Today 9.6, 9.5

$\boxed{L29}$

L29

Today 9.6, 9.5

Moment of inertia calculations

Today 9.6, 9.5

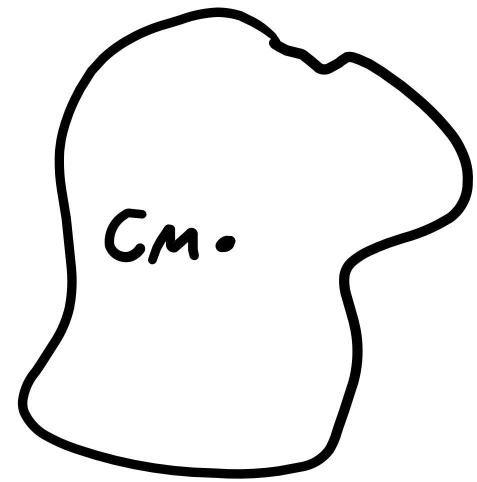
L29

Moment of inertia calculations

Parallel axis theorem

Moment of inertia

$$I_{cm} = \sum r_i^2 \Delta m_i$$



Moment of inertia

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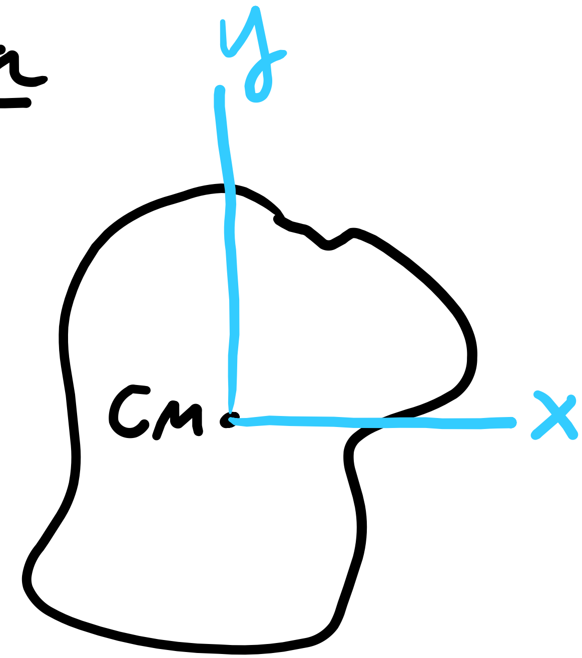

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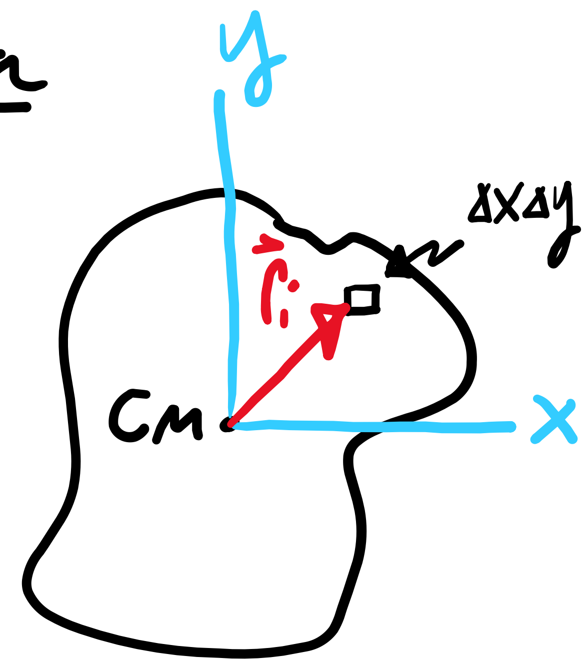
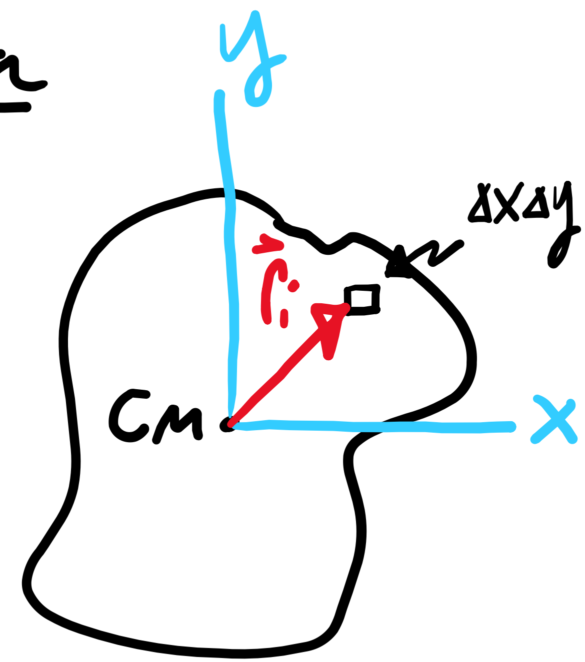


