Today 10.1, 10.2

$\boxed{L30}$

Today 10.1, 10.2

$\boxed{L30}$

Torque

Today 10.1, 10.2

L30

Torque

Torque and angular acceleration for a rigid body

Today 10.1, 10.2

$\boxed{L30}$

Monday 10.3, 10.4

Today 10.1, 10.2

L30

Monday 10.3, 10.4

Rigid body rotation about a moving axis

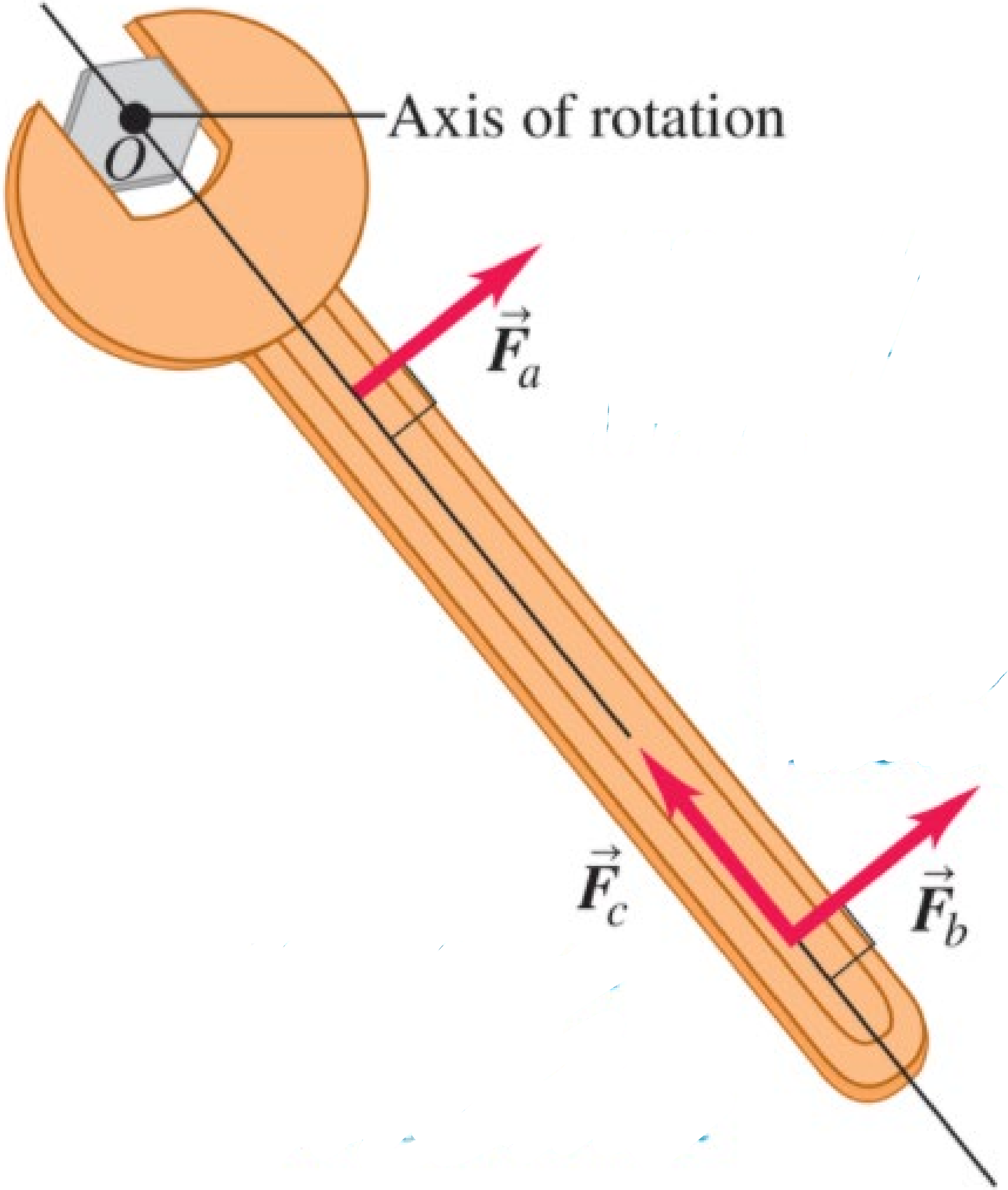
Today 10.1, 10.2

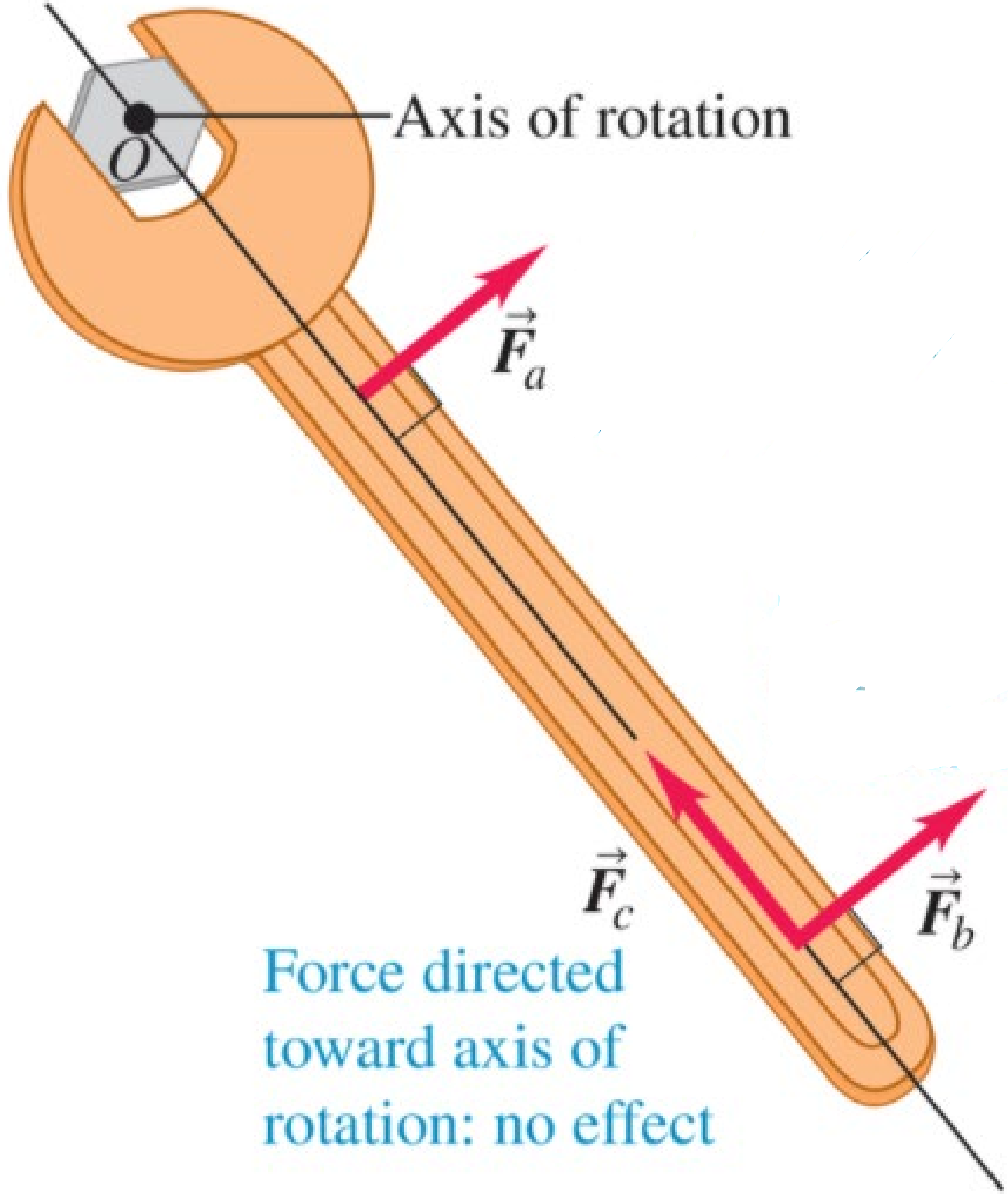
L30

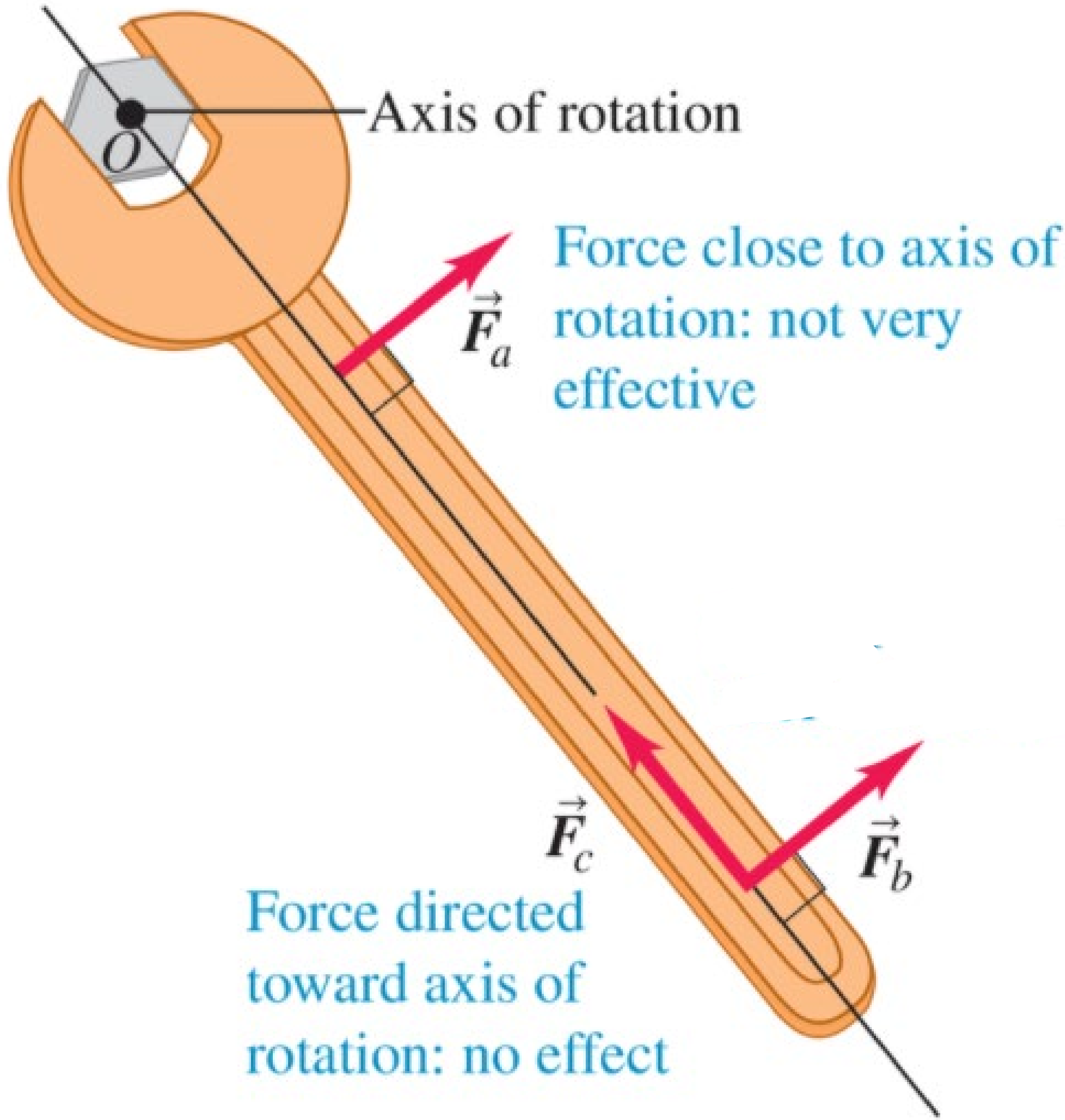
Monday 10.3, 10.4

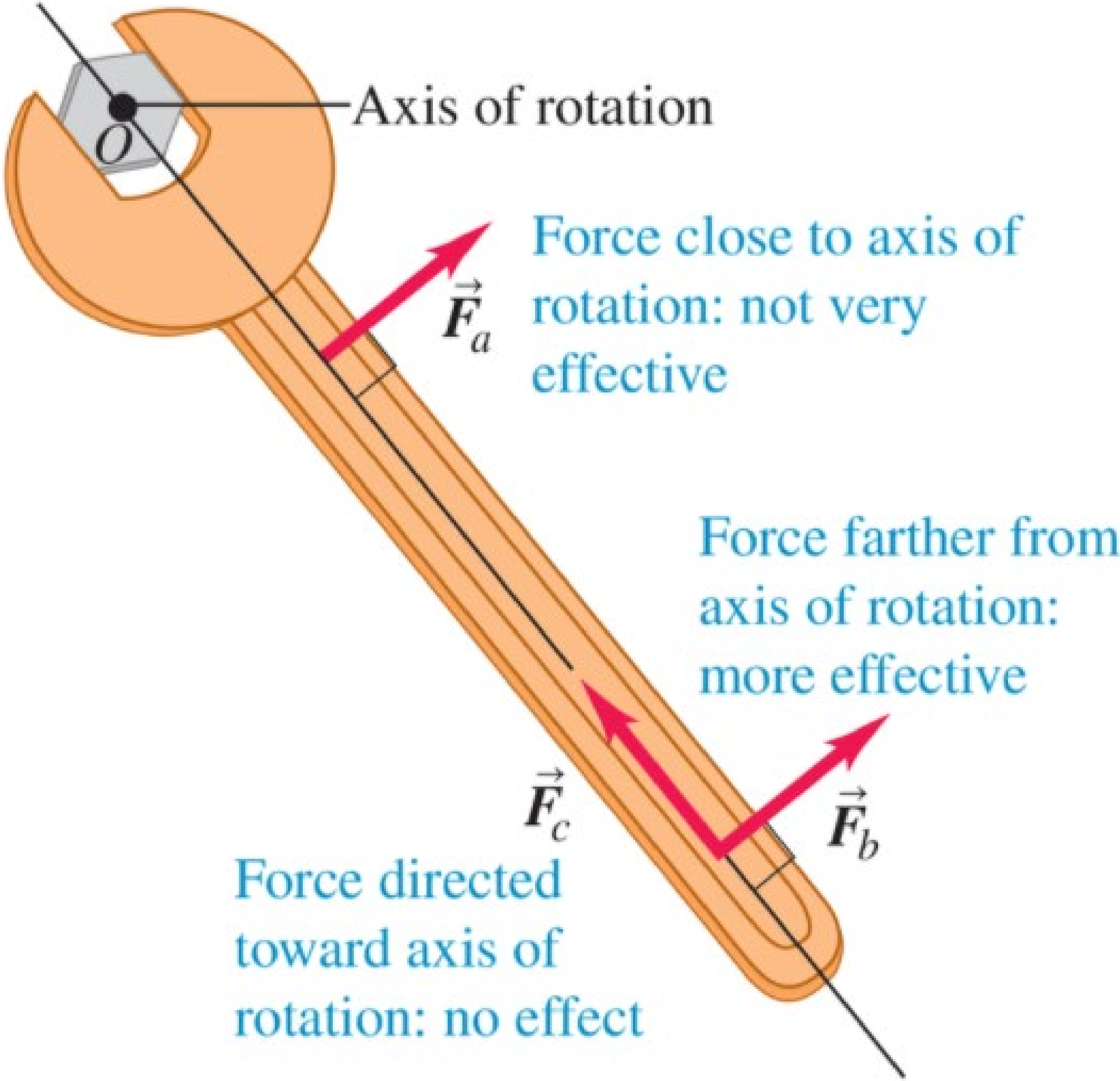
Rigid body rotation about a moving axis

Work & power in rotational motion


Axis of rotation
O
$\vec{F}_a$
$\vec{F}_c$
$\vec{F}_b$


Axis of rotation
O
$\vec{F}_a$
$\vec{F}_c$
$\vec{F}_b$
Force directed
toward axis of
rotation: no effect


Axis of rotation
O
Force close to axis of rotation: not very effective
$\vec{F}_a$
$\vec{F}_c$
$\vec{F}_b$
Force directed toward axis of rotation: no effect


Axis of rotation
O
Force close to axis of rotation: not very effective
$\vec{F}_a$
Force farther from axis of rotation: more effective
$\vec{F}_c$
$\vec{F}_b$
Force directed toward axis of rotation: no effect

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Torque vector
due to force $\vec{F}$
relative to point O

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

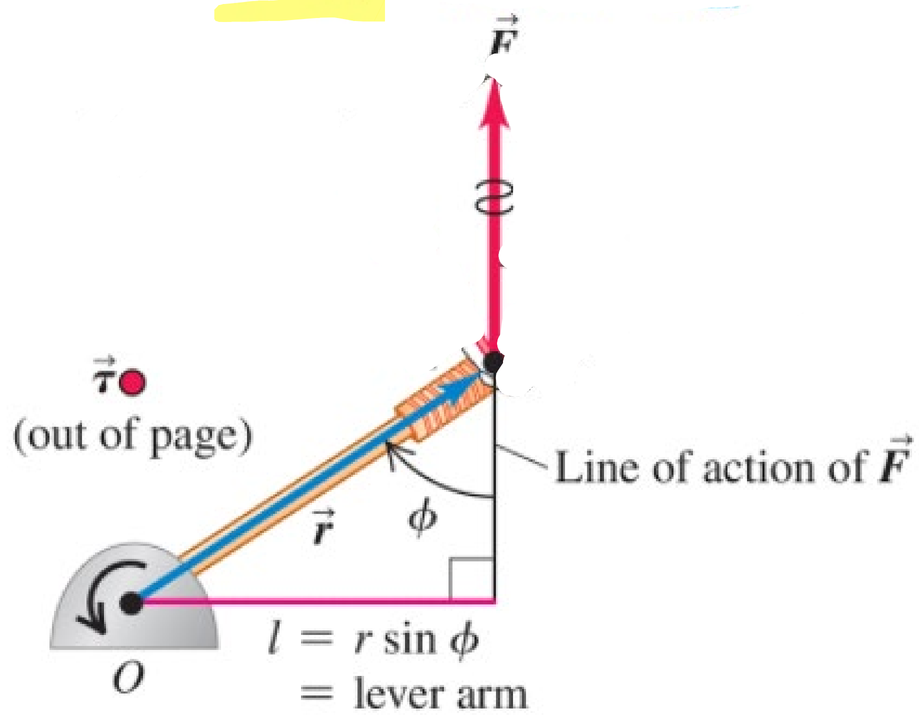
$$\vec{\tau} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

