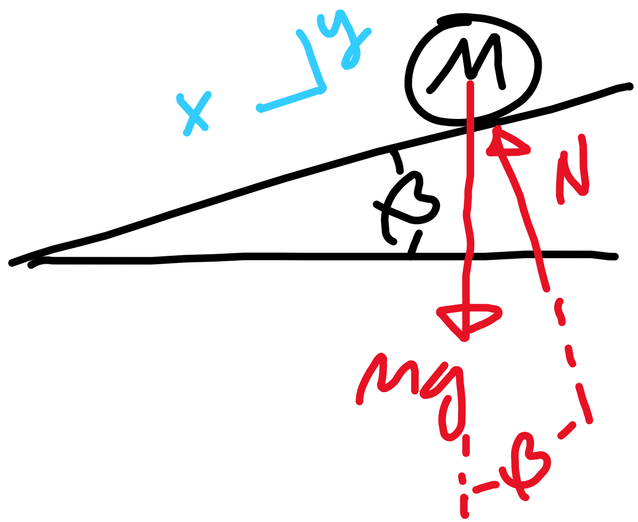


$F_F = N\mu_s$

$\sum F_y = 0 \Rightarrow$

$N = mg\cos\beta$

A bowling ball of mass M rolls without slipping down a ramp that is inclined at an angle β to the horizontal (Fig. 10.19a). What are the ball's acceleration and the magnitude of the friction force on the ball? Treat the ball as a uniform solid sphere, ignoring the finger holes.



$$F_F = N\mu_s$$

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$$\Sigma F_x = ma$$

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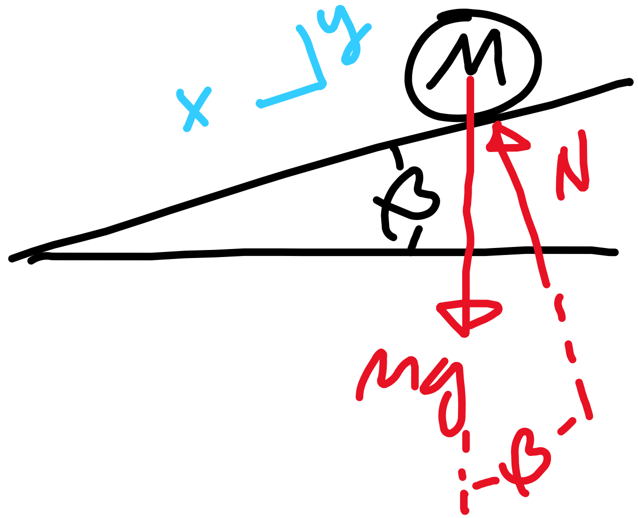

$F_F = N\mu_s$

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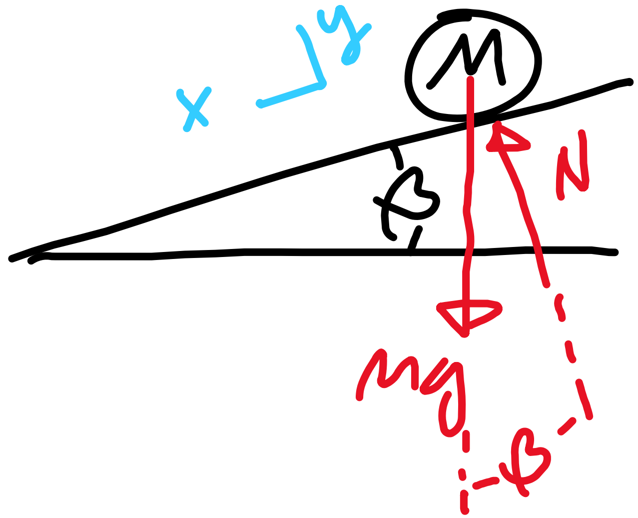