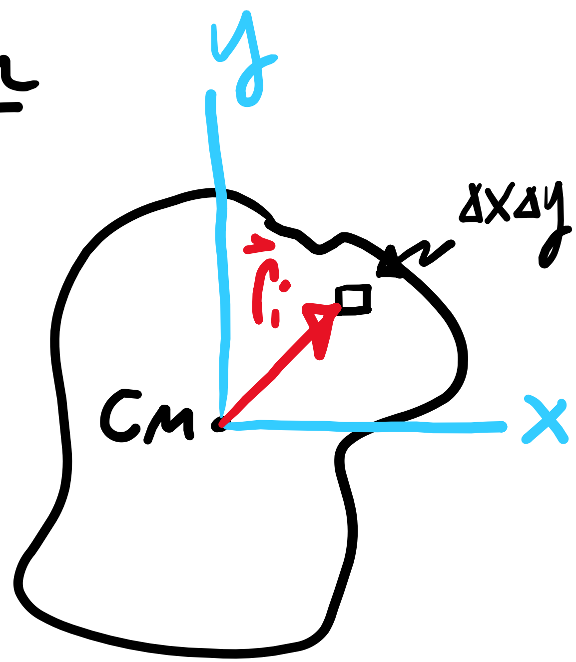
Moment of inertia

$I_{cm} = \sum r_i^2 \Delta M_i$. If mass is uniform over 2-d surface with total area A



Moment of inertia

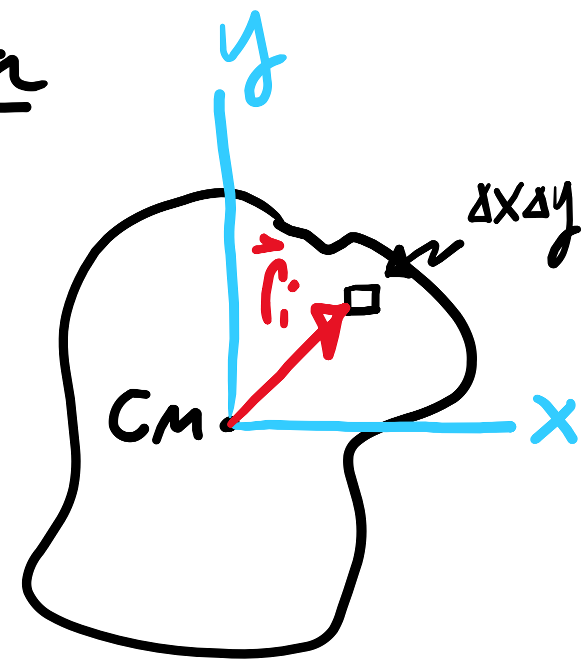
$I_{cm} = \sum r_i^2 \Delta M_i$. If mass is uniform over 2-d surface with total area A & total mass M



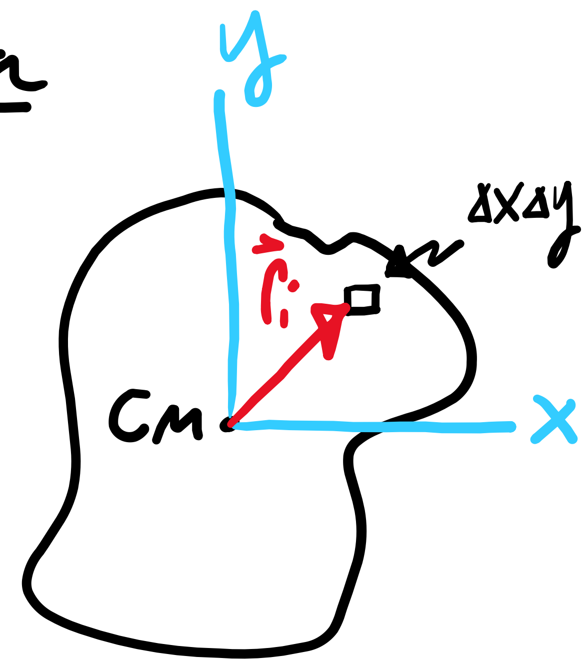
Moment of inertia

$I_{cm} = \sum r_i^2 \Delta M_i$. If mass is uniform over 2-d surface with total area A & total mass M, then

$\Delta M_i = \left(\frac{M}{A}\right) \Delta x_i \Delta y_i$



Moment of inertia



$I_{cm} = \sum r_i^2 \Delta M_i$. If mass is uniform over 2-d surface with total area A & total mass M, then

$\Delta M_i = \left(\frac{M}{A}\right)\Delta x \, \Delta y$ Now $I_{cm} = \left(\frac{M}{A}\right)\sum r_i^2 \Delta x \, \Delta y$

Moment of inertia

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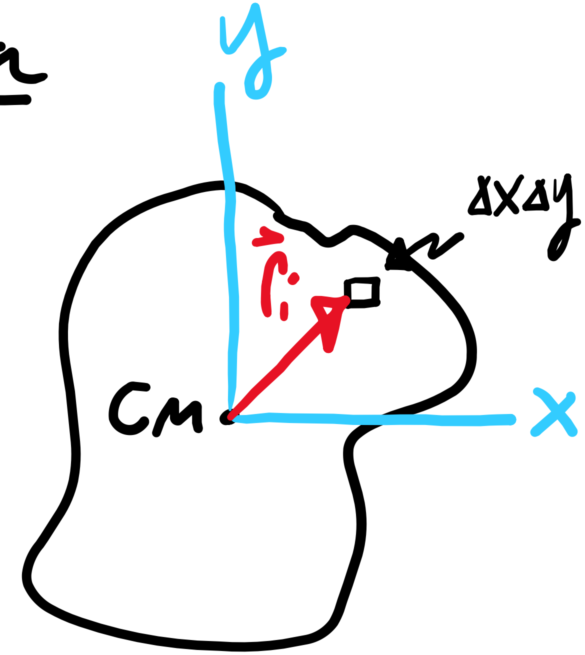