OR

Three ways to calculate torque:

$\tau = Fl$



ASU

Torque vector due to force $\vec{F}$ relative to point O

$$\vec{\tau} = \vec{r} \times \vec{F}$$

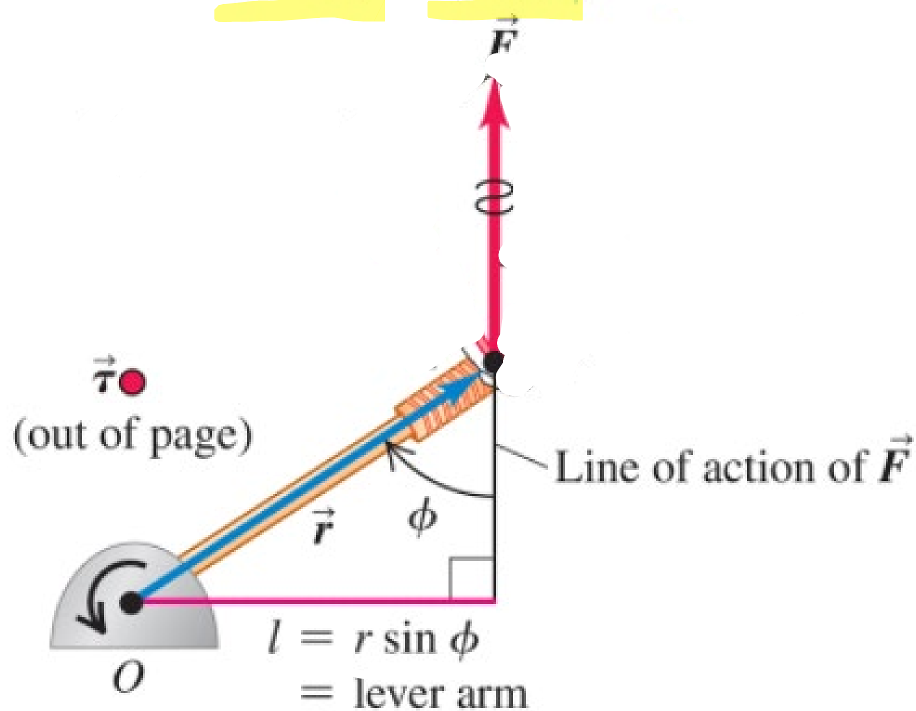
Vector from O to where $\vec{F}$ acts

Force $\vec{F}$

OR

Three ways to calculate torque:

$\tau = Fl = rF \sin\phi$



Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

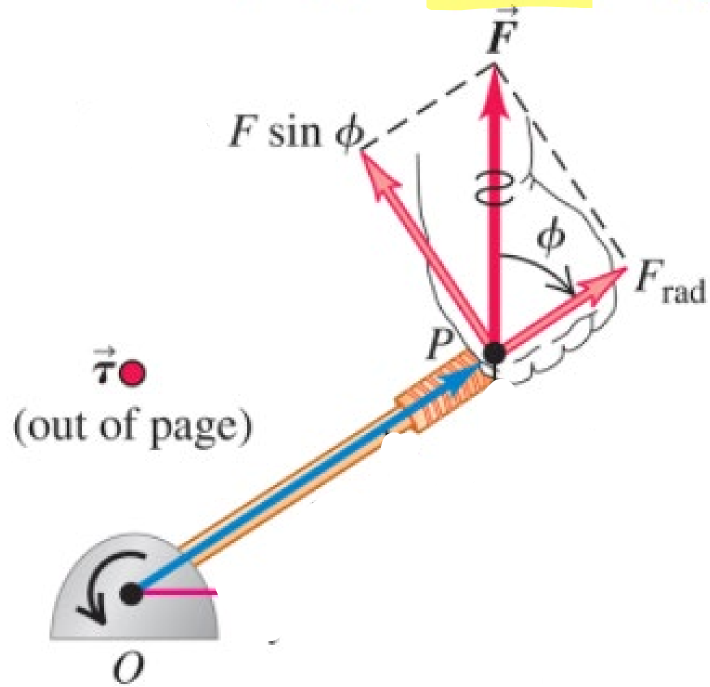
$$\vec{\tau} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

OR

Three ways to calculate torque:

$\tau = Fl = rF \sin\phi = F_{tan} r$



Torque vector due to force $\vec{F}$ relative to point O

$$\vec{\tau} = \vec{r} \times \vec{F}$$

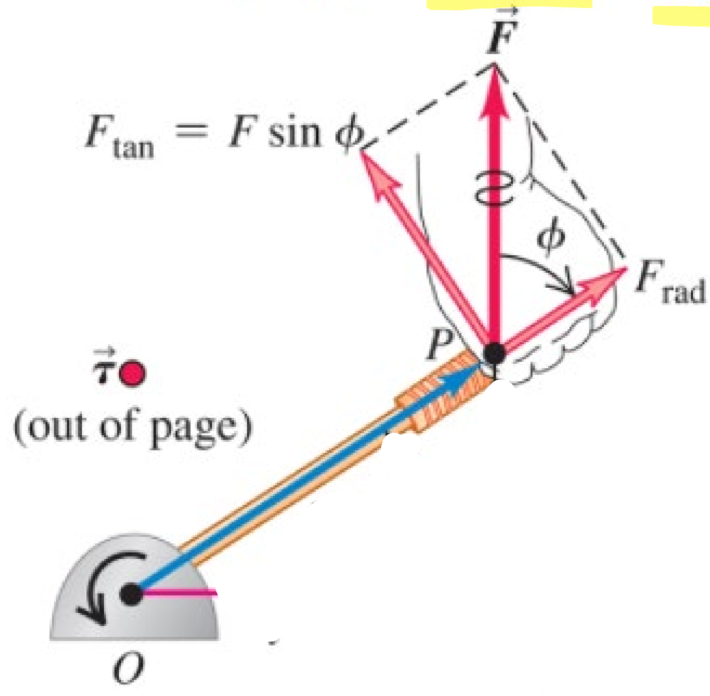
Vector from O to where $\vec{F}$ acts

Force $\vec{F}$

OR

Three ways to calculate torque:

$\tau = Fl = rF \sin\phi = F_{tan} r$



ASU

Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

OR

Three ways to calculate torque:

$$\tau = Fl = rF \sin\phi = F_{\tan} r$$

Torque vector due to force $\vec{F}$ relative to point O

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Vector from O to where $\vec{F}$ acts

Force $\vec{F}$

OR

$$\tau = Fl$$

Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

OR

Magnitude of torque due to force $\vec{F}$ relative to point O

$$\tau = Fl$$

Torque vector due to force $\vec{F}$ relative to point O

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Vector from O to where $\vec{F}$ acts

Force $\vec{F}$

OR

Magnitude of torque due to force $\vec{F}$ relative to point O

$$\tau = Fl$$

Magnitude of $\vec{F}$

ASU

Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

OR

Lever arm of $\vec{F}$

Magnitude of torque due to force $\vec{F}$ relative to point O

$$\tau = Fl$$

Magnitude of $\vec{F}$

Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

OR

Magnitude of torque due to force $\vec{F}$ relative to point O

Lever arm of $\vec{F}$

$$\tau = Fl = rF\sin\phi$$

Magnitude of $\vec{F}$

Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

$$\vec{\boldsymbol{\tau}} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

OR

Magnitude of torque due to force $\vec{F}$ relative to point O

Lever arm of $\vec{F}$

Magnitude of $\vec{r}$ (vector from O to where $\vec{F}$ acts)

$$\tau = Fl = rF\sin\phi$$

Magnitude of $\vec{F}$

Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

OR

Magnitude of $\vec{r}$ (vector from O to where $\vec{F}$ acts)

Lever arm of $\vec{F}$

Magnitude of torque due to force $\vec{F}$ relative to point O

$$\tau = Fl = rF\sin\phi$$

Magnitude of $\vec{F}$

Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

OR

Magnitude of torque due to force $\vec{F}$ relative to point O

Lever arm of $\vec{F}$

Magnitude of $\vec{r}$ (vector from O to where $\vec{F}$ acts)

$$\tau = Fl = rF\sin\phi$$

Magnitude of $\vec{F}$

Angle between $\vec{r}$ and $\vec{F}$

ASU

Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

OR

Magnitude of torque due to force $\vec{F}$ relative to point O

Lever arm of $\vec{F}$

Magnitude of $\vec{r}$ (vector from O to where $\vec{F}$ acts)

$$\tau = Fl = rF\sin\phi = F_{\tan}r$$

Magnitude of $\vec{F}$

Angle between $\vec{r}$ and $\vec{F}$

Torque vector due to force $\vec{F}$ relative to point O

Vector from O to where $\vec{F}$ acts

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

OR

Magnitude of $\vec{r}$ (vector from O to where $\vec{F}$ acts)

Lever arm of $\vec{F}$

Magnitude of torque due to force $\vec{F}$ relative to point O

$$\tau = Fl = rF\sin\phi = F_{\tan}r$$

Magnitude of $\vec{F}$

Angle between $\vec{r}$ and $\vec{F}$

Tangential component of $\vec{F}$

ASU

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a “cheater”) over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\ N,$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a “cheater”) over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

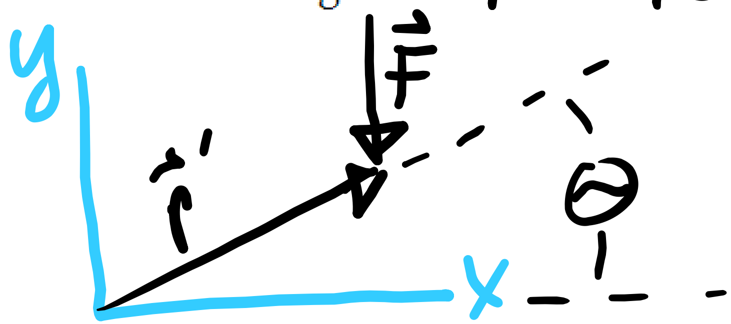
$F = 900\,N, \quad r = 0.8\,m,$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a “cheater”) over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$



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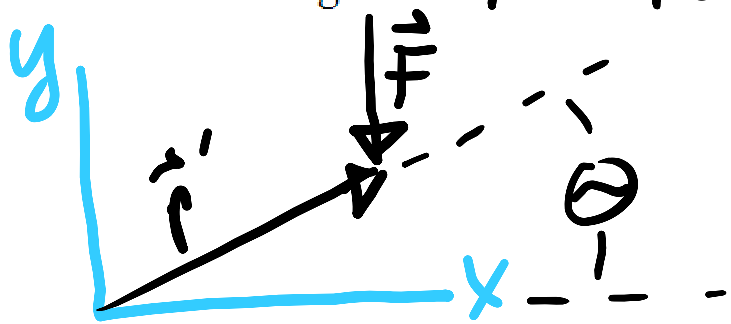
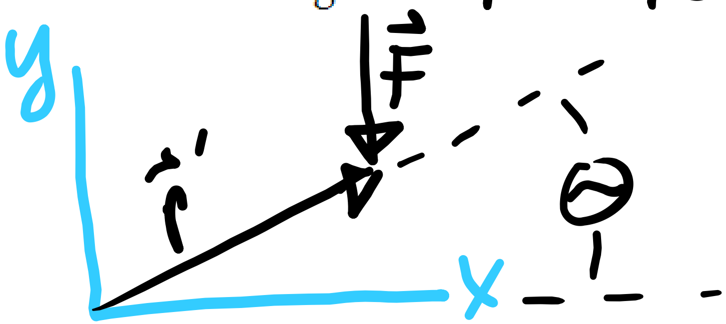
$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$



$\vec{\tau} = \vec{r} \times \vec{F}$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a “cheater”) over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N,\quad r = 0.8\,m,\quad \theta = 19^\circ$

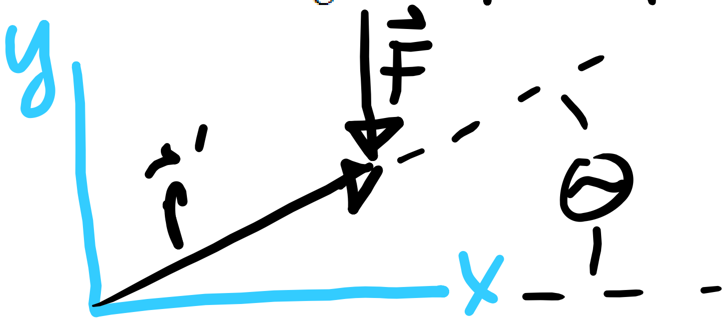


$\vec{\tau} = \vec{r} \times \vec{F}$ But

$\vec{r} = r[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}]$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$



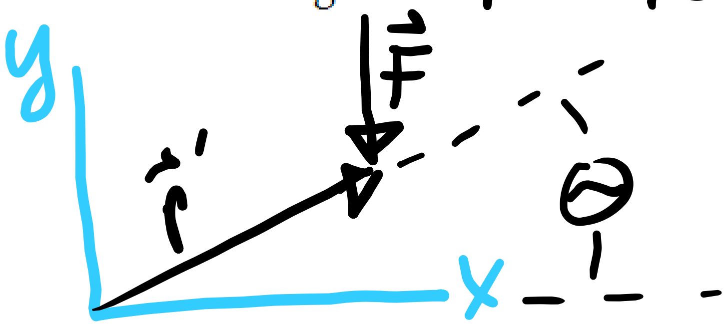
$\vec{\tau} = \vec{r} \times \vec{F}$ But

$\vec{r} = r[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}]$

& $\vec{F} = 900N(-\hat{j})$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$



$\vec{\tau} = \vec{r} \times \vec{F}$ But

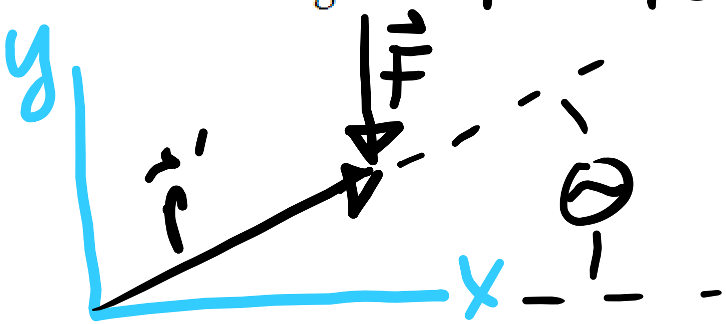
$\vec{r} = r[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}]$

& $\vec{F} = 900N(-\hat{j})$ so

$\vec{\tau} = (0.8m)[\cos 19^\circ\,\hat{i} + \sin 19^\circ\,\hat{j}] \times 900N(-\hat{j})$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$



$\vec{\tau} = \vec{r} \times \vec{F}$ But

$\vec{r} = r[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}]$

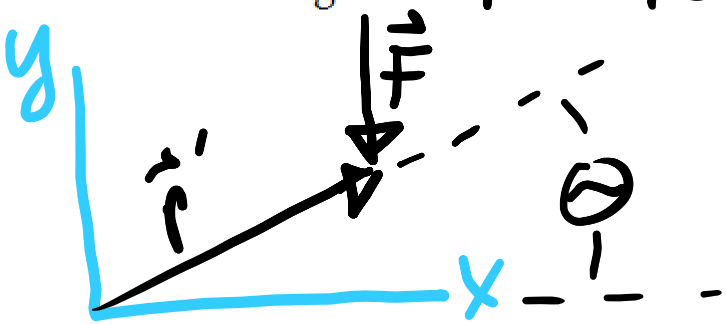
& $\vec{F} = 900N(-\hat{j})$ so

$\vec{\tau} = (0.8m)[\cos 19^\circ\,\hat{i} + \sin 19^\circ\,\hat{j}] \times 900N(-\hat{j}) \Rightarrow$

$\vec{\tau} = -(0.8)(900)N\cdot m\,[\cos 19^\circ\,\hat{i}\times\hat{j} + \sin 19^\circ\,\hat{j}\times\hat{j}]$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$



$\vec{\tau} = \vec{r} \times \vec{F}$ But

$\vec{r} = r[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}]$

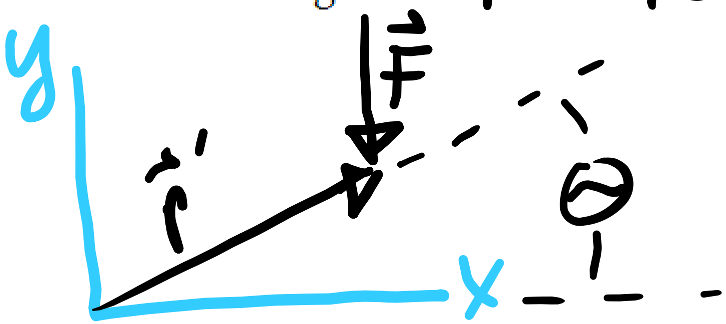
& $\vec{F} = 900N(-\hat{j})$ so

$\vec{\tau} = (0.8m)[\cos 19^\circ\,\hat{i} + \sin 19^\circ\,\hat{j}] \times 900N(-\hat{j}) \Rightarrow 0$

$\vec{\tau} = -(0.8)(900)N\cdot m\,[\cos 19^\circ\,\hat{i}\times\hat{j} + \cancel{\sin 19^\circ\,\hat{j}\times\hat{j}}]$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N,\quad r = 0.8\,m,\quad \theta = 19^\circ$



$\vec{\tau} = \vec{r} \times \vec{F}$ But

$\vec{r} = r[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}]$

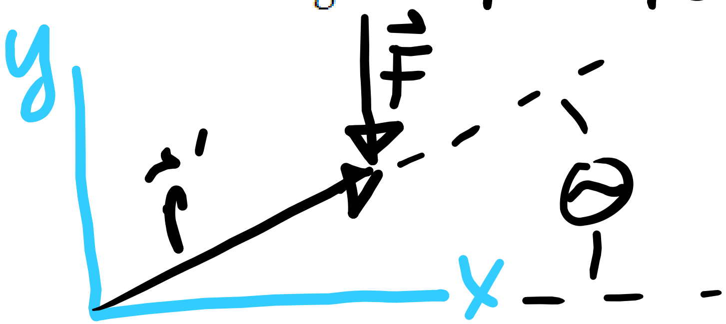
& $\vec{F} = 900N(-\hat{j})$ so

$$\vec{\tau} = (0.8m)[\cos 19^\circ\,\hat{i} + \sin 19^\circ\,\hat{j}] \times 900N(-\hat{j}) \Rightarrow$$

$$\vec{\tau} = -(0.8)(900)\,N\cdot m\,[\cos 19^\circ\,\cancelto{\hat{k}}{\hat{i}\times\hat{j}} + \sin 19^\circ\,\cancelto{0}{\hat{j}\times\hat{j}}]$$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N,\quad r = 0.8\,m,\quad \theta = 19^\circ$