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$$N = mg\cos\beta$$

$$\Sigma F_x = ma \Rightarrow mg\sin\beta - Mg\cos\beta\,\mu_s = ma$$

$$\Rightarrow g[\sin\beta - \mu_s\cos\beta] = a$$

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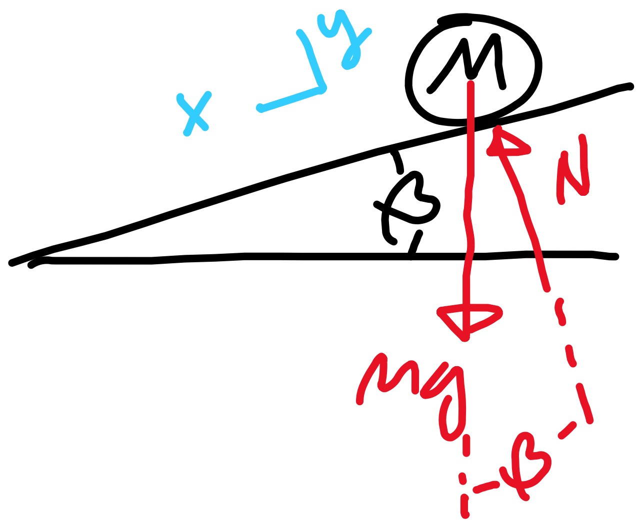
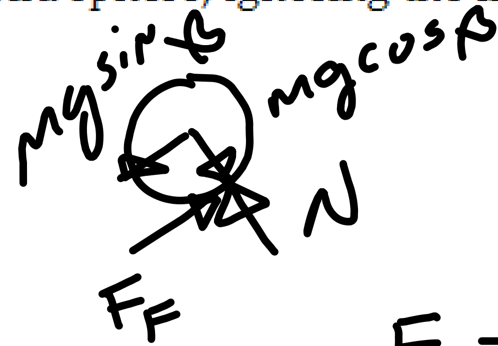




$F_F = N\mu_s$

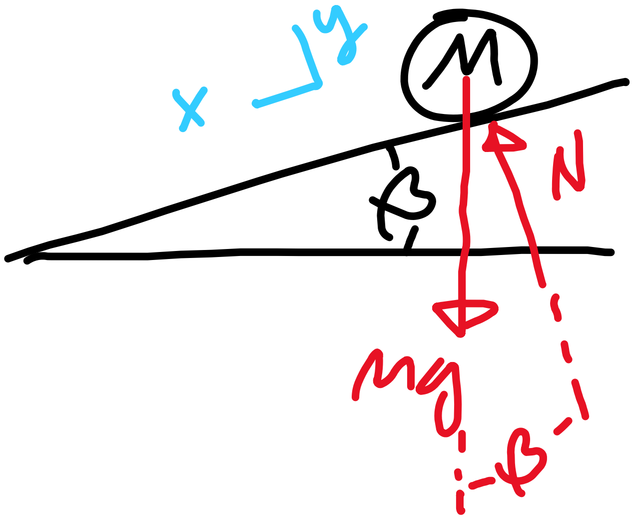
& $\sum F_y = 0 \Rightarrow$

$N = mg\cos\beta$

& $\sum F_x = ma \Rightarrow mg\sin\beta - Mg\cos\beta\,\mu_s = ma$

$\Rightarrow g[\sin\beta - \mu_s\cos\beta] = a$ & $\sum\tau = I\alpha$

A bowling ball of mass M rolls without slipping down a ramp that is inclined at an angle β to the horizontal (Fig. 10.19a). What are the ball's acceleration and the magnitude of the friction force on the ball? Treat the ball as a uniform solid sphere, ignoring the finger holes.