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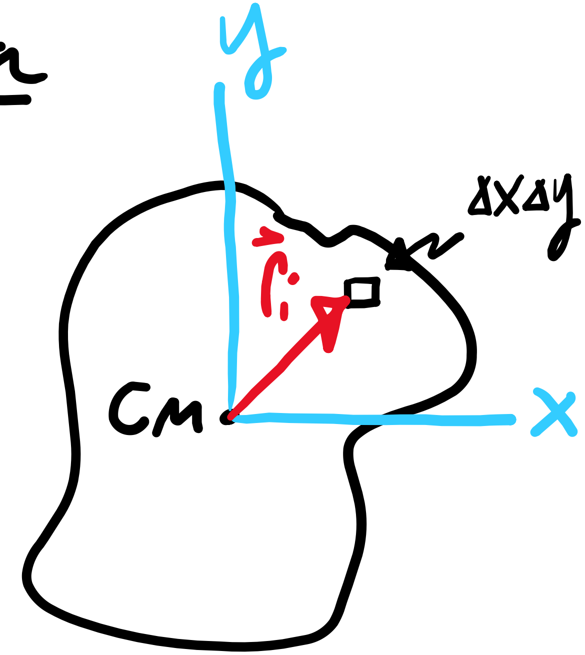

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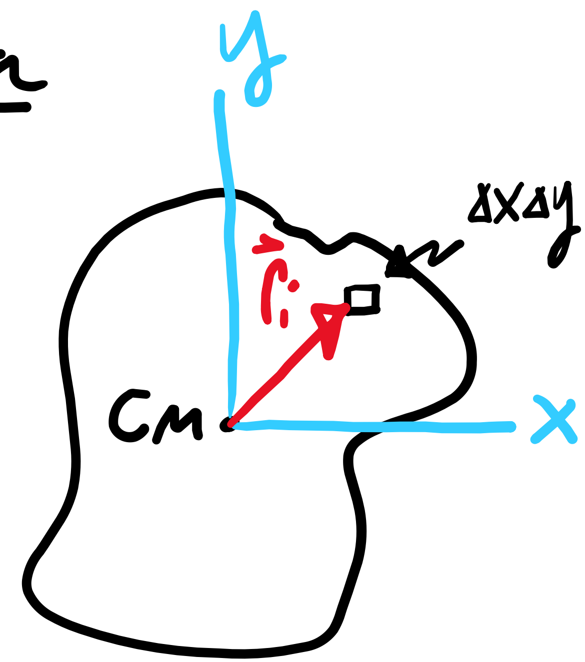
Moment of inertia



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For polar coordinates $dx\,dy = r\,dr\,d\theta$

For 1-2 :

For 1-2: replace $\frac{M}{A}$ with $\frac{M}{L}$

For 1-d: replace $\frac{M}{A}$ with $\frac{M}{L}$

& $dx\,dy$ with dx

For 1-d: replace $\frac{M}{A}$ with $\frac{M}{L}$

& $dx\,dy$ with dx

For 3-d:

For 1-d: replace $\frac{m}{A}$ with $\frac{m}{L}$

& $dx\,dy$ with dx

For 3-d: replace $\frac{m}{A}$ with $\frac{m}{V}$

For 1-d: replace $\frac{M}{A}$ with $\frac{M}{L}$

& $dx\,dy$ with dx

For 3-d: replace $\frac{M}{A}$ with $\frac{M}{V}$

& $dx\,dy$ with $dx\,dy\,dz$

For 1-d: replace $\frac{M}{A}$ with $\frac{M}{L}$

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For cylindrical coordinates

$dx\,dy\,dz \rightarrow r\,dr\,d\theta\,dz$

For 1-d: replace $\frac{M}{A}$ with $\frac{M}{L}$

& $dx\,dy$ with dx

For 3-d: replace $\frac{M}{A}$ with $\frac{M}{V}$

& $dx\,dy$ with $dx\,dy\,dz$

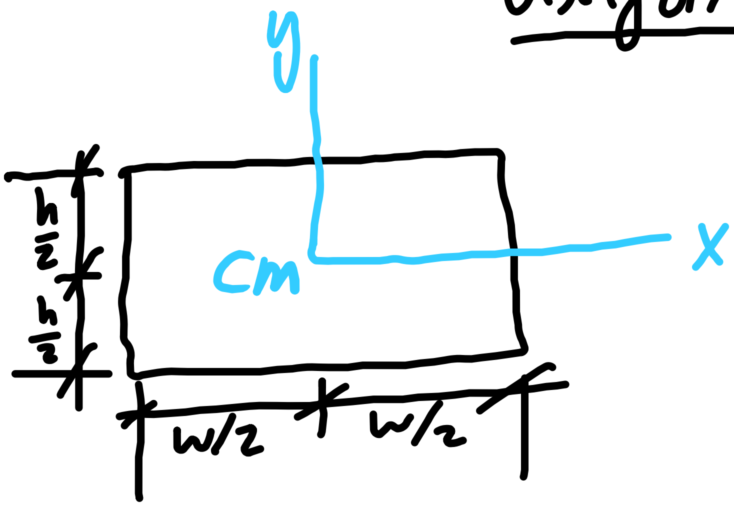
For cylindrical coordinates

$dx\,dy\,dz \rightarrow r\,dr\,d\theta\,dz$

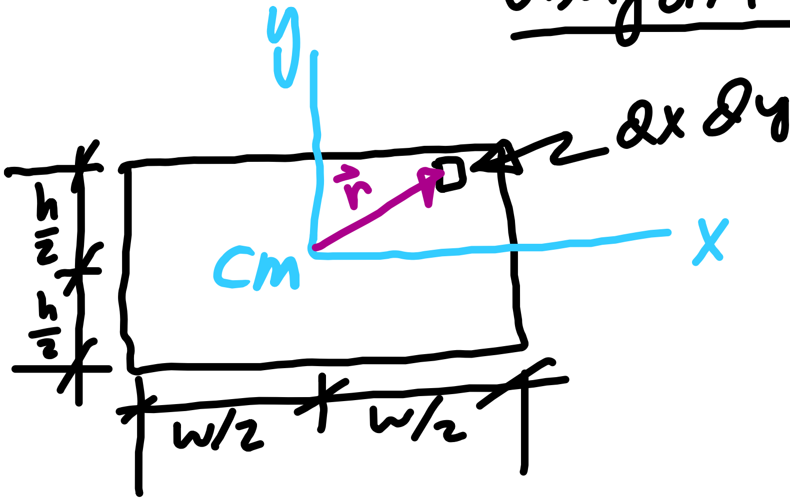
For polar coordinates

$dx\,dy\,dz \rightarrow r\sin\varphi\,dr\,d\varphi\,d\theta$

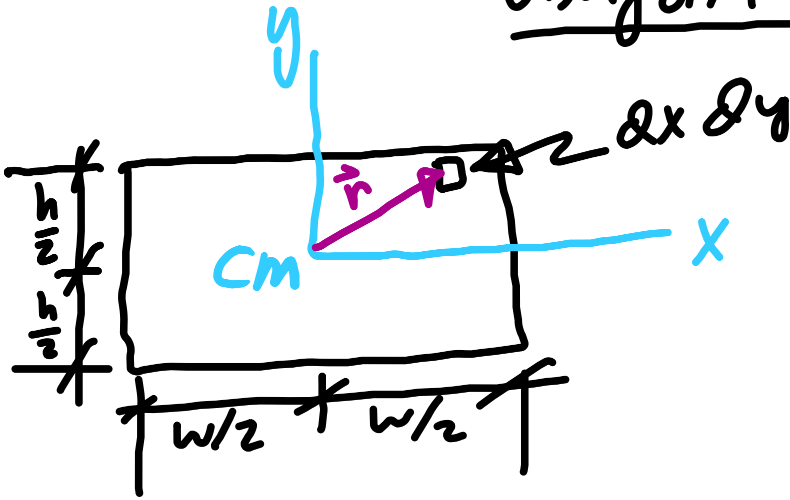
Uniform plate



Uniform plate

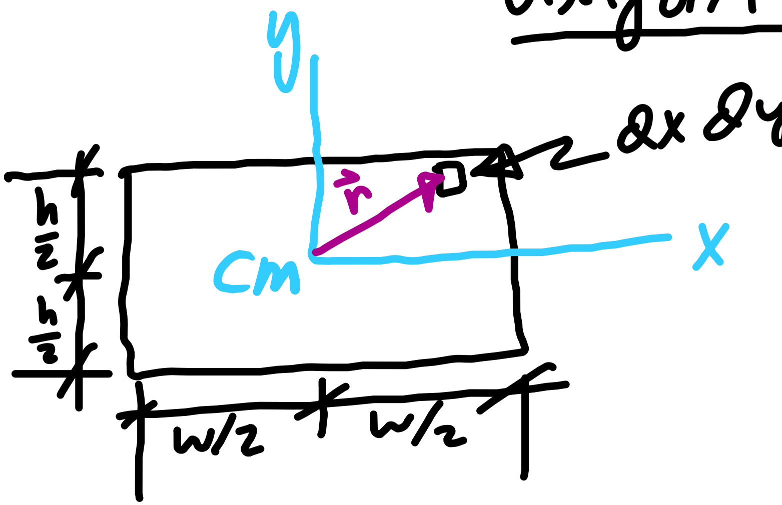


Uniform plate



$$I_{cm} = \left(\frac{M}{A}\right)\iint r^2\,dx\,dy$$

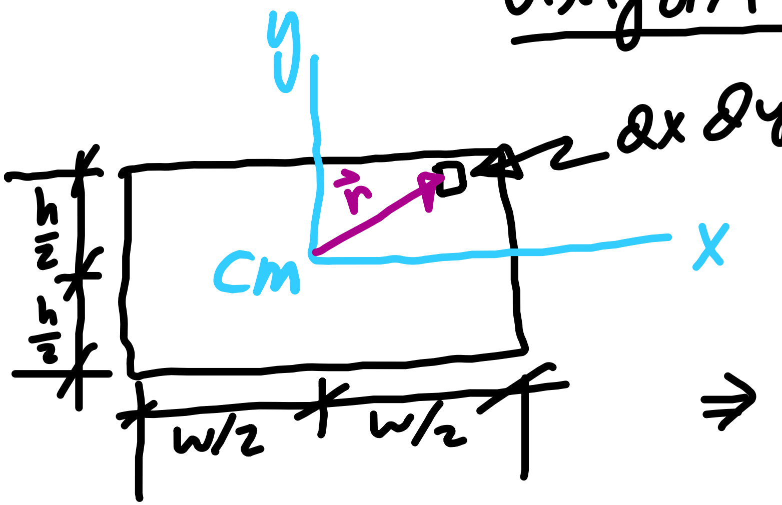
Uniform plate



$$I_{cm} = \left(\frac{M}{A}\right)\iint r^2\,dx\,dy$$

$$\Rightarrow I_{cm} = \left(\frac{M}{A}\right)\int_{-h/2}^{h/2}\int_{-w/2}^{w/2}(x^2+y^2)\,dx\,dy$$