$\vec\tau = \vec r \times \vec F$ But

$\vec r = r[\cos\theta\,\hat i + \sin\theta\,\hat j]$

& $\vec F = 900N(-\hat j)$ so

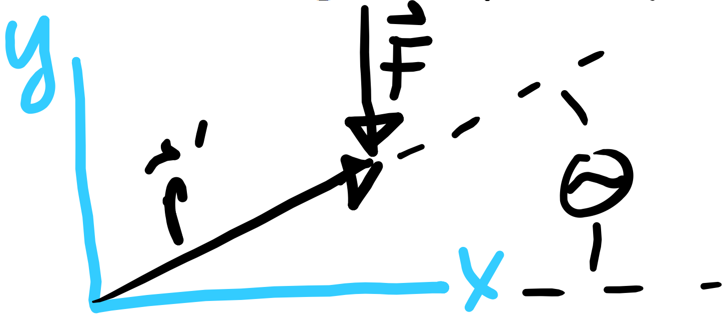
$\vec\tau = (0.8m)[\cos 19^\circ\,\hat i + \sin 19^\circ\,\hat j] \times 900N(-\hat j) \Rightarrow$

$\vec\tau = -(0.8)(900)N\cdot m\,[\cos 19^\circ\,\overset{\hat k}{\hat i \times \hat j} + \sin 19^\circ\,\overset{0}{\cancel{\hat j \times \hat j}}]$

$\Rightarrow \quad \tau = -720\,N\cdot m\cos 19^\circ\,\hat k$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N,\quad r = 0.8\,m,\quad \theta = 19°$



$\vec{\tau} = \vec{r} \times \vec{F}$ But

$\vec{r} = r[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}]$

& $\vec{F} = 900N(-\hat{j})$ so

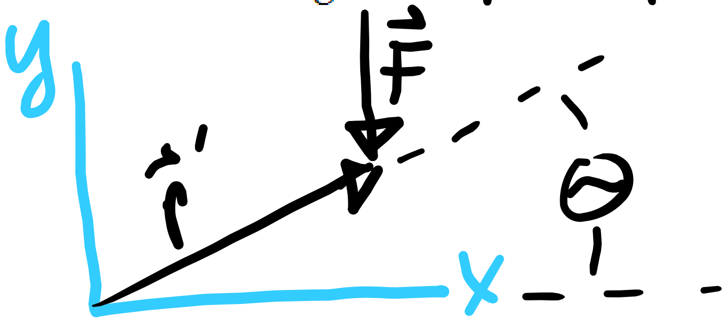
$\vec{\tau} = (0.8m)[\cos 19°\,\hat{i} + \sin 19°\,\hat{j}] \times 900N(-\hat{j}) \Rightarrow$

$\vec{\tau} = -(0.8)(900)N\cdot m\,[\cos 19°\,\underbrace{\hat{i}\times\hat{j}}_{\hat{k}} + \sin 19°\,\underbrace{\hat{j}\times\hat{j}}_{0}]$

$\Rightarrow \quad \tau = -720\,N\cdot m \cos 19°\,\hat{k} = -680\,N\cdot m\,\hat{k}$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

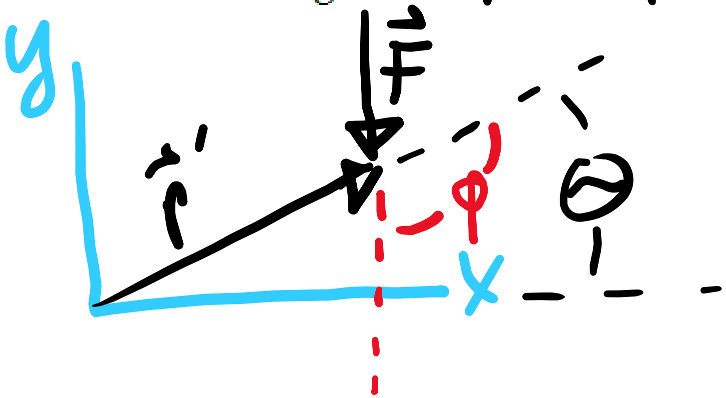
$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$



$\tau = rF\sin\phi$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

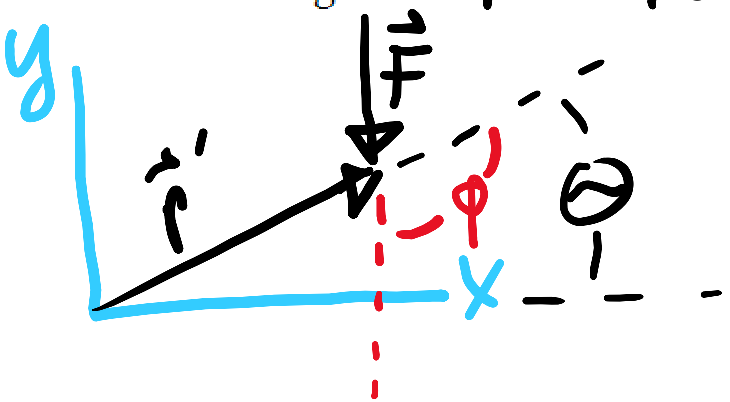
$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$



$\tau = rF\sin\phi$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19°$

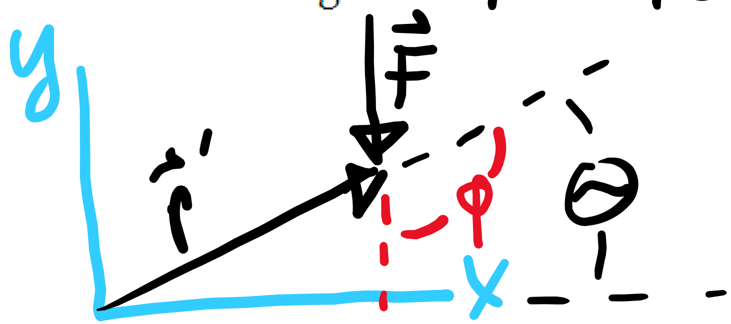


$\tau = rF\sin\phi$, where

$\phi = 90° + \theta = 109°$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$



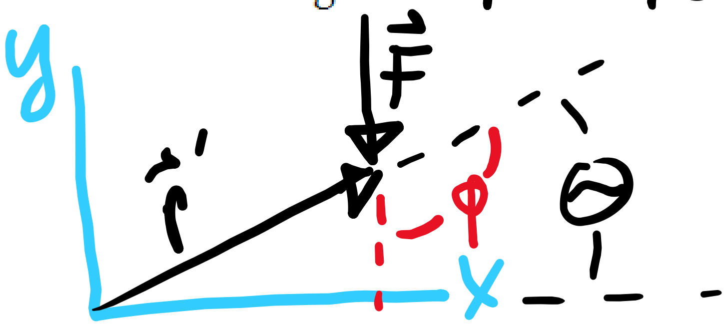
$\tau = rF\sin\varphi$, where

$\varphi = 90^\circ + \theta = 109^\circ$

so $\tau = (0.8\,m)(900\,N)\sin(109^\circ)$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N,\quad r = 0.8\,m,\quad \theta = 19^\circ$



$\tau = rF\sin\varphi$, where

$\varphi = 90^\circ + \theta = 109^\circ$

so $\tau = (0.8\,m)(900\,N)\sin(109^\circ)$

$\Rightarrow \quad \vec{\tau} = 680\,N\cdot m\,(-\hat{k})$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

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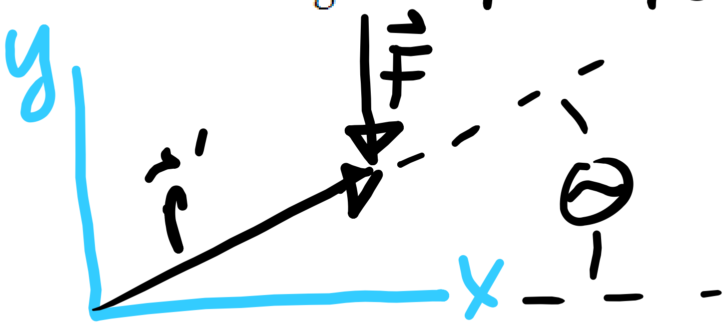

$\tau = F\ell$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

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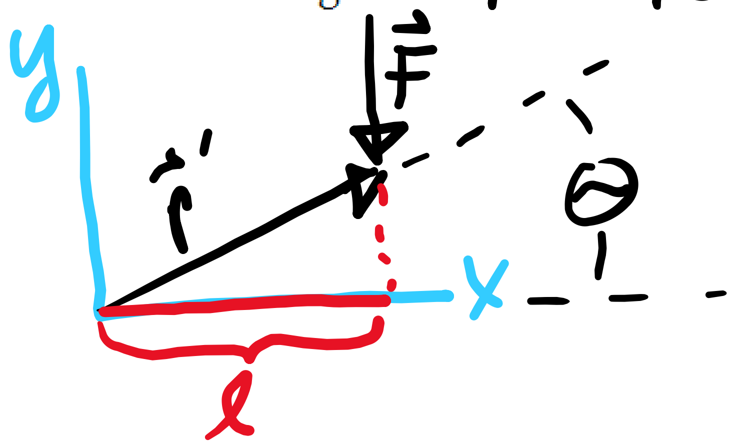

$\tau = F\ell$

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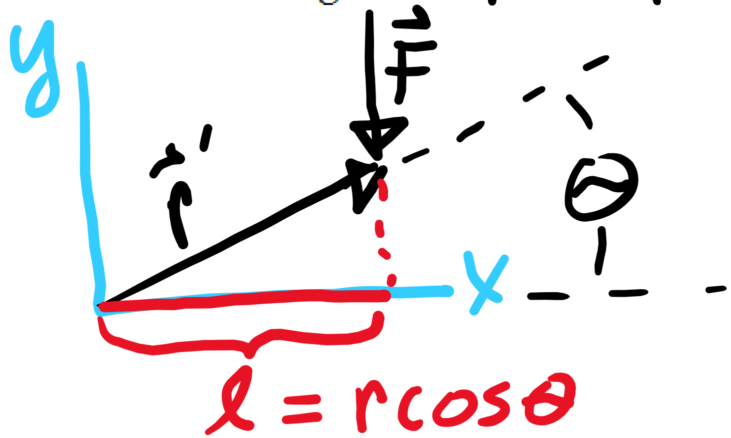
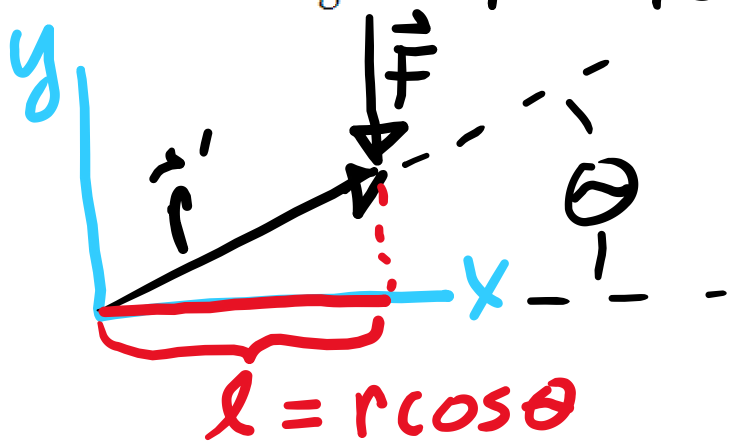
$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$



$\tau = Fl$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$

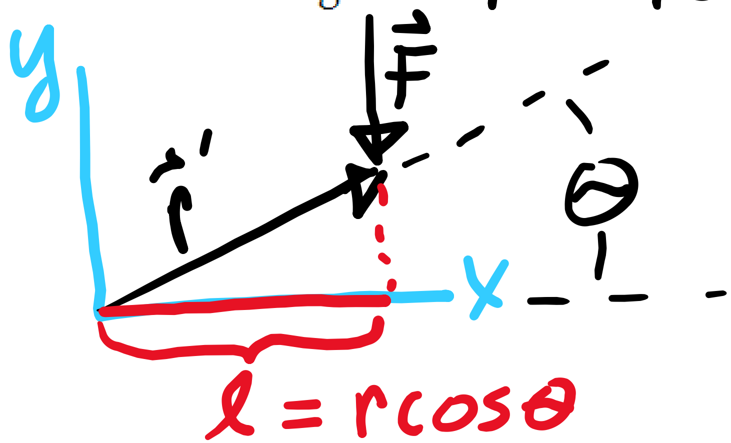


$\tau = F\ell$ so

$\tau = (900\,N)(0.8\,m)\cos 19^\circ$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\,N, \quad r = 0.8\,m, \quad \theta = 19^\circ$



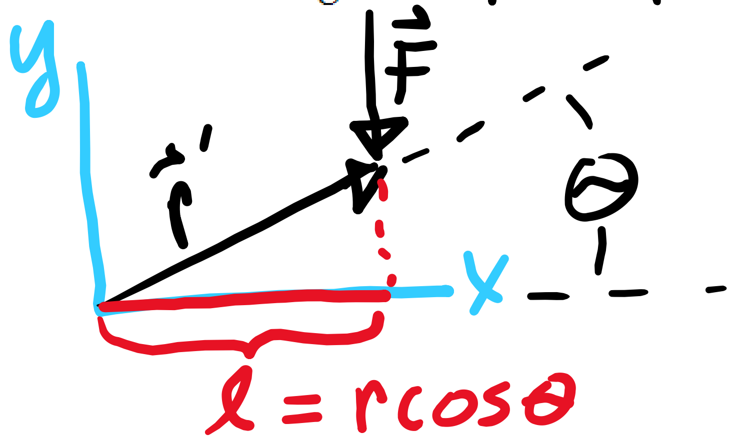
$\tau = F\ell$ so

$\tau = (900N)(0.8m)\cos 19^\circ$

$\Rightarrow \quad \tau = 680\,N\cdot m$

To loosen a pipe fitting, a plumber slips a piece of scrap pipe (a “cheater”) over his wrench handle. He stands on the end of the cheater, applying his 900 N weight at a point 0.80 m from the center of the fitting (~~Fig. 10.5a~~). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