$F_F = N\mu_s$

$\cancel{\&}$ $\Sigma F_y = \cancel{0} \Rightarrow$

$N = mg\cos\beta$

$\& \quad \Sigma F_x = ma \Rightarrow mg\sin\beta - Mg\cos\beta\,\mu_s = ma$

$\Rightarrow \quad g[\sin\beta - \mu_s\cos\beta] = a \quad \& \ \Sigma\tau = I\alpha \Rightarrow$

$-F_F R = -\frac{2}{5}MR^2\alpha$

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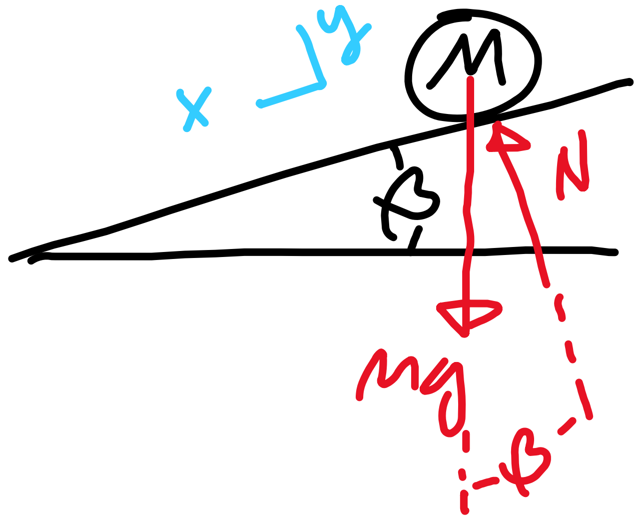

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$-F_F R = -\frac{2}{5}MR^2\alpha \Rightarrow Rmg\mu_s\cos\beta = \frac{2}{5}mRa$

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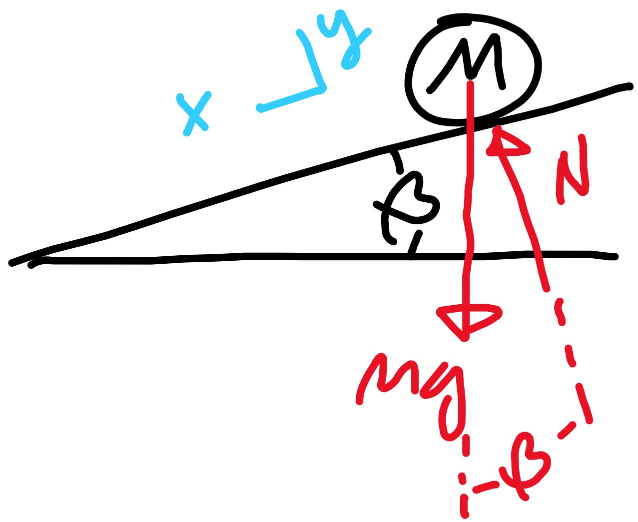

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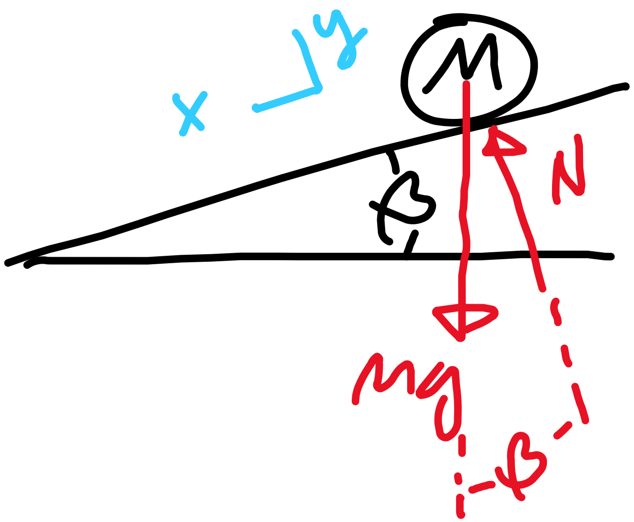

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$-F_F R = -\frac{2}{5}MR^2\alpha \Rightarrow \cancel{R}\cancel{m}g\mu_s\cos\beta = \frac{2}{5}\cancel{m}\cancel{R}a$ Sub