Uniform plate



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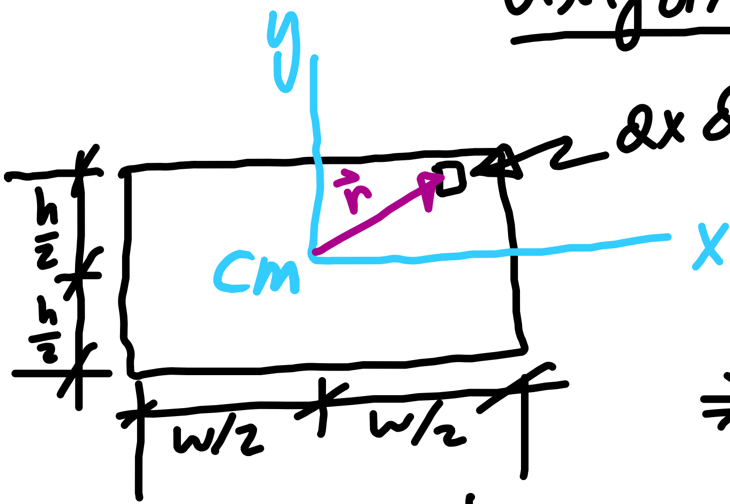
Uniform plate



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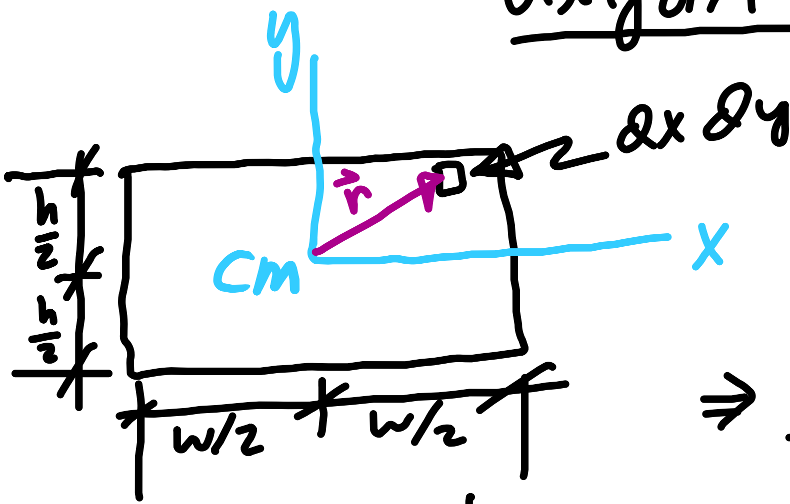
$$\Rightarrow I_{cm} = \left(\frac{M}{A}\right) \int_{-h/2}^{h/2} \int_{-w/2}^{w/2} (x^2 + y^2) \, dx \, dy$$

$$\Rightarrow I_{cm} = \left(\frac{M}{A}\right) \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\frac{x^3}{3} + y^2 x'\right) \Big|_{-\frac{w}{2}}^{w/2} dy$$

$$\Rightarrow I_{cm} = \left(\frac{M}{A}\right) \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(w^3/12 + w y^2\right) dy$$

Uniform plate



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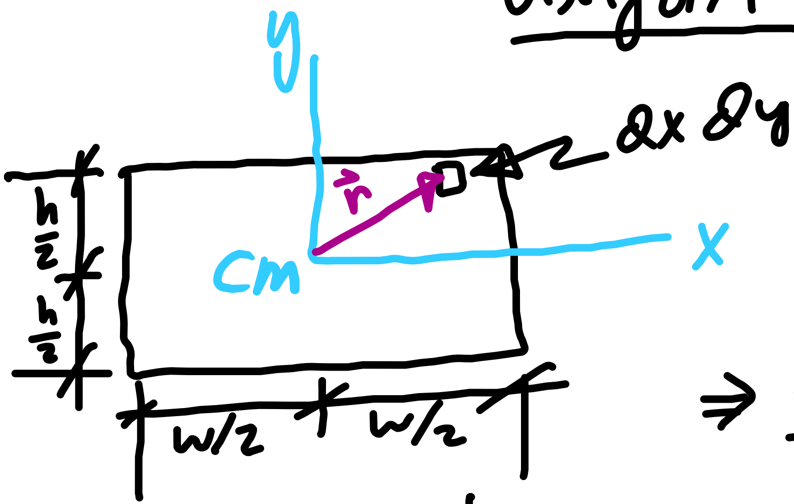
Since

$$x^3\Big|_{-w/2}^{w/2} = \frac{w^3}{2^3} - \left(-\frac{w^3}{2^3}\right)$$

Uniform plate



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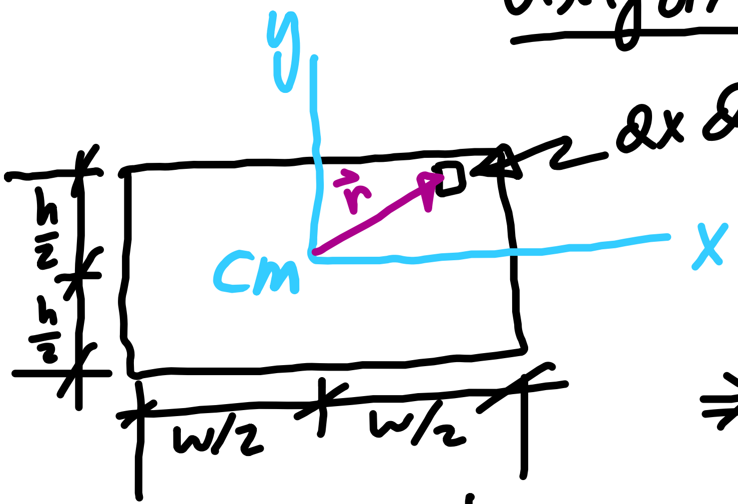
Uniform plate