$F = 900\ N,\quad r = 0.8\,m,\quad \theta = 19°$



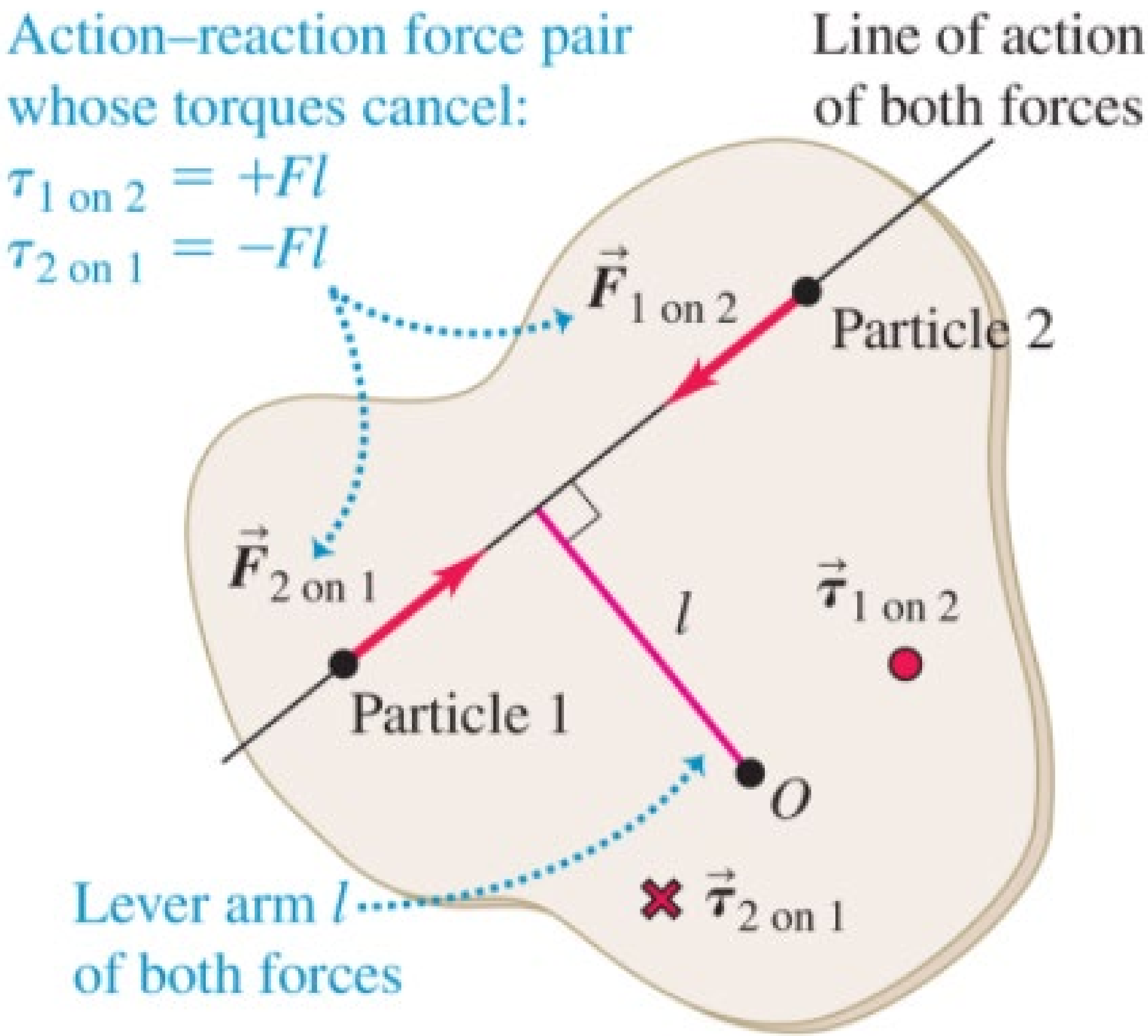
$\tau = F\ell$ so

$\tau = (900N)(0.8m)\cos 19°$

$\Rightarrow \quad \vec{\tau} = 680\,N\cdot m\,(-\hat{k})$

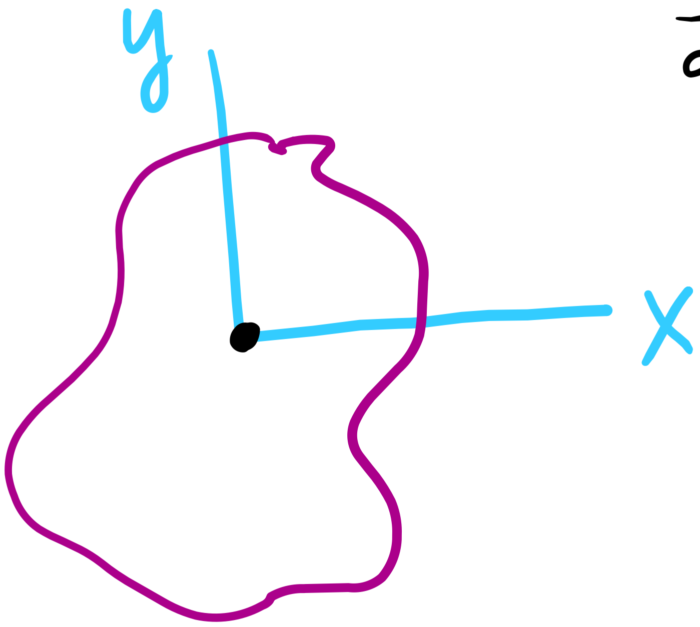
10.2

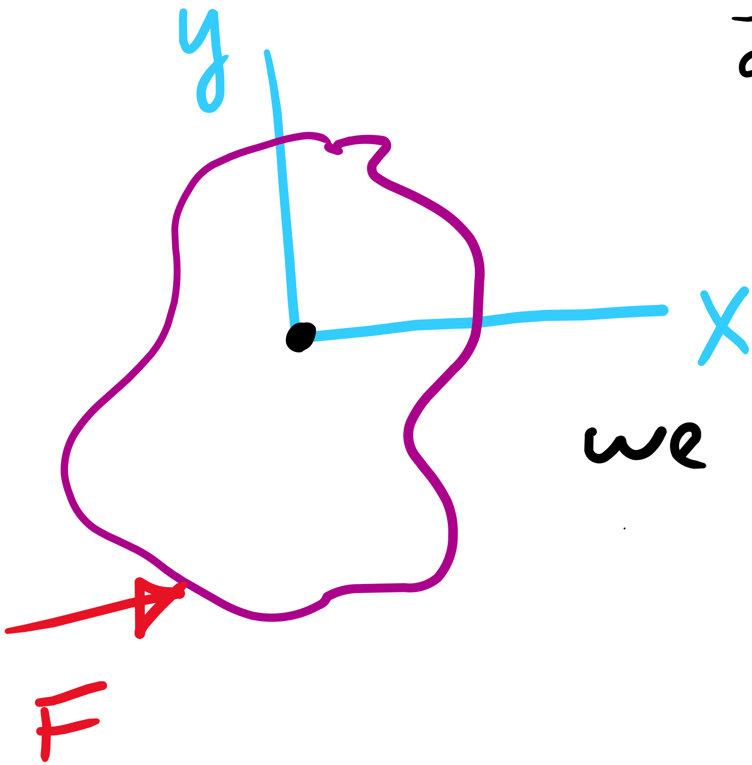
Torque and angular acceleration for a rigid body



Action–reaction force pair
whose torques cancel:
$\tau_{1\text{ on }2} = +Fl$
$\tau_{2\text{ on }1} = -Fl$
Line of action
of both forces
$\vec{F}_{1\text{ on }2}$
Particle 2
$\vec{F}_{2\text{ on }1}$
l
$\vec{\tau}_{1\text{ on }2}$
Particle 1
O
Lever arm l
of both forces
$\vec{\tau}_{2\text{ on }1}$

Let rigid body rotate about z-axis

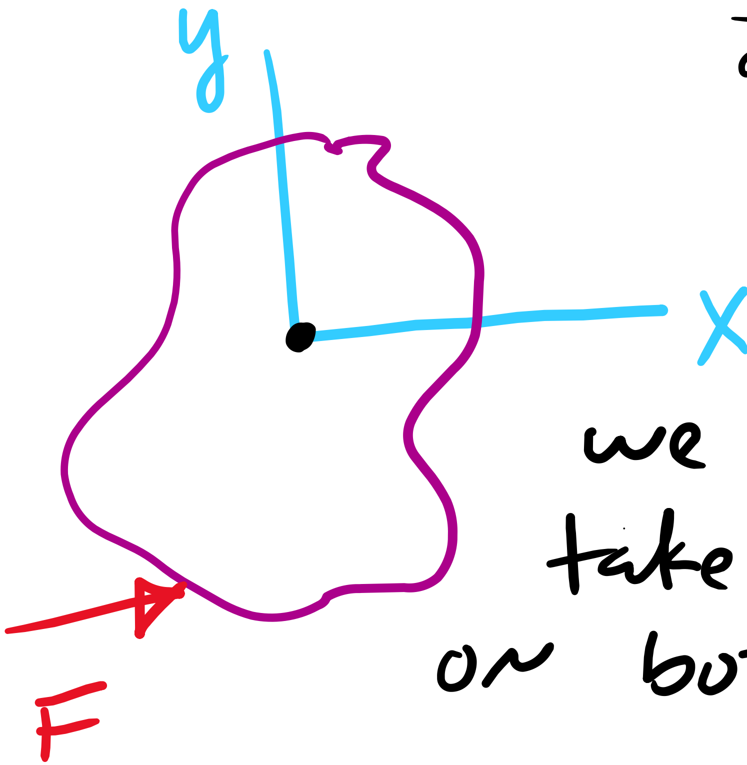




Let rigid body rotate about z-axis

Apply a force

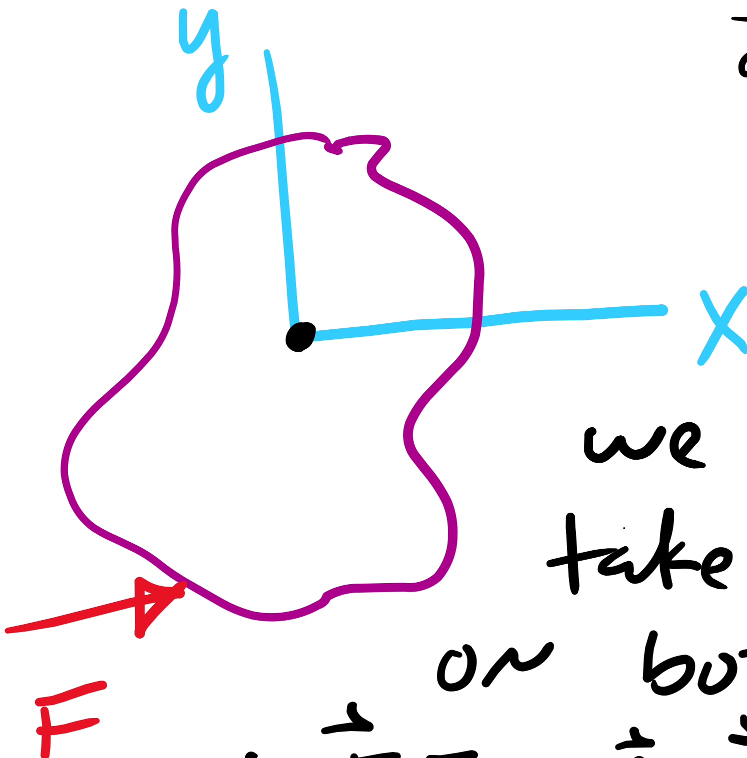
we know $\vec{F} = m\vec{a}$



Let rigid body rotate about z-axis

Apply a force

we know $\vec{F} = m\vec{a}$ Now take cross product $[\vec{r}\times]$ on both sides :

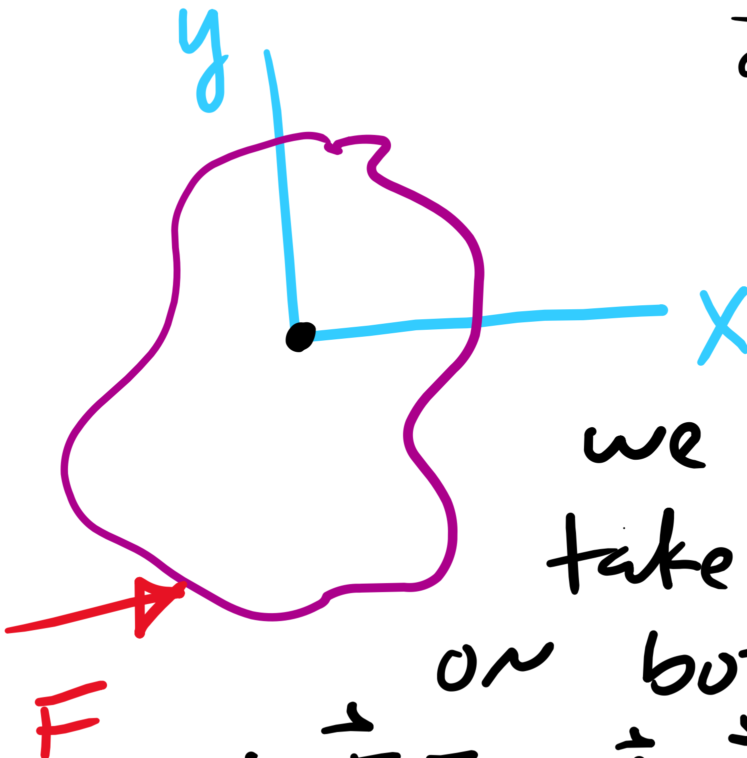


Let rigid body rotate about z-axis

Apply a force

we know $\vec{F} = m\vec{a}$ Now take cross product $[\vec{r}\times]$ on both sides :

$$\vec{r}\times\vec{F} = m\,\vec{r}\times\vec{a}$$

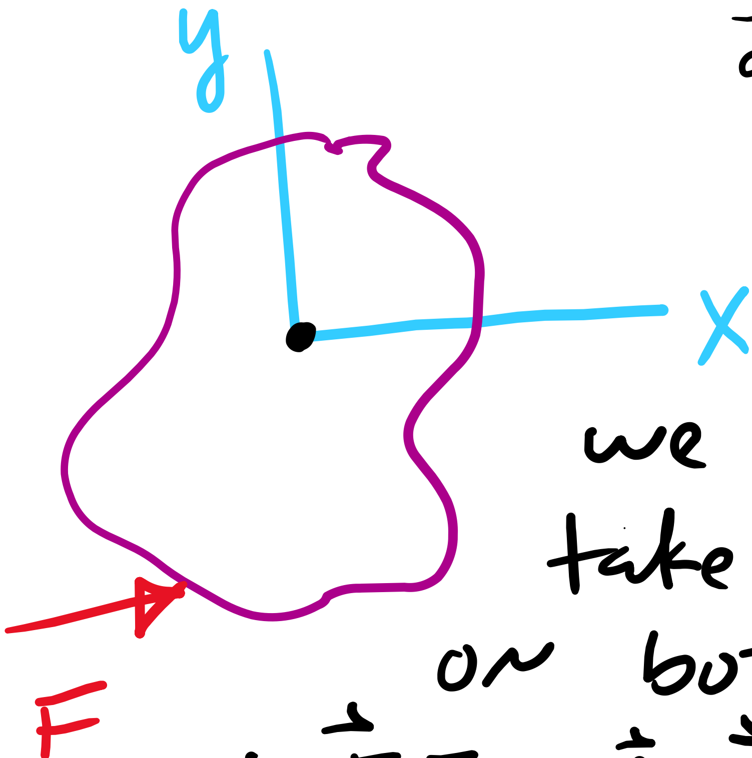


Let rigid body rotate about z-axis

Apply a force

we know $\vec{F} = m\vec{a}$ Now take cross product $[\vec{r}\times]$ on both sides:

$\vec{r}\times\vec{F} = m\,\vec{r}\times\vec{a}$ But if rotating $\vec{a}$ is not constant.



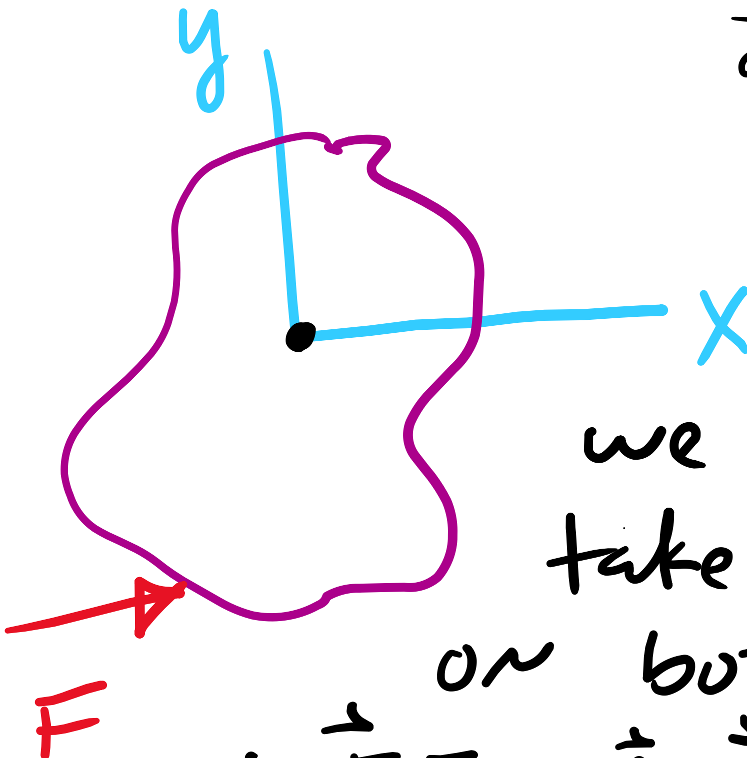
Let rigid body rotate about z-axis

Apply a force

we know $\vec{F} = m\vec{a}$ Now take cross product $[\vec{r}\times]$ on both sides:

$\vec{r}\times\vec{F} = m\,\vec{r}\times\vec{a}$ But if rotating, $\vec{a}$ is not constant. Need to break up into small mass chunks. So

$\vec{\tau} = \Sigma m_i \vec{r}_i \times \vec{a}_i$



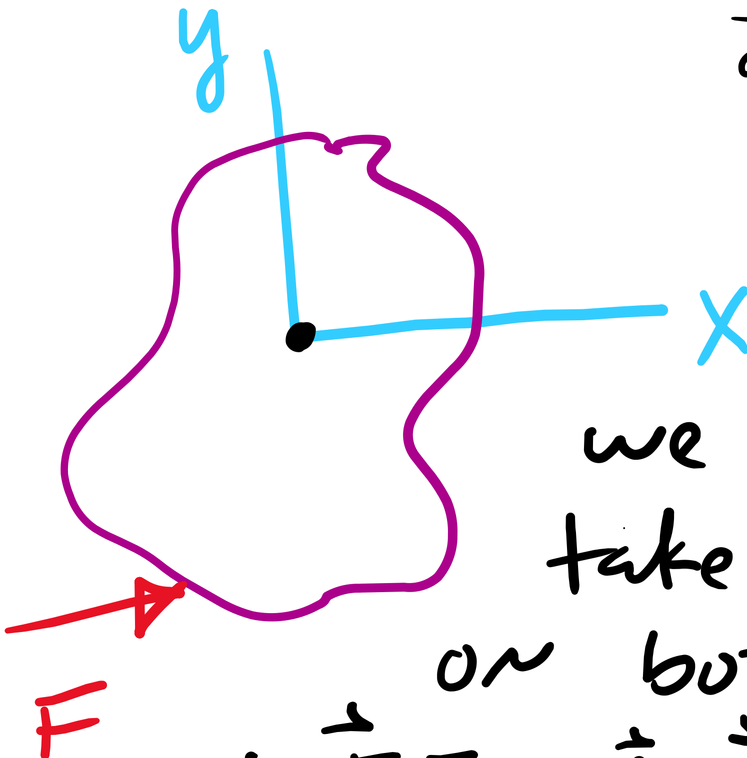
Let rigid body rotate about z-axis

Apply a force

we know $\vec{F} = m\vec{a}$ Now take cross product [$\vec{r}\times$] on both sides:

$\vec{r}\times\vec{F} = m\,\vec{r}\times\vec{a}$ But if rotating, $\vec{a}$ is not constant. Need to break up into small mass chunks. So

$\vec{\tau} = \Sigma m_i \vec{r}_i \times \vec{a}_i$ & since $a_{t_i} = r_i \alpha_i$



Let rigid body rotate about z-axis

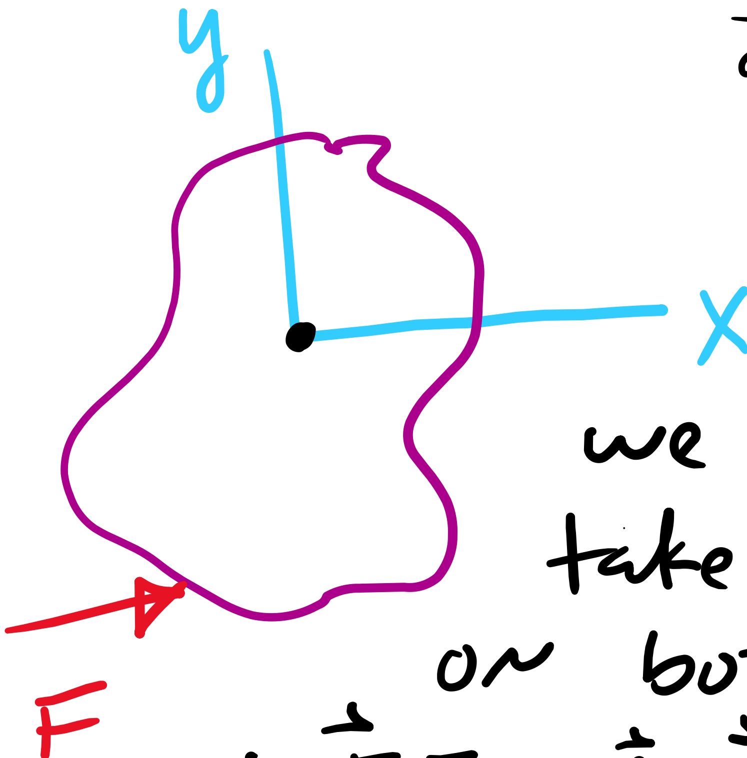
Apply a force

we know $\vec{F} = m\vec{a}$ Now take cross product $[\vec{r}\times]$ on both sides:

$\vec{r}\times\vec{F} = m\vec{r}\times\vec{a}$ But if rotating, $\vec{a}$ is not constant. Need to break up into small mass chunks. So

$\vec{\tau} = \Sigma m_i \vec{r}_i \times \vec{a}_i$ & since $a_{t_i} = r_i \alpha_i$ &

$|\vec{r}_i \times \vec{a}_i| = |r_i a_{t_i}|$



Let rigid body rotate about z-axis

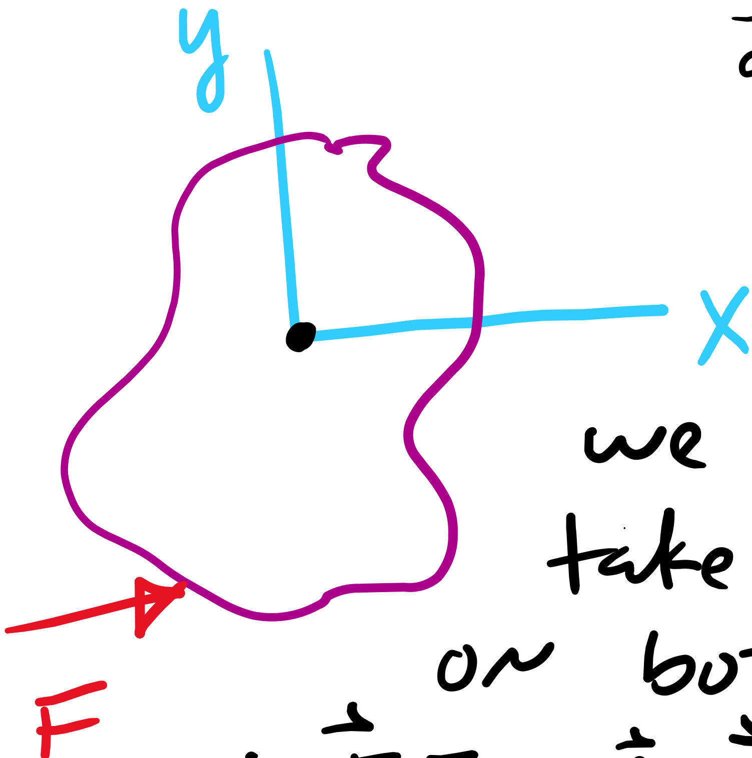
Apply a force

we know $\vec{F}=m\vec{a}$. Now take cross product $[\vec{r}\times]$ on both sides:

$\vec{r}\times\vec{F}=m\,\vec{r}\times\vec{a}$ But if rotating, $\vec{a}$ is not constant. Need to break up into small mass chunks. So

$\vec{\tau}=\sum m_i\vec{r}_i\times\vec{a}_i$ & since $a_{t_i}=r_i\alpha_i$ &

$|\vec{r}_i\times\vec{a}_i|=|r_i a_{t_i}|$, then $|\vec{\tau}|=\sum m_i r_i^2\alpha_i$



Let rigid body rotate about z-axis

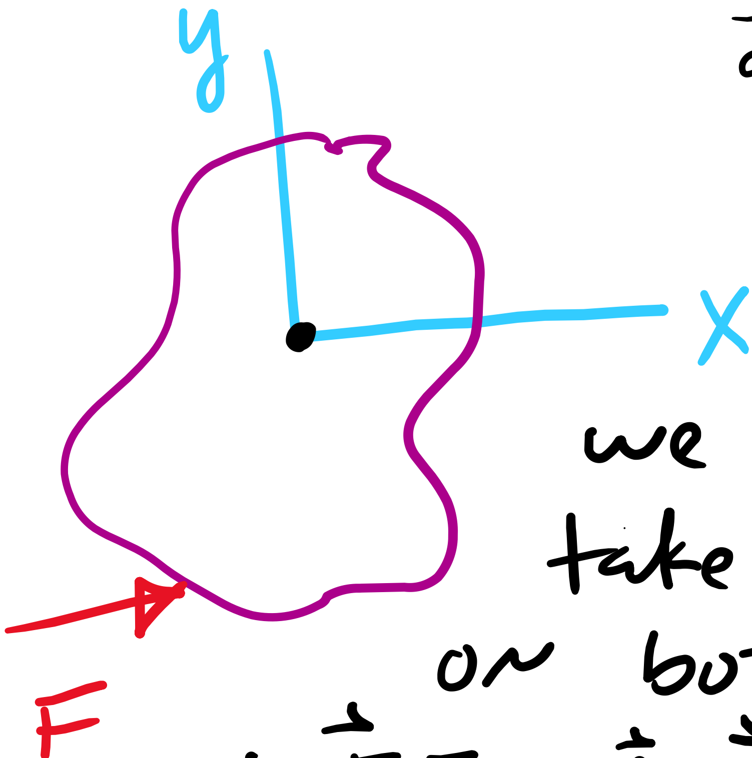
Apply a force

we know $\vec{F} = m\vec{a}$ Now take cross product $[\vec{r}\times]$ on both sides:

$\vec{r}\times\vec{F} = m\,\vec{r}\times\vec{a}$ But if rotating, $\vec{a}$ is not constant. Need to break up into small mass chunks. So

$\vec{\tau} = \Sigma m_i \vec{r}_i \times \vec{a}_i$ & since $a_{t_i} = r_i \alpha_i$ & $|\vec{r}_i \times \vec{a}_i| = |r_i a_{t_i}|$, then $|\vec{\tau}| = \Sigma m_i r_i^2 \alpha_i$ But

$I = \Sigma m_i r_i^2$



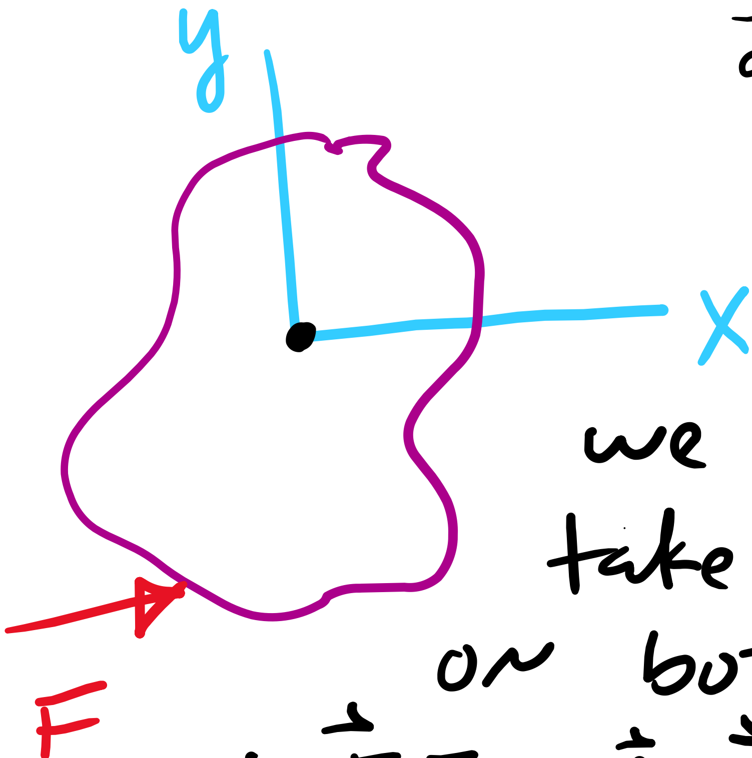
Let rigid body rotate about z-axis

Apply a force

we know $\vec{F} = m\vec{a}$ Now take cross product $[\vec{r}\times]$ on both sides:

$\vec{r}\times\vec{F} = m\vec{r}\times\vec{a}$ But if rotating, $\vec{a}$ is not constant. Need to break up into small mass chunks. So

$\vec{\tau} = \Sigma m_i \vec{r}_i \times \vec{a}_i$ & since $a_{t_i} = r_i \alpha_i$ & $|\vec{r}_i \times \vec{a}_i| = |r_i a_{t_i}|$, then $|\vec{\tau}| = \Sigma m_i r_i^2 \alpha_i$ But $I = \Sigma m_i r_i^2$ so $\tau = I\alpha$



Let rigid body rotate about z-axis

Apply a force

we know $\vec{F} = m\vec{a}$ Now take cross product $[\vec{r}\times]$ on both sides:

$\vec{r}\times\vec{F} = m\,\vec{r}\times\vec{a}$ But if rotating, $\vec{a}$ is not constant. Need to break up into small mass chunks. So

$\vec{\tau} = \sum m_i \vec{r}_i \times \vec{a}_i$ & since $a_{t_i} = r_i \alpha_i$ & $|\vec{r}_i \times \vec{a}_i| = |r_i a_{t_i}|$, then $|\vec{\tau}| = \sum m_i r_i^2 \alpha_i$ But $I = \sum m_i r_i^2$ so $\vec{\tau} = I\vec{\alpha}$

Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?

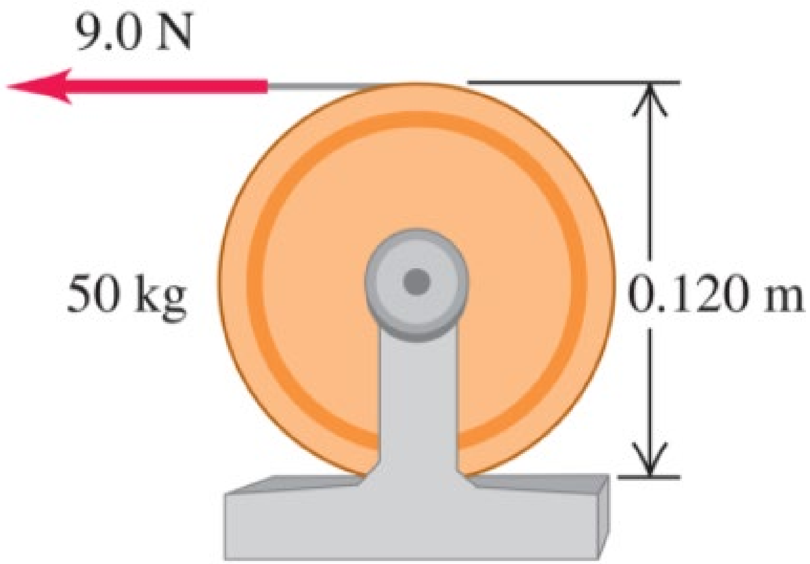


Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods.

What is the cable's acceleration?

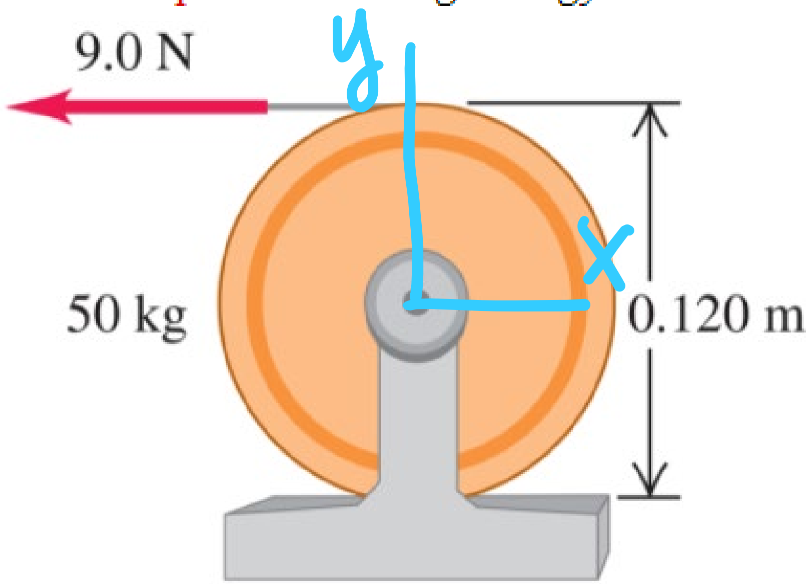


Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?

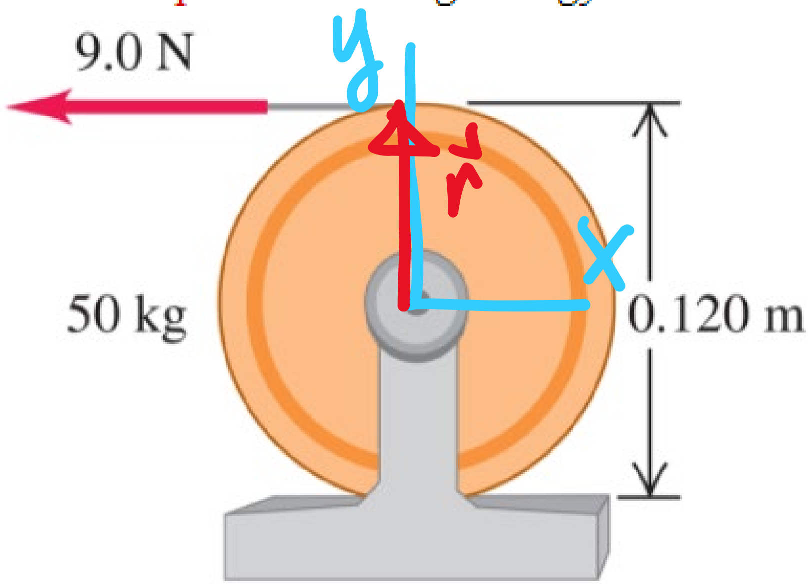


Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?

$\vec{r} = R\hat{j}$

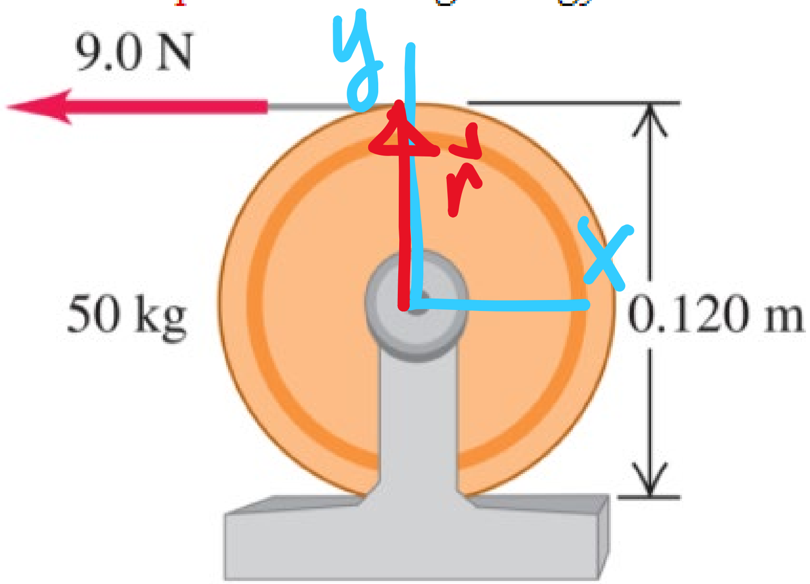


Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?

$\vec{r} = R\hat{j}$, where $R = \frac{0.120\,m}{2}$

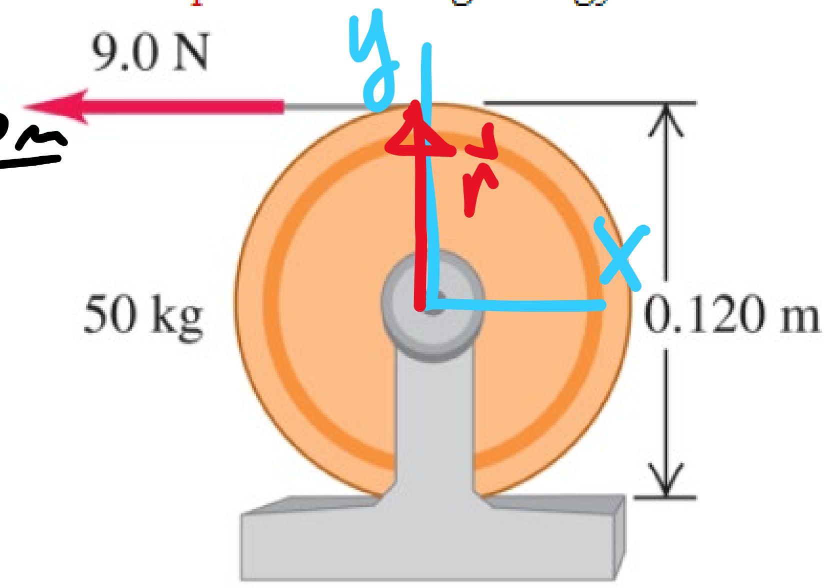
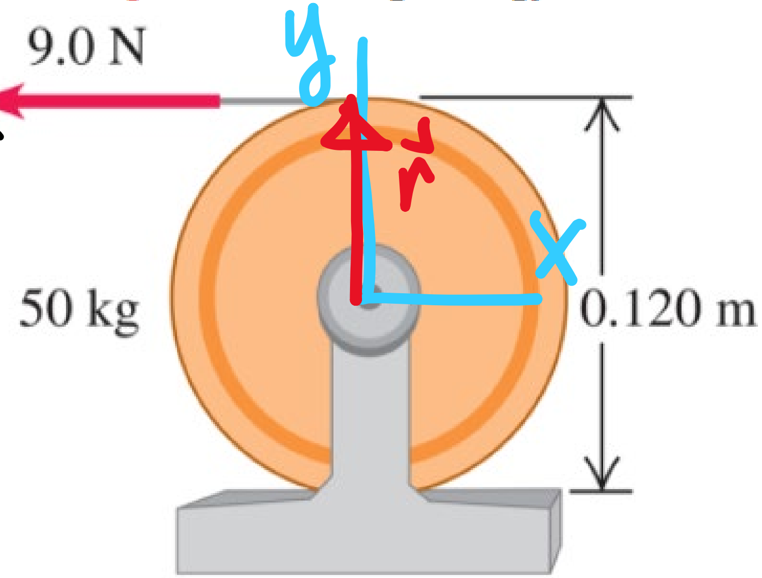


Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods.