into

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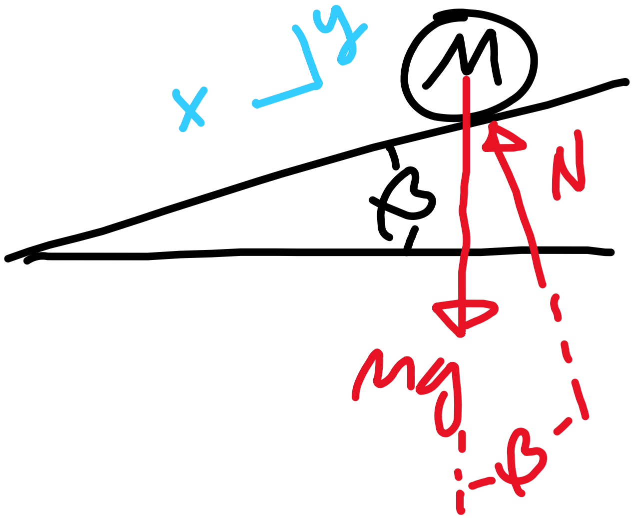

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& $\Sigma F_y = 0 \Rightarrow$
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& $\Sigma F_x = ma \Rightarrow mg\sin\beta - Mg\cos\beta\,\mu_s = ma$

$\Rightarrow \underline{g[\sin\beta - \mu_s\cos\beta] = a}$ & $\Sigma\tau = I\alpha \Rightarrow$

$-F_F R = -\frac{2}{5}MR^2\alpha \Rightarrow \cancel{R}\,\cancel{m}g\mu_s\cos\beta = \frac{2}{5}\cancel{m}\cancel{R}a$ Sub into to get $g\sin\beta - \frac{2}{5}a = a$

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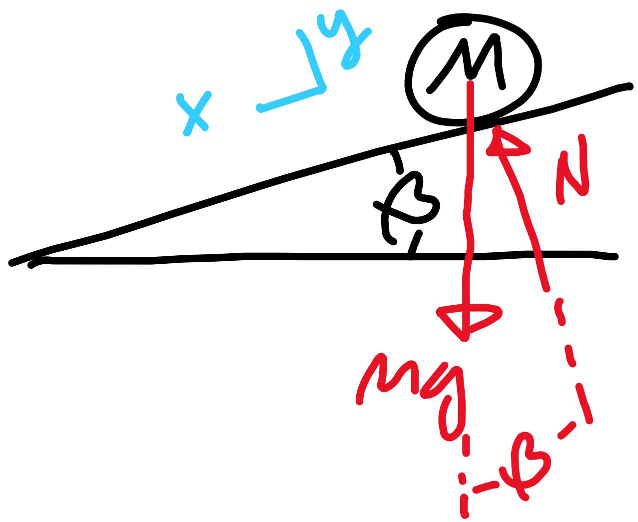

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$\boxed{\frac{5}{7}g\sin\beta = a}$

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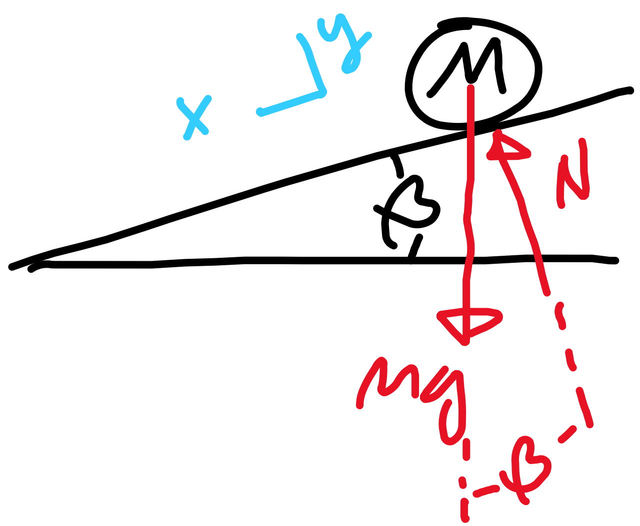

$F_F = N\mu_s$

& $\sum F_y = 0 \Rightarrow N = mg\cos\beta$

& $\sum F_x = ma \Rightarrow mg\sin\beta - Mg\cos\beta\,\mu_s = ma$

$\Rightarrow g[\sin\beta - \mu_s\cos\beta] = a$ & $\sum \tau = I\alpha \Rightarrow$