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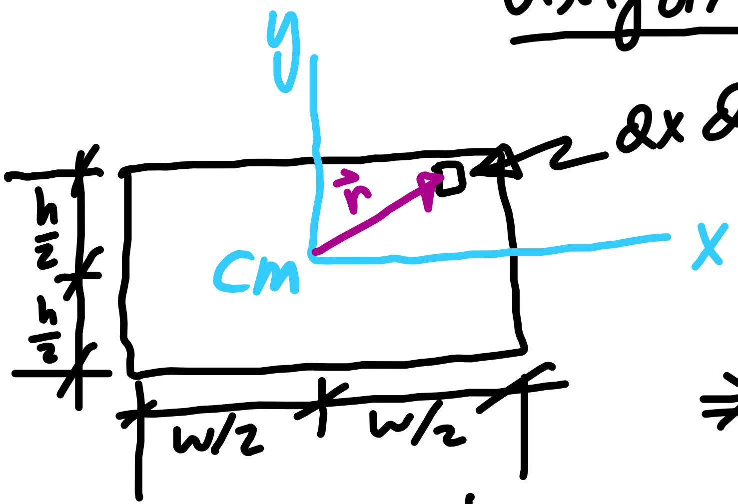
Uniform plate



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&

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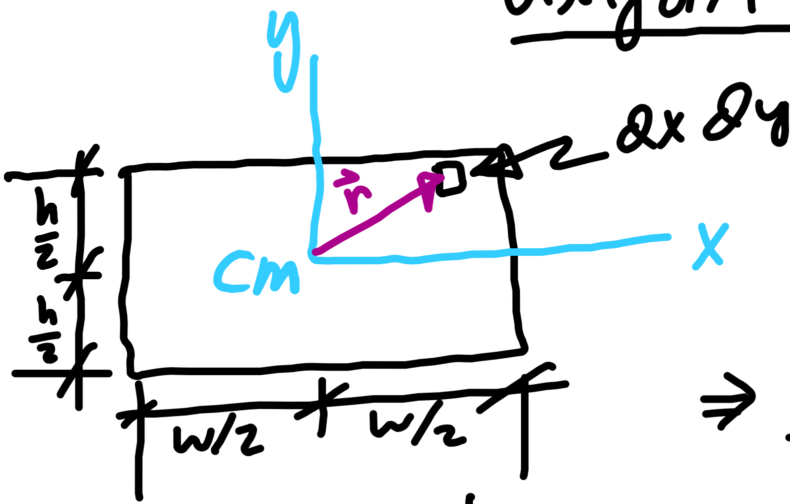
Uniform plate



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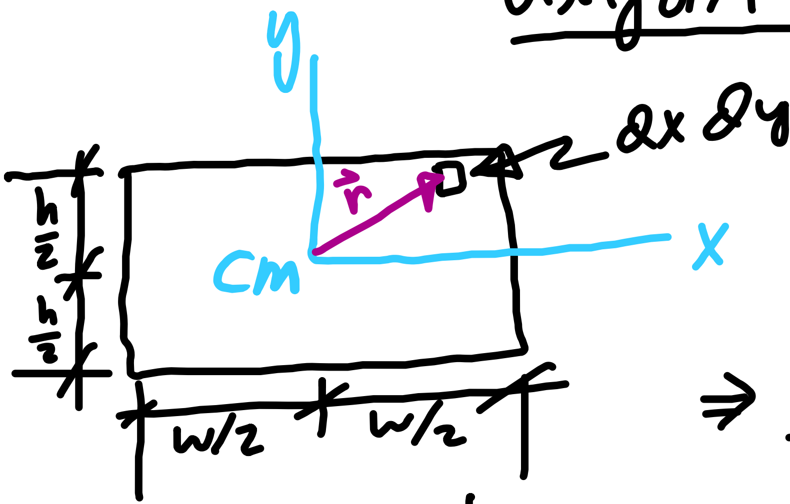
&

$$x\Big|_{-w/2}^{w/2} = \frac{w}{2} - \left(-\frac{w}{2}\right) = w$$

Uniform plate



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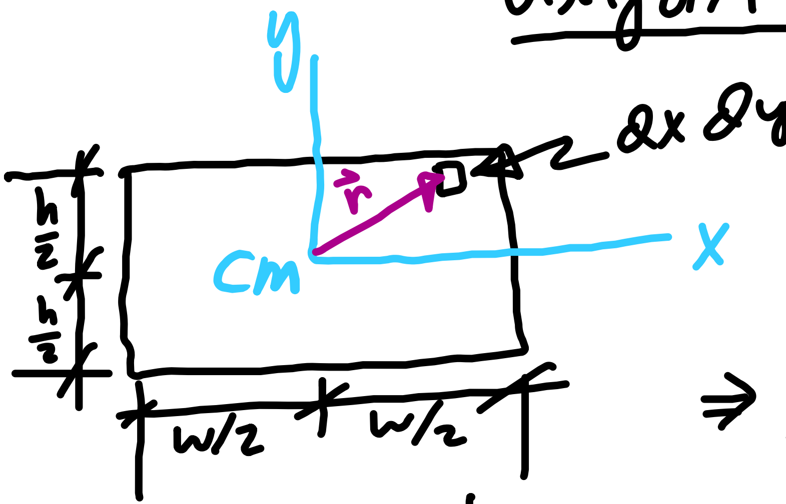
Uniform plate