What is the cable's acceleration?

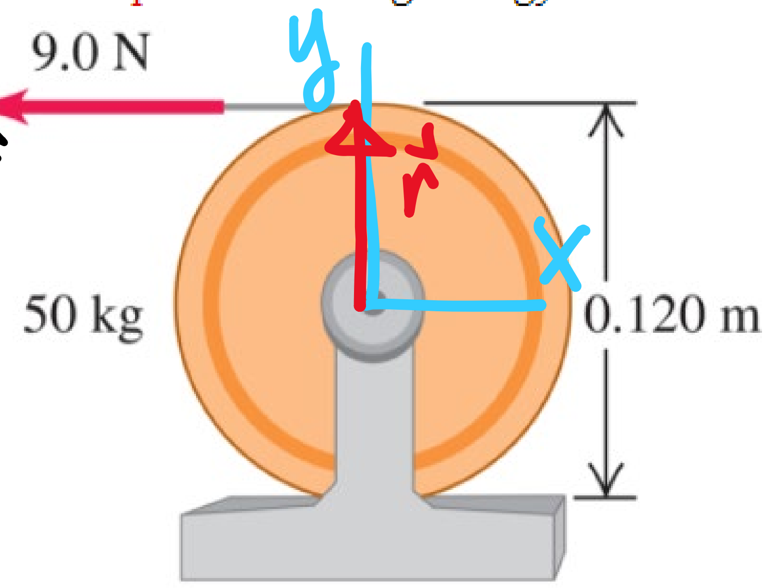


$\vec{r} = R\hat{j}$, where $R = \frac{0.120\,m}{2}$

& $\vec{F} = F(-\hat{i})$

Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods.

What is the cable's acceleration?

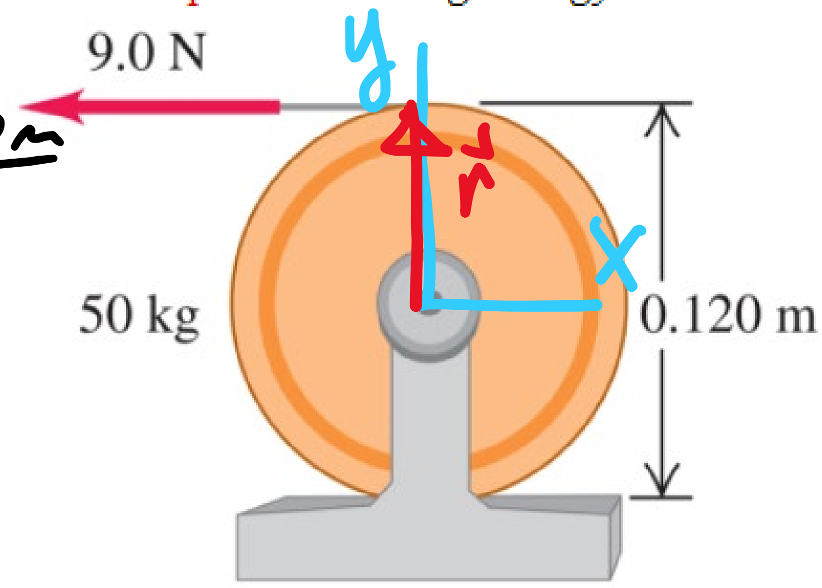


$\vec{r} = R\hat{j}$, where $R = \frac{0.120\,m}{2}$

& $\vec{F} = F(-\hat{i})$ So

$\vec{r} \times \vec{F} = -RF(\hat{j} \times \hat{i})$

Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?

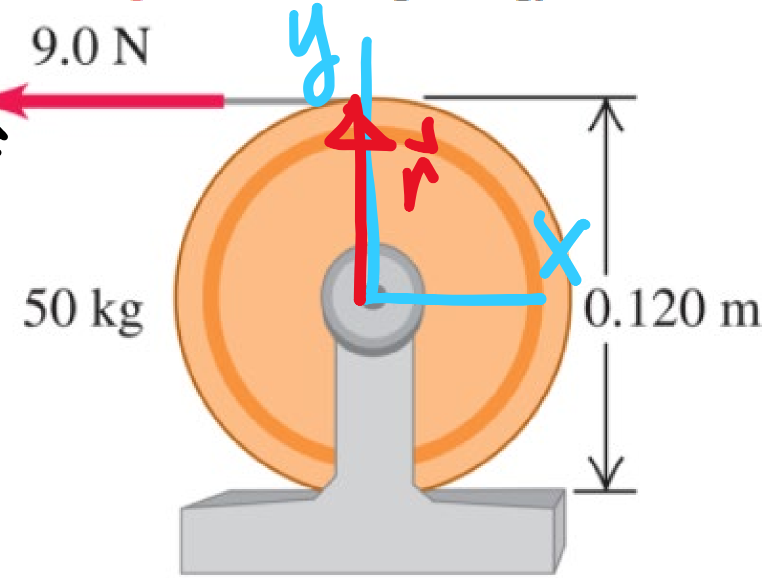


$\vec{r} = R\hat{j}$, where $R = \frac{0.120\,m}{2}$

& $\vec{F} = F(-\hat{i})$ so

$\vec{r} \times \vec{F} = -RF\cancelto{-\hat{k}}{(\hat{j} \times \hat{i})}$

Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?

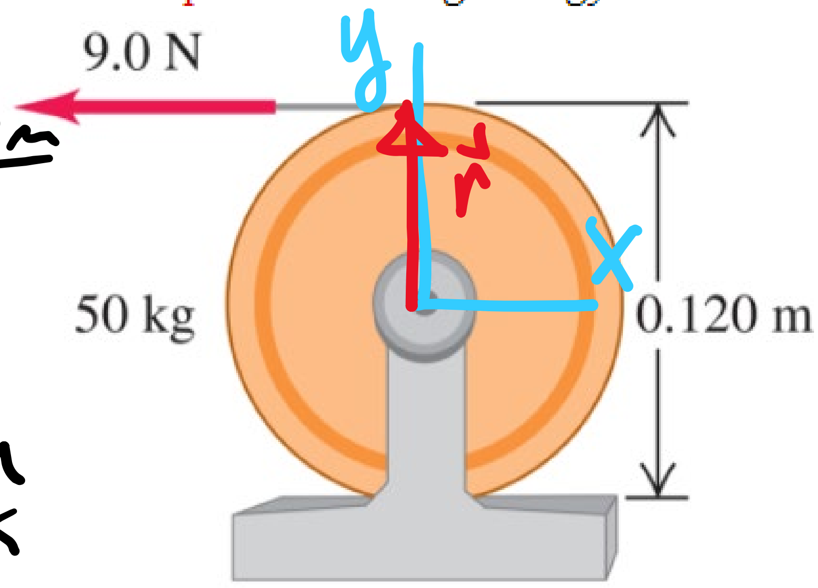


$\vec{r} = R\hat{j}$, where $R = \frac{0.120\,m}{2}$

& $\vec{F} = F(-\hat{i})$ So

$\vec{r} \times \vec{F} = -RF\underbrace{(\hat{j} \times \hat{i})}_{-\hat{k}} = RF\hat{k}$

Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?



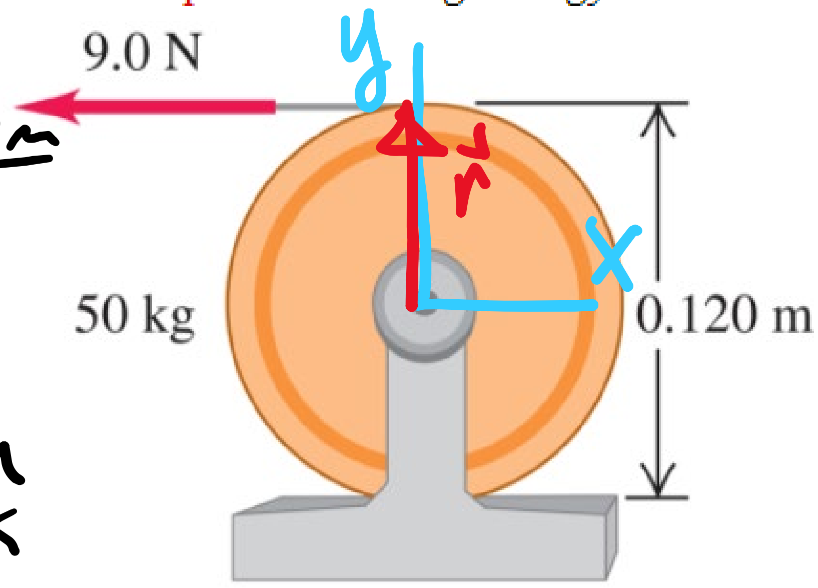
$\vec{r} = R\hat{j}$, where $R = \frac{0.120\,m}{2}$

& $\vec{F} = F(-\hat{i})$ So

$\vec{r} \times \vec{F} = -RF(\hat{j} \times \hat{i}) = RF\hat{k}$ $\quad -\hat{k}$

Now $\vec{\tau} = I_{cm}\vec{\alpha} \Rightarrow$

Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?



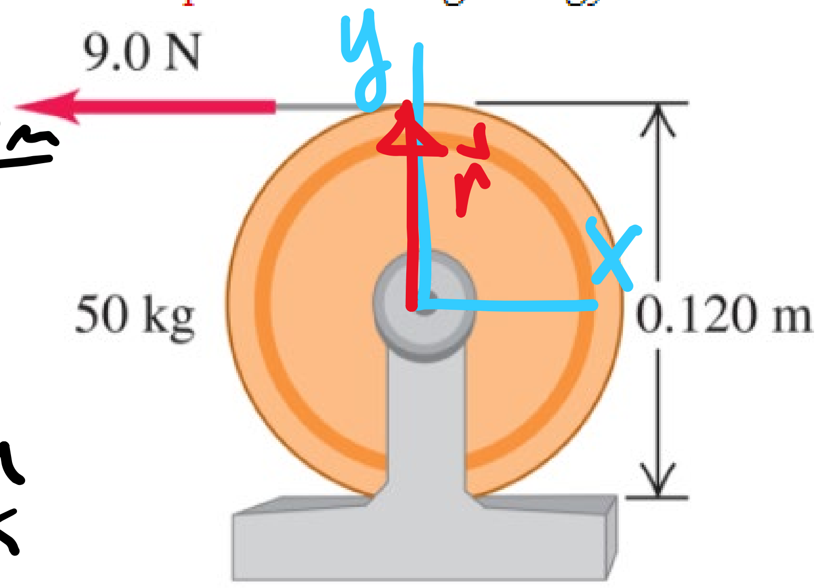
$\vec{r} = R\hat{j}$, where $R = \frac{0.120\,m}{2}$

& $\vec{F} = F(-\hat{i})$ So

$$\vec{r} \times \vec{F} = -RF(\overbrace{\hat{j} \times \hat{i}}^{-\hat{k}}) = RF\hat{k}$$

Now $\vec{\tau} = I_{cm}\vec{\alpha} \Rightarrow RF\hat{k} = I_{cm}\vec{\alpha}$

Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?



$\vec r = R\hat j$, where $R = \frac{0.120\,m}{2}$

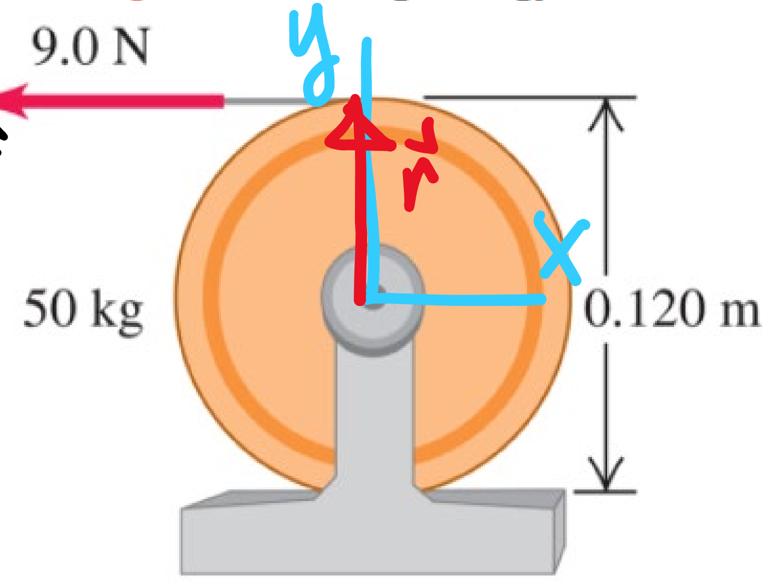
& $\vec F = F(-\hat i)$ So

$\vec r \times \vec F = -RF(\overbrace{\hat j \times \hat i}^{-\hat k}) = RF\hat k$

Now $\vec \tau = I_{cm}\vec \alpha \Rightarrow RF\hat k = I_{cm}\vec\alpha$

But $a = R\alpha$

Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?



$\vec{r} = R\hat{j}$, where $R = \frac{0.120\,m}{2}$

& $\vec{F} = F(-\hat{i})$ so

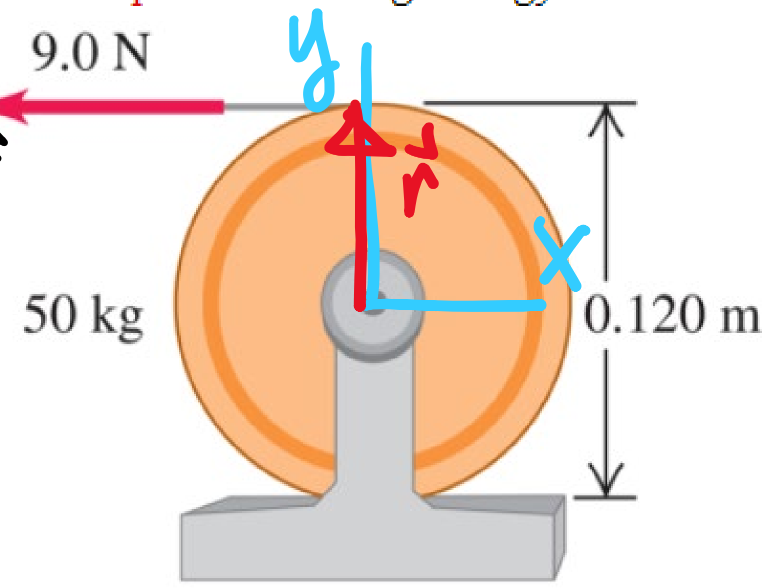
$\vec{r} \times \vec{F} = -RF(\hat{j} \times \hat{i}) = RF\hat{k}$

$-\hat{k}$

Now $\vec{\tau} = I_{cm}\vec{\alpha} \Rightarrow RF\hat{k} = I_{cm}\vec{\alpha}$

But $a = R\alpha$ so $RF = I_{cm}\left(\frac{a}{R}\right)$

Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?



$\vec{r} = R\hat{j}$, where $R = \frac{0.120\,m}{2}$

& $\vec{F} = F(-\hat{i})$ so

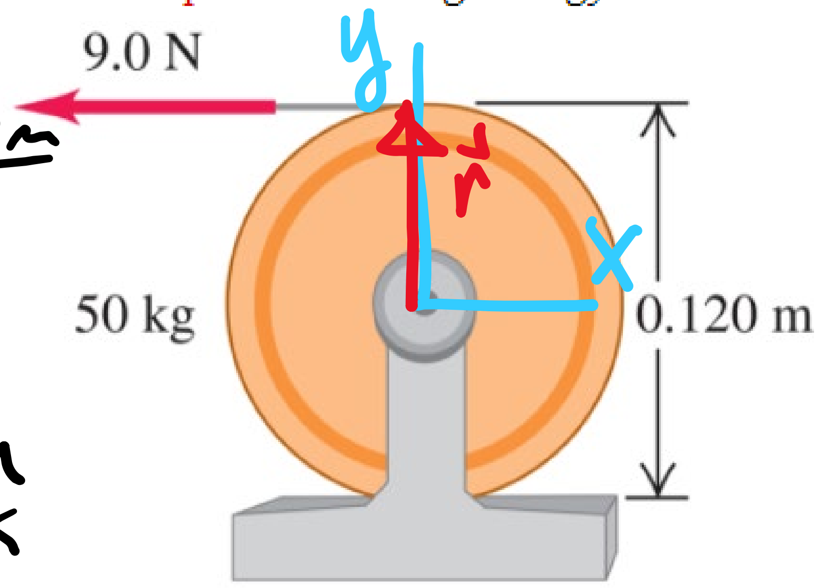
$\vec{r} \times \vec{F} = -RF(\underbrace{\hat{j} \times \hat{i}}_{-\hat{k}}) = RF\hat{k}$

Now $\vec{\tau} = I_{cm}\vec{\alpha} \Rightarrow RF\hat{k} = I_{cm}\vec{\alpha}$

But $a = R\alpha$ so $RF = I_{cm}\left(\frac{a}{R}\right)$

$\Rightarrow a = \frac{R^2 F}{I_{cm}}$

Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?