$\boxed{-F_F R = -\frac{2}{5}MR^2\alpha} \Rightarrow \cancel{R}\cancel{m}g\mu_s\cos\beta = \frac{2}{5}\cancel{M}\cancel{R}a$ Sub into to get $g\sin\beta - \frac{2}{5}a = a \Rightarrow$

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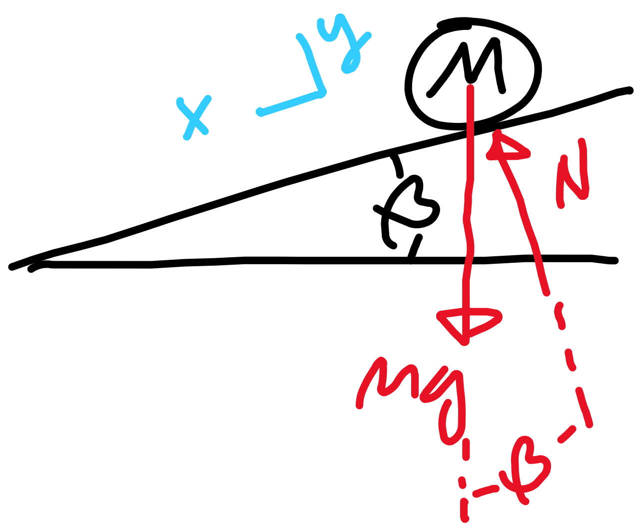
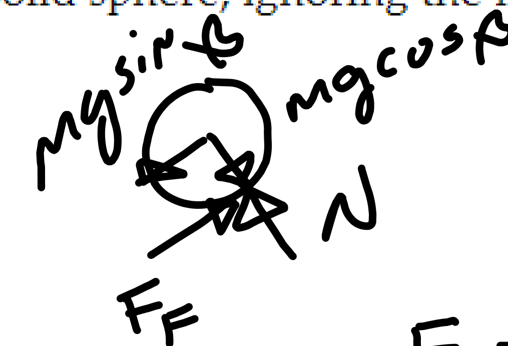
A bowling ball of mass M rolls without slipping down a ramp that is inclined at an angle β to the horizontal (Fig. 10.19a). What are the ball's acceleration and the magnitude of the friction force on the ball? Treat the ball as a uniform solid sphere, ignoring the finger holes.





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& $\Sigma F_y = 0 \Rightarrow$

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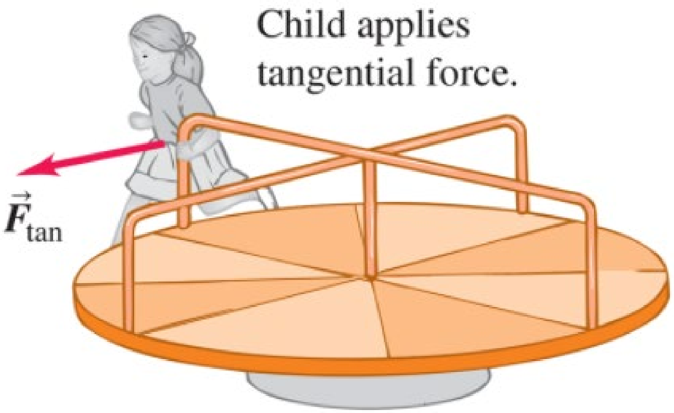
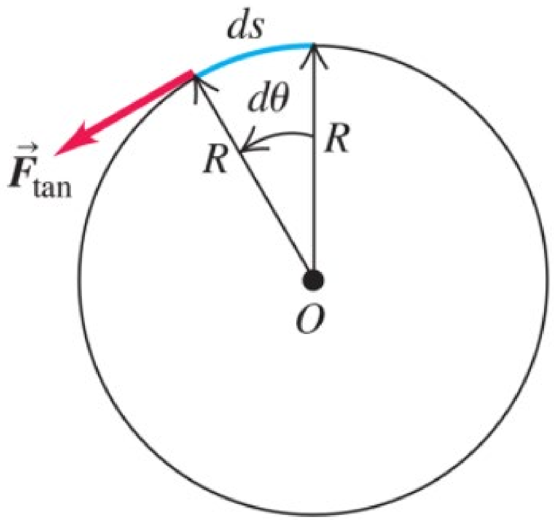
$\boxed{\frac{5}{7}g\sin\beta = a}$ & $\boxed{F = \frac{2}{7}Mg\sin\beta}$