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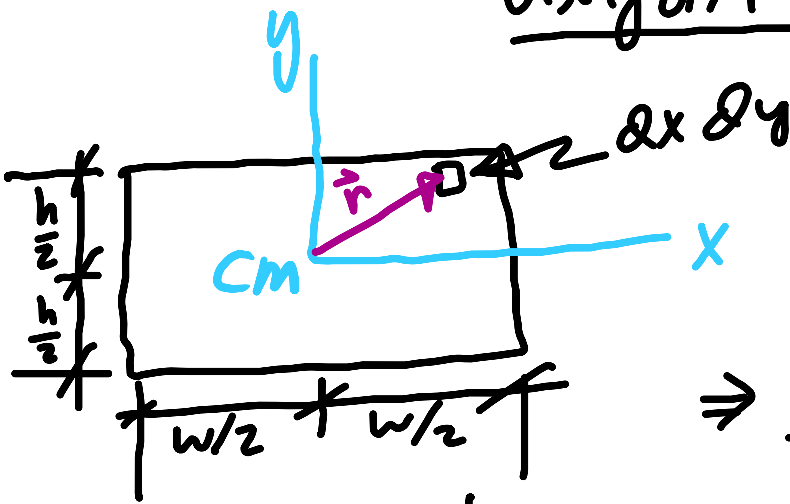
$$\Rightarrow I_{cm} = \left(\frac{M}{A}\right)\left[\frac{w^3h}{12} + \frac{wh^3}{12}\right]$$

Uniform plate



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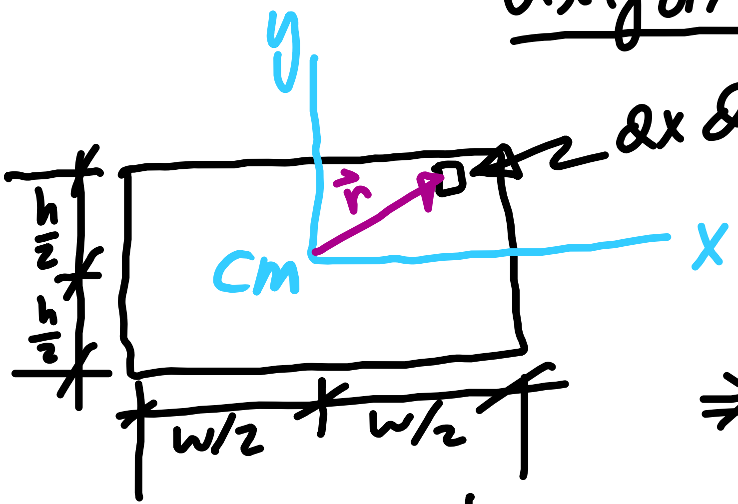
Uniform plate



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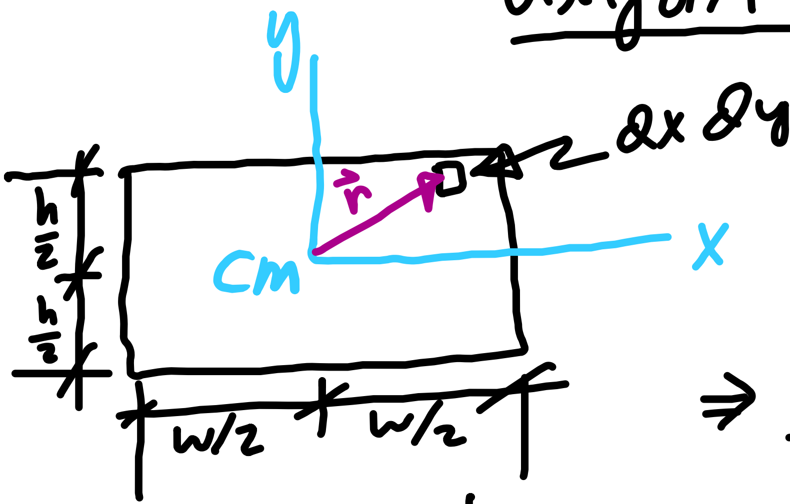
But $A = wh$

Uniform plate



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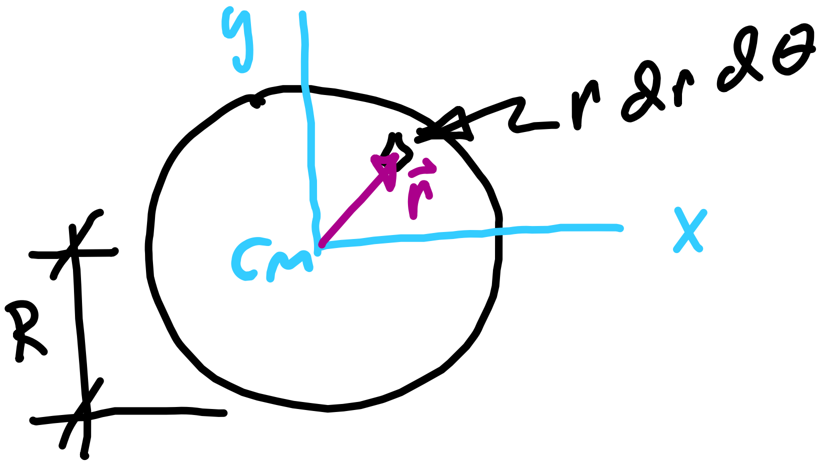
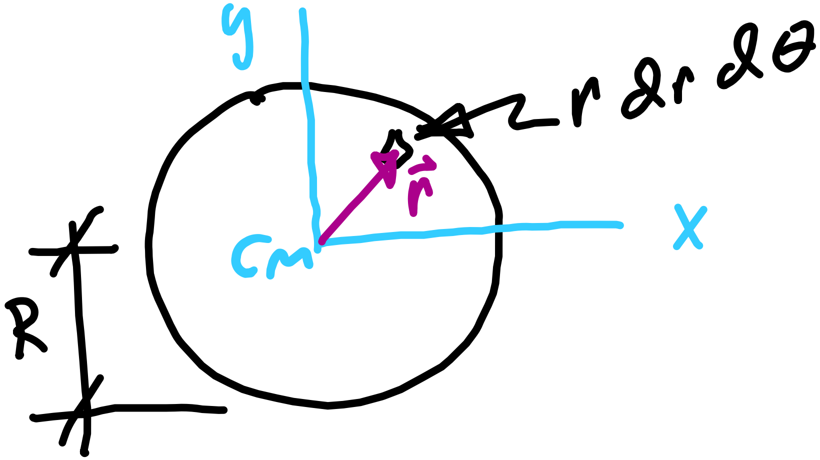
But $A = wh$ so $\boxed{I = \frac{M}{12}\left[w^2+h^2\right]}$