$\vec{r} = R\hat{j}$, where $R = \frac{0.120\,m}{2}$

& $\vec{F} = F(-\hat{i})$ so

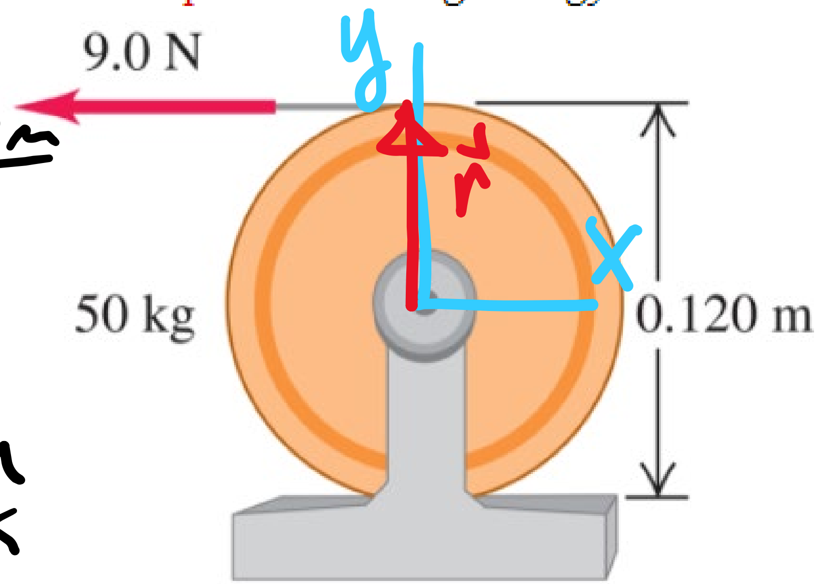
$\vec{r} \times \vec{F} = -RF\underbrace{(\hat{j} \times \hat{i})}_{-\hat{k}} = RF\hat{k}$

Now $\vec{\tau} = I_{cm}\vec{\alpha} \Rightarrow RF\hat{k} = I_{cm}\vec{\alpha}$

But $a = R\alpha$ so $RF = I_{cm}\left(\frac{a}{R}\right)$

$\Rightarrow a = \frac{R^2 F}{I_{cm}}$ But $I_{cm} = MR^2/2$

Figure 10.9a shows the situation that we analyzed in Example 9.7 using energy methods. What is the cable's acceleration?



$$\vec{r} = R\hat{j}, \text{ where } R = \frac{0.120\,\text{m}}{2}$$

$$\& \quad \vec{F} = F(-\hat{i}) \quad \text{so}$$

$$\vec{r}\times\vec{F} = -RF\overbrace{(\hat{j}\times\hat{i})}^{-\hat{k}} = RF\hat{k}$$

$$\text{Now} \quad \vec{\tau} = I_{cm}\vec{\alpha} \Rightarrow RF\hat{k} = I_{cm}\vec{\alpha}$$

$$\text{But} \quad a = R\alpha \quad \text{so} \quad RF = I_{cm}\left(\frac{a}{R}\right)$$

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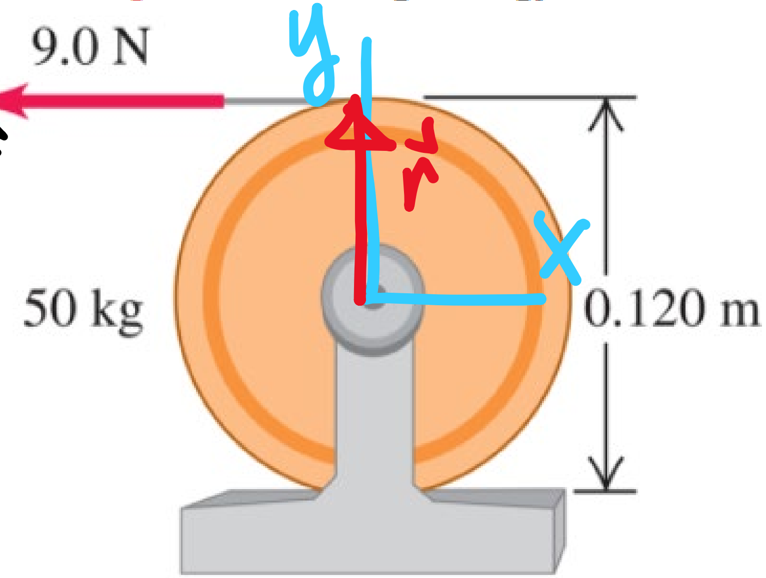

$\vec{r} = R\hat{j}$, where $R = \frac{0.120\,m}{2}$

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$\vec{r} \times \vec{F} = -RF\underbrace{(\hat{j}\times\hat{i})}_{-\hat{k}} = RF\hat{k}$

Now $\vec{\tau} = I_{cm}\vec{\alpha} \Rightarrow RF\hat{k} = I_{cm}\vec{\alpha}$

But $a = R\alpha$ so $RF = I_{cm}\left(\frac{a}{R}\right)$

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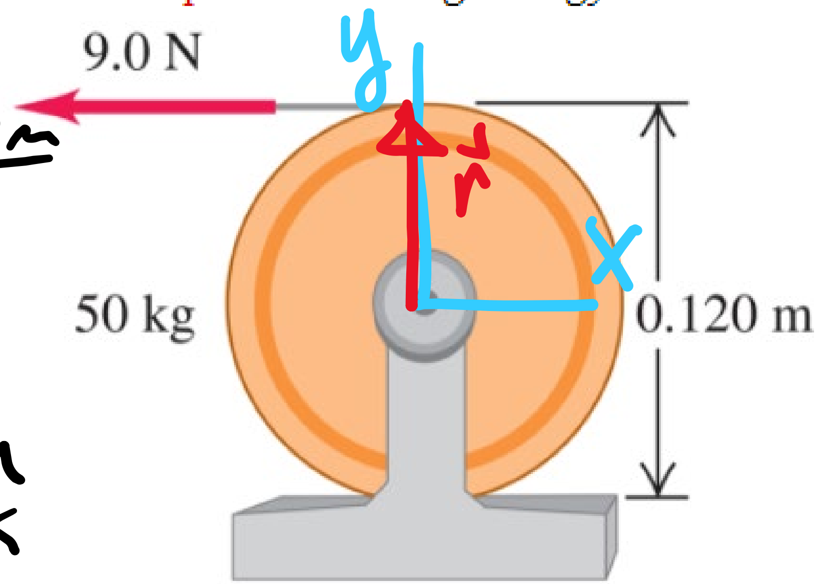

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Now $\vec{\tau} = I_{cm}\vec{\alpha} \Rightarrow RF\hat{k} = I_{cm}\vec{\alpha}$

But $a = R\alpha$ so $RF = I_{cm}\left(\frac{a}{R}\right)$

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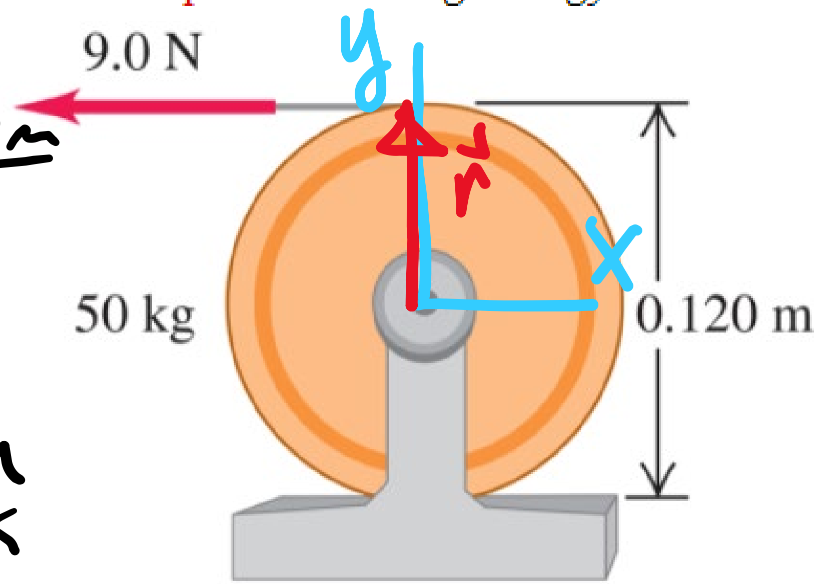

$\vec{r} = R\hat{j}$, where $R = \frac{0.120\,m}{2}$

& $\vec{F} = F(-\hat{i})$ so

$\vec{r} \times \vec{F} = -RF(\hat{j} \times \hat{i}) = RF\hat{k}$

$-\hat{k}$

Now $\vec{\tau} = I_{cm}\vec{\alpha} \Rightarrow RF\hat{k} = I_{cm}\vec{\alpha}$

But $a = R\alpha$ so $RF = I_{cm}\left(\frac{a}{R}\right)$

$\Rightarrow a = \frac{R^2F}{I_{cm}}$ But $I_{cm} = MR^2/2$

$$\Rightarrow \boxed{a = \frac{R^2F}{MR^2/2} = \frac{2F}{m} = \frac{18\,m}{50\,s^2} = 0.36\frac{m}{s^2}}$$

We wrap a light, nonstretching cable around a solid cylinder with mass M and radius R. The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass m and release the block from rest at a distance h above the floor. As the block falls, the cable unwinds without stretching or slipping. Find acceleration of falling block & tension in cable

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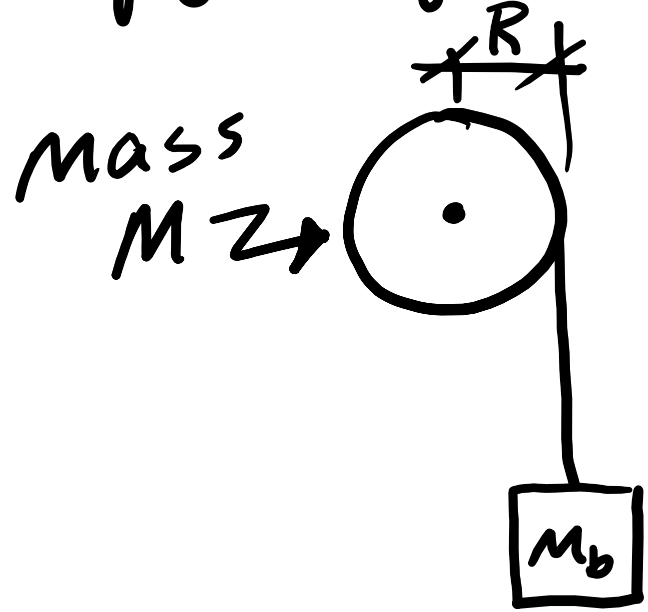

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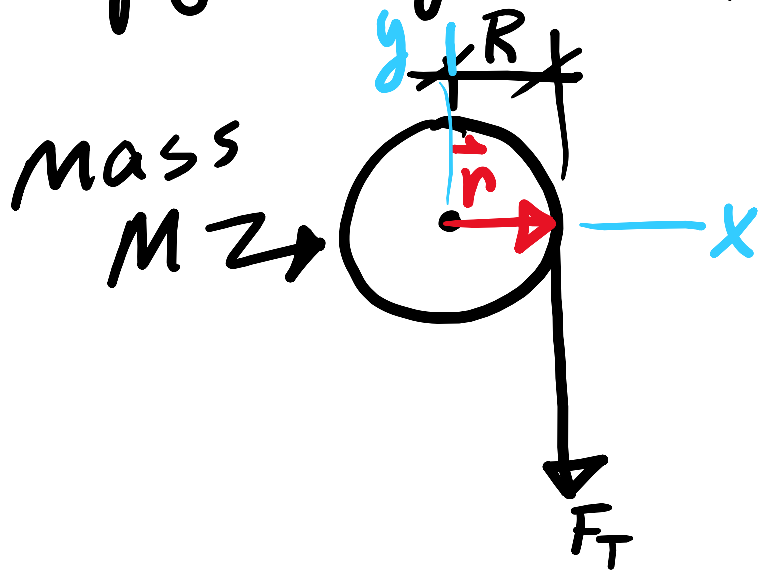
$$\vec{\tau} = \vec{r} \times \vec{F}_T$$



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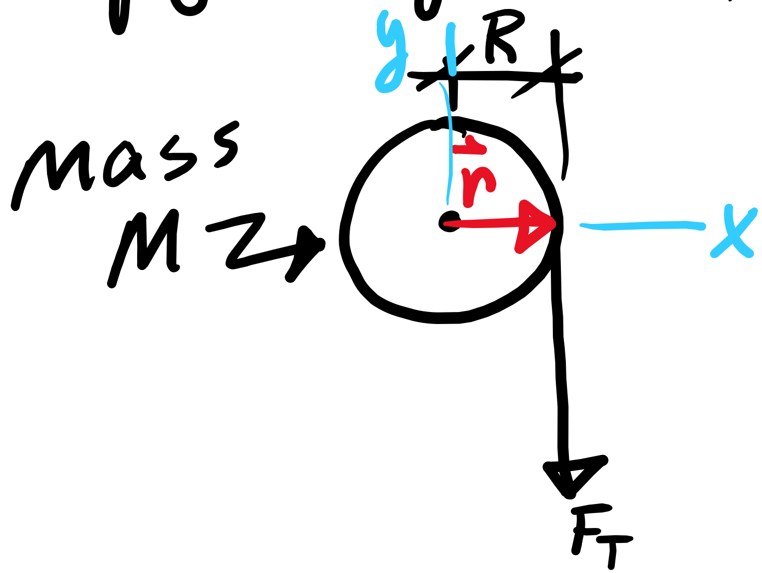
$$\vec{\tau} = \vec{r} \times \vec{F}_T = R F_T(-\hat{k})$$



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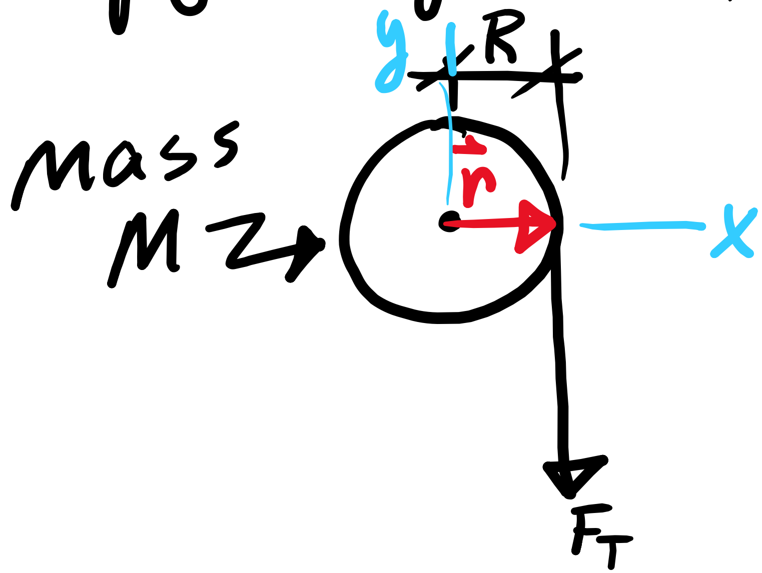
$\vec{\tau} = I_{cm} \vec{\alpha}$



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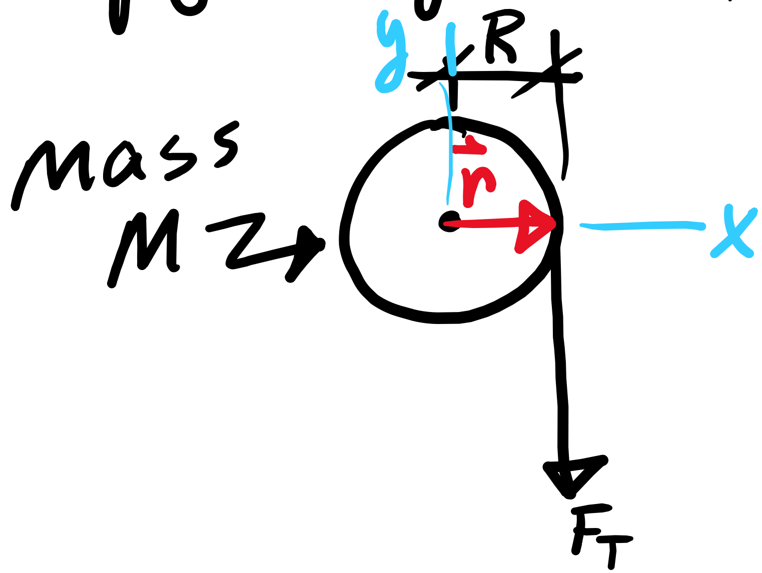

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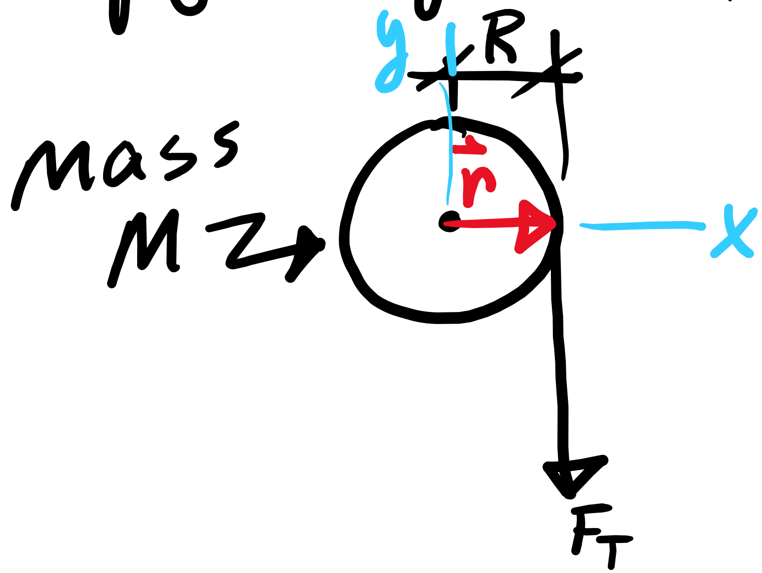

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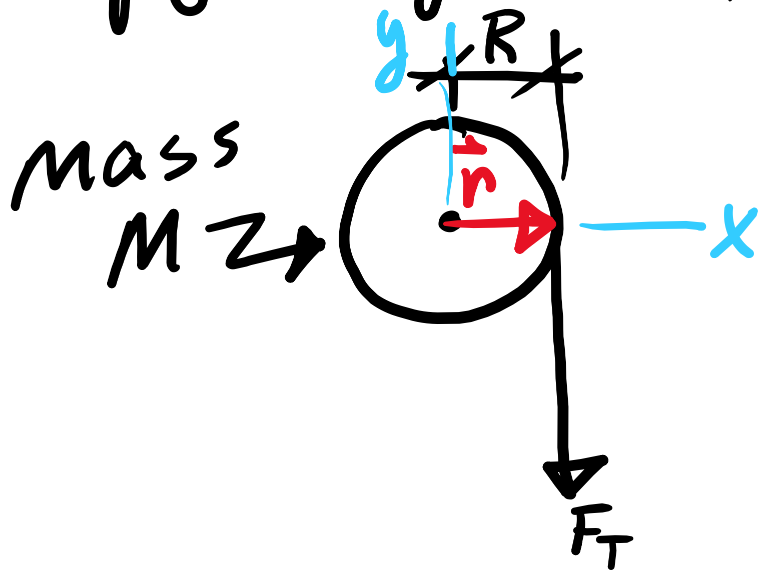