(a)



(b) Overhead view of merry-go-round

$dW = F_{tan} R\, d\theta$



$$\Rightarrow \quad dW = \tau \, d\theta$$

$$\Rightarrow \quad dW = \tau \, d\theta$$

$$\Rightarrow \quad W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

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If $\tau = \text{constant}$ $\quad W = \tau \Delta\theta$

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$$\Rightarrow \quad W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

If $\tau = \text{constant}$ $\quad W = \tau \Delta\theta$

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta = \int_{\theta_1}^{\theta_2} I\alpha \, d\theta$$

$$\Rightarrow \quad dW = \tau \, d\theta$$

$$\Rightarrow \quad W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

If $\tau = \text{constant}$ $\quad W = \tau \Delta\theta$

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta = \int_{\theta_1}^{\theta_2} I \alpha \, d\theta$$ but

$$\alpha = \omega \frac{d\omega}{d\theta}$$

$$\Rightarrow \quad dW = \tau\, d\theta$$

$$\Rightarrow \quad W = \int_{\theta_1}^{\theta_2} \tau\, d\theta$$

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$$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta = \int_{\theta_1}^{\theta_2} I \alpha\, d\theta$$ but

$\alpha = \omega \dfrac{d\omega}{d\theta}$ so $W = \displaystyle\int_{\omega_1}^{\omega_2} I \omega\, d\omega$

$$\Rightarrow \quad dW = \tau\, d\theta$$

$$\Rightarrow \quad W = \int_{\theta_1}^{\theta_2} \tau\, d\theta$$

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$$\Rightarrow \quad W = I \int_{\omega_1}^{\omega_2} \omega\, d\omega$$

$$\Rightarrow \quad dW = \tau\, d\theta$$

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$\alpha = \omega \frac{d\omega}{d\theta}$ so $W = \int_{\omega_1}^{\omega_2} I \omega\, d\omega$

$$\Rightarrow \quad W = I \int_{\omega_1}^{\omega_2} \omega\, d\omega = I \tfrac{1}{2}(\omega_2^2 - \omega_1^2)$$

$$\Rightarrow \quad dW = \tau\, d\theta$$

$$\Rightarrow \quad W = \int_{\theta_1}^{\theta_2} \tau\, d\theta$$

If $\tau = \text{constant}$ $\quad W = \tau \Delta\theta$

$$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta = \int_{\theta_1}^{\theta_2} I \alpha\, d\theta \quad \text{but}$$

$\alpha = \omega \dfrac{d\omega}{d\theta}$ so $W = \displaystyle\int_{\omega_1}^{\omega_2} I \omega\, d\omega$

$$\Rightarrow \quad W = I \int_{\omega_1}^{\omega_2} \omega\, d\omega = I \tfrac{1}{2}(\omega_2^2 - \omega_1^2)$$

$$W = \tfrac{1}{2} I \omega_2^2 - \tfrac{1}{2} I \omega_1^2$$

$$\Rightarrow \quad dW = \tau \, d\theta$$

$$\Rightarrow \quad W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

If $\tau = \text{constant}$ $\quad W = \tau \Delta\theta$

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta = \int_{\theta_1}^{\theta_2} I \alpha \, d\theta \quad \text{but}$$

$$\alpha = \omega \frac{d\omega}{d\theta} \quad \text{so} \quad W = \int_{\omega_1}^{\omega_2} I \omega \, d\omega$$

$$\Rightarrow \quad W = I \int_{\omega_1}^{\omega_2} \omega \, d\omega = I \tfrac{1}{2}(\omega_2^2 - \omega_1^2)$$

$$W = \tfrac{1}{2} I \omega_2^2 - \tfrac{1}{2} I \omega_1^2 \quad \Rightarrow$$

$$W = \quad K_2 \quad - \quad K_1$$

Power

$P = \tau_{ell}$

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

An electric motor exerts a constant $10\ \mathrm{N} \cdot \mathrm{m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N \cdot m$

An electric motor exerts a constant $10 \text{ N} \cdot \text{m}$ torque on a grindstone, which has a moment of inertia of $2.0 \text{ kg}{\cdot}\text{m}^2$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10 N \cdot m$, $I = 2.0 \, kg \cdot m^2$

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N \cdot m$, $I = 2.0\,kg \cdot m^2$, $K_1 = 0$

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N \cdot m$, $I = 2.0\ kg \cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$:

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in $8.0\ \mathrm{s}$ and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N\cdot m$, $I = 2.0\,kg\cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N\cdot m$, $I = 2.0\,kg\cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ 10

$\alpha = \frac{\tau}{I}$

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg \cdot m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N \cdot m$, $I = 2.0\,kg \cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ 10

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2$

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N\cdot m$, $I = 2.0\ kg\cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$

$\Delta\theta = \frac{\tau}{2I}(\Delta t)^2$

ASU

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in $8.0\ \mathrm{s}$ and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N\cdot m\ ,\ I = 2.0\,kg\cdot m^2\ ,\ K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$

$\Delta\theta = \frac{\tau}{2I}(\Delta t)^2$ & $W = \int \tau\, d\theta$

An electric motor exerts a constant $10\ \mathrm{N\cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

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Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$

$\Delta\theta = \frac{\tau}{2I}(\Delta t)^2$ & $W = \int \tau\, d\theta = \tau\Delta\theta$

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg \cdot m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N \cdot m$, $I = 2.0\ kg \cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$

$\Delta\theta = \frac{\tau}{2I}(\Delta t)^2$ & $W = \int \tau\, d\theta = \tau\Delta\theta$

So $W = \frac{(\tau\Delta t)^2}{2I}$

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N \cdot m$, $I = 2.0\,kg \cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$

$\Delta\theta = \frac{\tau}{2I}(\Delta t)^2$ & $W = \int \tau\, d\theta = \tau \Delta\theta$

So $W = \frac{(\tau \Delta t)^2}{2I} = \frac{(10*8)^2}{2*2} J$

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N\cdot m$, $I = 2.0\,kg\cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$

$\Delta\theta = \frac{\tau}{2I}(\Delta t)^2$ & $W = \int \tau\, d\theta = \tau\Delta\theta$

So $W = \frac{(\tau\Delta t)^2}{2I} = \frac{(10*8)^2}{2*2} J = \frac{6400}{4} J$

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg \cdot m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in $8.0\ \mathrm{s}$ and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N \cdot m$, $I = 2.0\,kg \cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$

$\Delta\theta = \frac{\tau}{2I}(\Delta t)^2$ & $W = \int \tau\, d\theta = \tau\Delta\theta$

So $W = \frac{(\tau\Delta t)^2}{2I} = \frac{(10*8)^2}{2*2} J = \frac{6400}{4} J$

$\Rightarrow$ $\boxed{W = 1600\ J}$

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg \cdot m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\ N \cdot m$, $I = 2.0\ kg \cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$

$\Delta\theta = \frac{\tau}{2I}(\Delta t)^2$ & $W = \int \tau\, d\theta = \tau \Delta\theta$

So $W = \frac{(\tau \Delta t)^2}{2I} = \frac{(10 * 8)^2}{2 * 2} J = \frac{6400}{4} J$

$\Rightarrow$ $\boxed{W = 1600\ J}$ Find K_2:

ASU

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg \cdot m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N\cdot m$, $I = 2.0\,kg\cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$

$\Delta\theta = \frac{\tau}{2I}(\Delta t)^2$ & $W = \int \tau\, d\theta = \tau\Delta\theta$