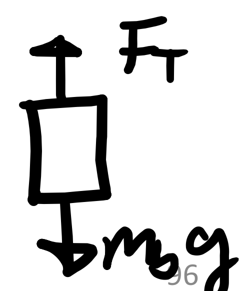

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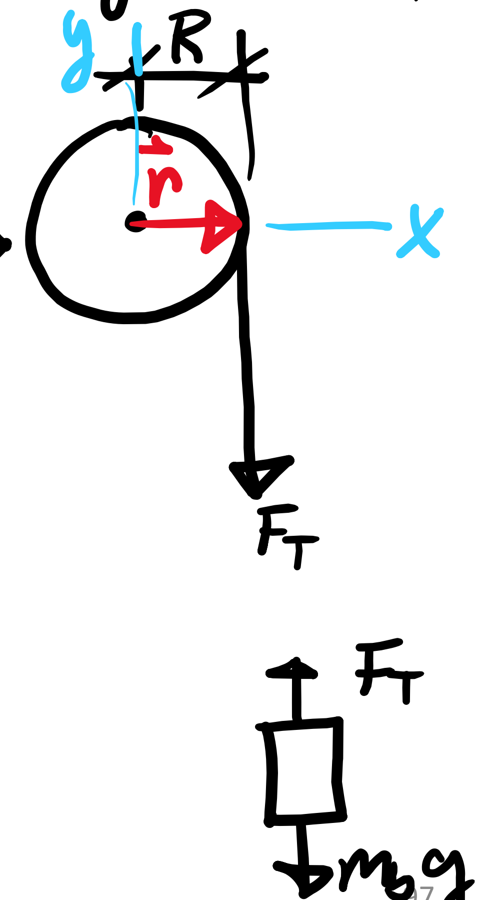
$$R F_T = I_{cm} \alpha \quad (1) \qquad \sum F_y = m_b a$$

mass M



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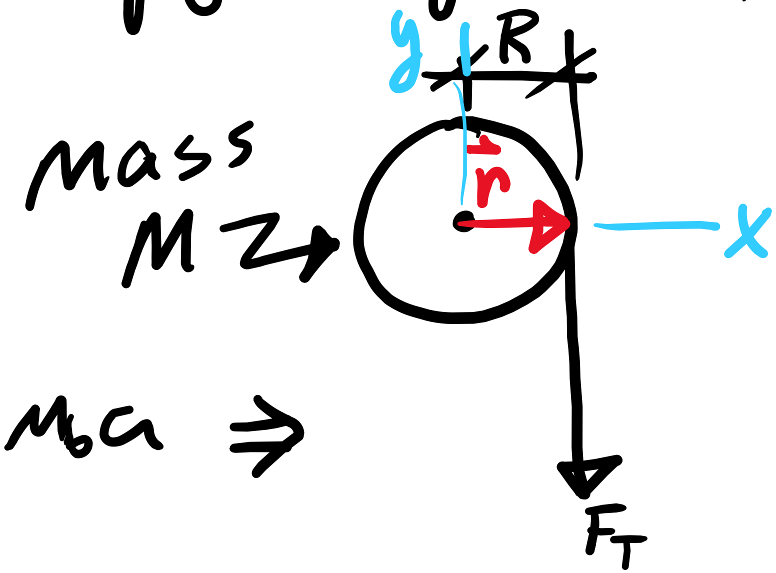

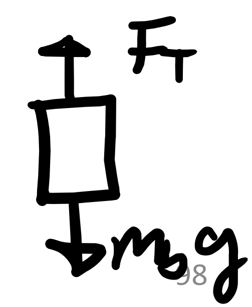

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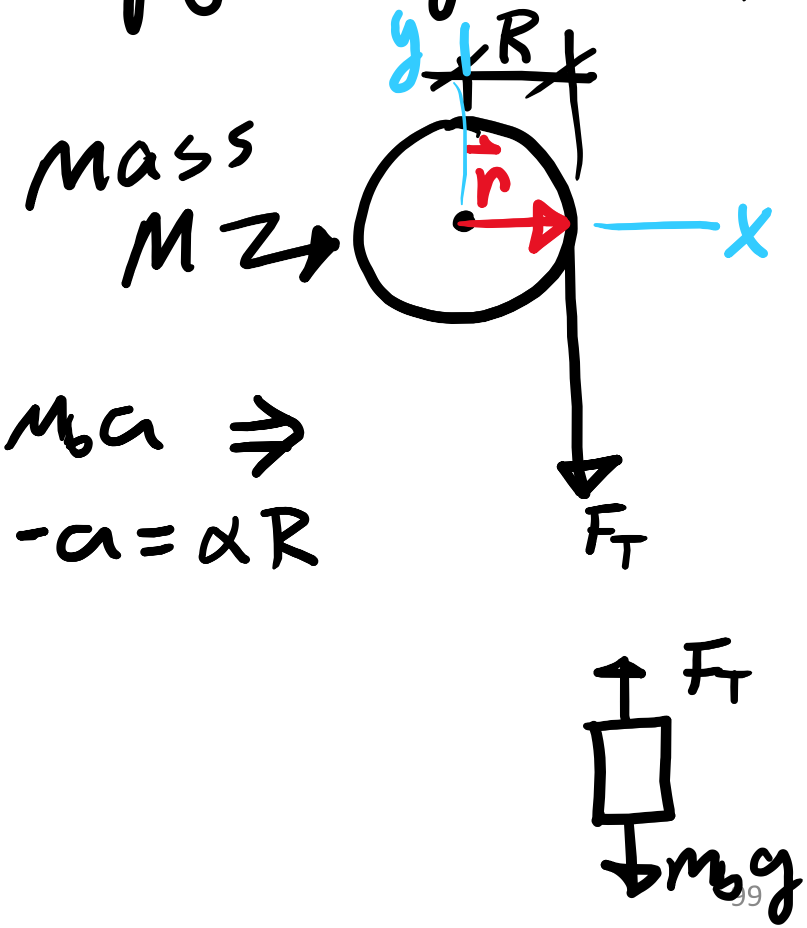

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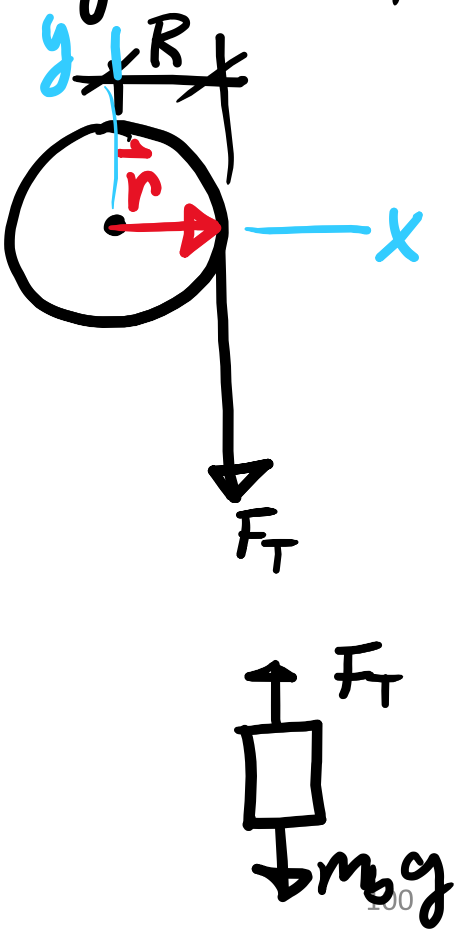

$$R F_T = I_{cm}\alpha \quad (1) \qquad \sum F_y = m_b a \Rightarrow$$

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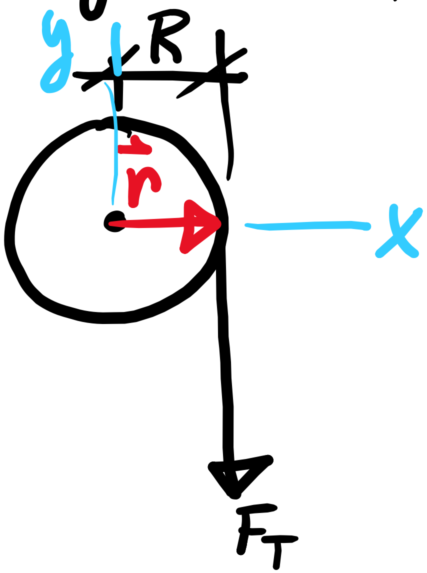

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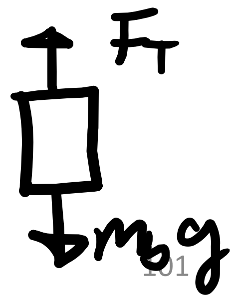

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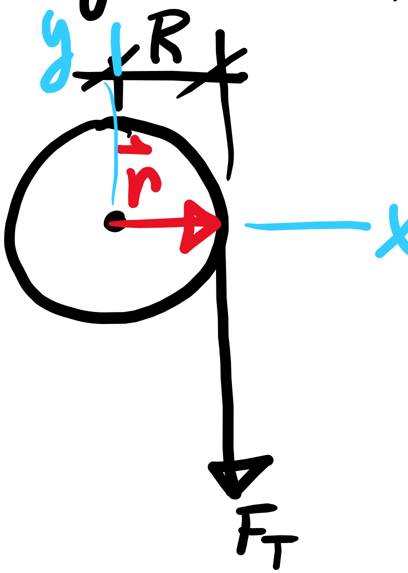

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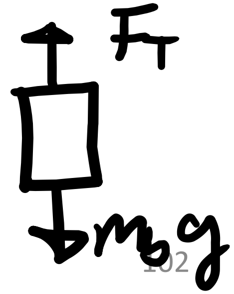

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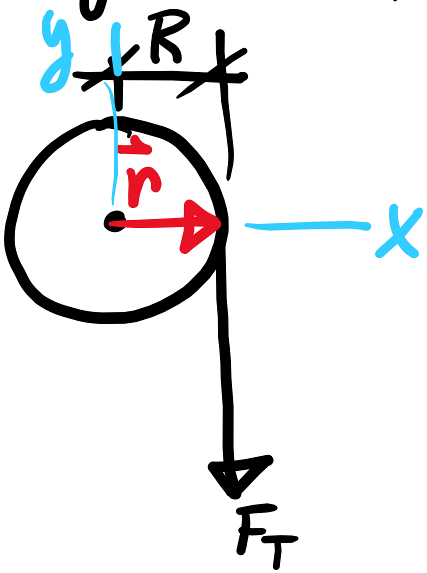

$R F_T = I_{cm}\alpha$ (1)

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$F_T - M_b g = M_b a$ But $-a = \alpha R$

So $-\alpha M_b R = F_T - M_b g \Rightarrow$

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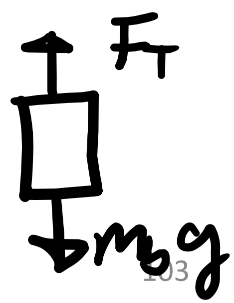

From previous slide we have

$$RF_T = I_{cm}\alpha \quad (1)$$

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$$RF_T = I_{cm}\alpha \quad (1) \qquad \& \quad -\alpha = \frac{F_T - m_b g}{M_b R} \quad (2)$$

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Sub Eqn 2 into Eqn 1

From previous slide we have

$$RF_T = I_{cm}\alpha \quad (1) \qquad \& \quad -\alpha = \frac{F_T - M_b g}{M_b R} \quad (2)$$

Sub Eqn 2 into Eqn 1 $\Rightarrow$

$$-RF_T = I_{cm}\left[\frac{F_T - M_b g}{M_b R}\right]$$

From previous slide we have

$$RF_T = I_{cm}\alpha \quad (1) \qquad \& \; -\alpha = \frac{F_T - M_b g}{M_b R} \quad (2)$$

Sub Eqn 2 into Eqn 1 $\Rightarrow$

$$-RF_T = I_{cm}\left[\frac{F_T - M_b g}{M_b R}\right] \Rightarrow -RF_T - \frac{I_{cm}}{M_b R}F_T = -I_{cm}\frac{g}{R}$$

From previous slide we have

$$R F_T = I_{cm} \alpha \quad (1) \qquad \& \quad -\alpha = \frac{F_T - M_b g}{M_b R} \quad (2)$$

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so $\left[\frac{M_b R^2 + \frac{1}{2}MR^2}{M_b R}\right] F_T = \frac{1}{2}MRg$

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so $$\left[\frac{M_b R^{\cancel{2}} + \frac{1}{2}MR^{\cancel{2}}}{M_b \cancel{R}}\right]F_T = \frac{1}{2}MRg$$

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so

$$\left[\frac{M_b \cancel{R^2} + \frac{1}{2}M\cancel{R^2}}{M_b \cancel{R}}\right]F_T = \frac{1}{2}M\cancel{R}g \Rightarrow$$

$$F_T = \frac{MgM_b}{2\left(M_b + \frac{1}{2}M\right)}$$

From previous slide we have

$$RF_T = I_{cm}\alpha \quad (1) \qquad \& \quad -\alpha = \frac{F_T - M_b g}{M_b R} \quad (2)$$

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$$\text{so } \left[\frac{M_b \cancel{R}\cancel{^2} + \frac{1}{2}M\cancel{R}\cancel{^2}}{M_b \cancel{R}}\right]F_T = \frac{1}{2}M\cancel{R}g \Rightarrow$$

$$F_T = \frac{MgM_b}{2\left(M_b + \frac{1}{2}M\right)} = \frac{MM_b g}{2M_b + M}$$

ASU

From previous slide we have

$RF_T = I_{cm}\alpha$ (1) & $-\alpha = \dfrac{F_T - M_b g}{M_b R}$ (2)

Sub Eqn 2 into Eqn 1 $\Rightarrow$

$$-RF_T = I_{cm}\left[\frac{F_T - M_b g}{M_b R}\right] \Rightarrow -RF_T - \frac{I_{cm}}{M_b R}F_T = -I_{cm}\frac{g}{R}$$

$$\Rightarrow -\left[R - \frac{I_{cm}}{M_b R}\right]F_T = -I_{cm}\left(\frac{g}{R}\right)$$ But $I_{cm} = MR^2/2$

so $\left[\dfrac{M_b \cancel{R^2} + \frac{1}{2}M\cancel{R^2}}{M_b \cancel{R}}\right]F_T = \frac{1}{2}M\cancel{R}g \Rightarrow$

$$\boxed{F_T = \frac{MgM_b}{2(M_b + \frac{1}{2}M)} = \frac{MM_b g}{2M_b + M} = \frac{M_b g}{2M_b/M + 1}}$$

ASU

From previous slide we have

$$RF_T = I_{cm}\alpha \quad (1) \qquad \& \quad -\alpha = \frac{F_T - M_b g}{M_b R} \quad (2)$$

Sub Eqn 2 into Eqn 1 $\Rightarrow$

$$-RF_T = I_{cm}\left[\frac{F_T - M_b g}{M_b R}\right] \Rightarrow -RF_T - \frac{I_{cm}}{M_b R}F_T = -I_{cm}\frac{g}{R}$$

$$\Rightarrow -\left[R - \frac{I_{cm}}{M_b R}\right]F_T = -I_{cm}\left(\frac{g}{R}\right)$$ But $I_{cm} = MR^2/2$

so $$\left[\frac{M_b \cancel{R^2} + \frac{1}{2}M\cancel{R^2}}{M_b \cancel{R}}\right]F_T = \frac{1}{2}M\cancel{R}g \Rightarrow$$

$$\boxed{F_T = \frac{MgM_b}{2\left(M_b + \frac{1}{2}M\right)} = \frac{MM_b g}{2M_b + M} = \frac{M_b g}{2M_b/M + 1}} \quad (3)$$

From previous

$$R F_T = I_{cm} \alpha \quad (1)$$

From previous

$R F_T = I_{cm}\alpha$ (1) & $F_T = \dfrac{m_b g}{2m_b/m + 1}$ (3)

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From previous

$RF_T = I_{cm}\alpha$ (1) & $F_T = \dfrac{m_b g}{2m_b/m + 1}$ (3)

Also $\alpha R = -a$ so Eqn 1: $-R^2 F_T = I_{cm} a$

From previous

$R F_T = I_{cm}\alpha$ (1) & $F_T = \dfrac{M_b g}{2M_b/M + 1}$ (3)

Also $\alpha R = -a$ so Eqn 1: $-R^2 F_T = I_{cm} a$

$\Rightarrow a = -\dfrac{R^2 F_T}{I_{cm}}$

From previous

$$R F_T = I_{cm}\alpha \quad (1) \quad \& \quad F_T = \frac{M_b g}{2M_b/M + 1} \quad (3)$$

Also $\alpha R = -a$ so Eqn 1: $-R^2 F_T = I_{cm} a$

$$\Rightarrow \quad a = -\frac{R^2 F_T}{I_{cm}} = -\frac{R^2 F_T}{\frac{1}{2} M R^2}$$

From previous

$R F_T = I_{cm}\alpha$ (1) & $F_T = \frac{M_b g}{2M_b/M + 1}$ (3)

Also $\alpha R = -a$ so Eqn 1: $-R^2 F_T = I_{cm} a$

$\Rightarrow a = -\frac{R^2 F_T}{I_{cm}} = -\frac{R^2 F_T}{\frac{1}{2}MR^2} = \frac{2F_T}{M}$

From previous

$$R F_T = I_{cm}\alpha \quad (1) \quad \& \quad F_T = \frac{M_b g}{2M_b/M + 1} \quad (3)$$

Also $\alpha R = -a$ so Eqn 1: $-R^2 F_T = I_{cm} a$

$$\Rightarrow \quad a = -\frac{R^2 F_T}{I_{cm}} = -\frac{R^2 F_T}{\frac{1}{2} M R^2} = \frac{2F_T}{M}$$

From previous

$RF_T = I_{cm}\alpha$ (1) & $F_T = \dfrac{M_b g}{2M_b/M + 1}$ (3)

Also $\alpha R = -a$ so Eqn 1: $-R^2 F_T = I_{cm} a$

$$\Rightarrow \boxed{a = -\frac{R^2 F_T}{I_{cm}} = -\frac{R^2 F_T}{\frac{1}{2}MR^2} = \frac{2F_T}{M} = \frac{2M_b g}{2M_b + M}}$$