So $W = \frac{(\tau\Delta t)^2}{2I} = \frac{(10*8)^2}{2*2} J = \frac{6400}{4} J$

$\Rightarrow$ $\boxed{W = 1600\,J}$ Find K_2: $W = K_2 - \cancel{K_1}$ 0

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg \cdot m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\ \mathrm{N \cdot m}$, $I = 2.0\ \mathrm{kg \cdot m^2}$, $K_1 = 0$

Find W after $\Delta t = 8\,s$: $\tau = I\alpha$ so

$$\alpha = \frac{\tau}{I} \quad \& \quad \Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$$

$$\Delta\theta = \frac{\tau}{2I}(\Delta t)^2 \quad \& \quad W = \int \tau\, d\theta = \tau\Delta\theta$$

so $$W = \frac{(\tau\Delta t)^2}{2I} = \frac{(10*8)^2}{2*2}\,J = \frac{6400}{4}\,J$$

$\Rightarrow$ $\boxed{W = 1600\,J}$ Find K_2 : $W = K_2 - \cancel{K_1}$ 0

so $\boxed{K_2 = 1600\,J}$

An electric motor exerts a constant $10 \text{ N} \cdot \text{m}$ torque on a grindstone, which has a moment of inertia of $2.0 \text{ kg} \cdot \text{m}^2$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10 N \cdot m$, $I = 2.0 \, kg \cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$

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So $W = \frac{(\tau \Delta t)^2}{2I} = \frac{(10 * 8)^2}{2 * 2} J = \frac{6400}{4} J$

$\Rightarrow$ $\boxed{W = 1600 \, J}$ Find K_2: $W = K_2 - \cancel{K_1}$

So $\boxed{K_2 = 1600 J}$ Find P_{ave}:

An electric motor exerts a constant $10\ \mathrm{N\cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N\cdot m$, $I = 2.0\,kg\cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$

$\Delta\theta = \frac{\tau}{2I}(\Delta t)^2$ & $W = \int \tau\, d\theta = \tau\Delta\theta$

So $W = \frac{(\tau\Delta t)^2}{2I} = \frac{(10*8)^2}{2*2}J = \frac{6400}{4}J$

$\Rightarrow \boxed{W = 1600\,J}$ Find K_2: $W = K_2 - \cancel{K_1}$

So $\boxed{K_2 = 1600\,J}$ Find P_{ave}:

$P_{ave} = \frac{\Delta W}{\Delta t}$

ASU

An electric motor exerts a constant $10\ \mathrm{N \cdot m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \mathrm{kg{\cdot}m^2}$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N\cdot m$, $I = 2.0\,kg\cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

$\alpha = \frac{\tau}{I}$ & $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow$

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So $W = \frac{(\tau\Delta t)^2}{2I} = \frac{(10*8)^2}{2*2}J = \frac{6400}{4}J$

$\Rightarrow$ $\boxed{W = 1600\,J}$ Find K_2: $W = K_2 - \cancel{K_1}$

So $\boxed{K_2 = 1600J}$ Find P_{AVE}:

$P_{AVE} = \frac{\Delta W}{\Delta t} = \frac{1600J}{8s}$

An electric motor exerts a constant $10\ \text{N}\cdot\text{m}$ torque on a grindstone, which has a moment of inertia of $2.0\ \text{kg}\cdot\text{m}^2$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone's kinetic energy K at this time. What average power P_{av} is delivered by the motor?

$\tau = 10\,N\cdot m$, $I = 2.0\,kg\cdot m^2$, $K_1 = 0$

Find W after $\Delta t = 8s$: $\tau = I\alpha$ so

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So $W = \frac{(\tau\Delta t)^2}{2I} = \frac{(10*8)^2}{2*2}J = \frac{6400}{4}J$

$\Rightarrow$ $\boxed{W = 1600\,J}$ Find K_2: $W = K_2 - \cancel{K_1}$

So $\boxed{K_2 = 1600\,J}$ Find P_{AVE}:

$$\boxed{P_{AVE} = \frac{\Delta W}{\Delta t} = \frac{1600\,J}{8\,s} = 200\,W}$$

If roll no slip,

ASU

If roll no slip, then rolling friction causes

<u>no</u> work