Uniform disk

Uniform disk

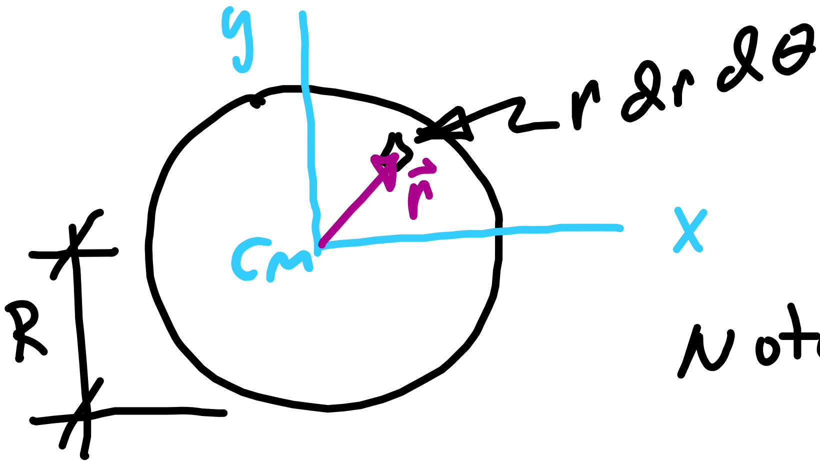


Uniform disk



$$I_{cm} = \left(\frac{M}{A}\right) \int_0^R \int_0^{2\pi} (r)^2 \, r \, dr \, d\theta$$

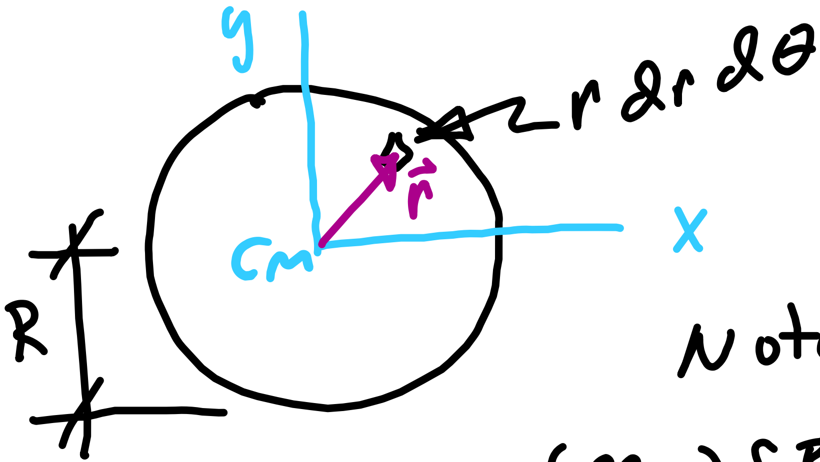
Uniform disk



$$I_{cm} = \left(\frac{M}{A}\right)\int_0^R\int_0^{2\pi}(r)^2\, r\, dr\, d\theta$$

Note: $A = \pi R^2$

Uniform disk



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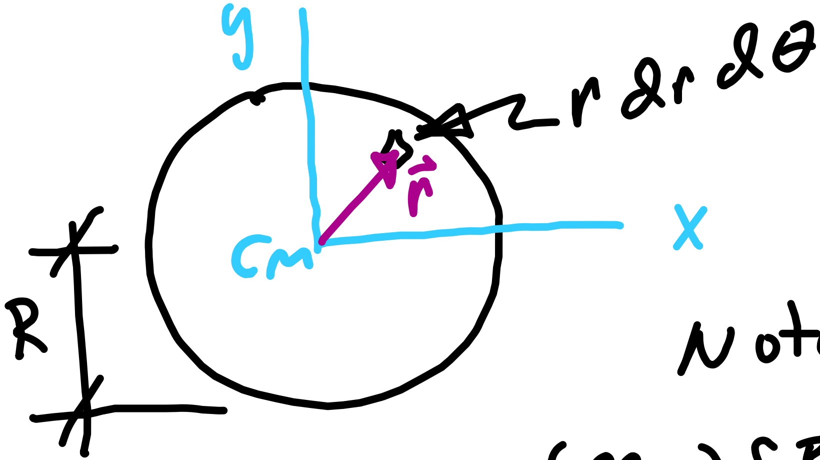
Note: $A = \pi R^2$ so

$$I_{cm} = \left(\frac{M}{\pi R^2}\right)\int_0^R \int_0^{2\pi} r^3\, dr\, d\theta$$

Uniform disk



$$I_{cm} = \left(\frac{M}{A}\right)\int_0^R \int_0^{2\pi} (r)^2\, r\, dr\, d\theta$$

Note: $A = \pi R^2$ so

$$I_{cm} = \left(\frac{M}{\pi R^2}\right)\int_0^R \int_0^{2\pi} r^3\, dr\, d\theta = \left(\frac{2M}{R^2}\right)\int_0^R r^3\, dr$$

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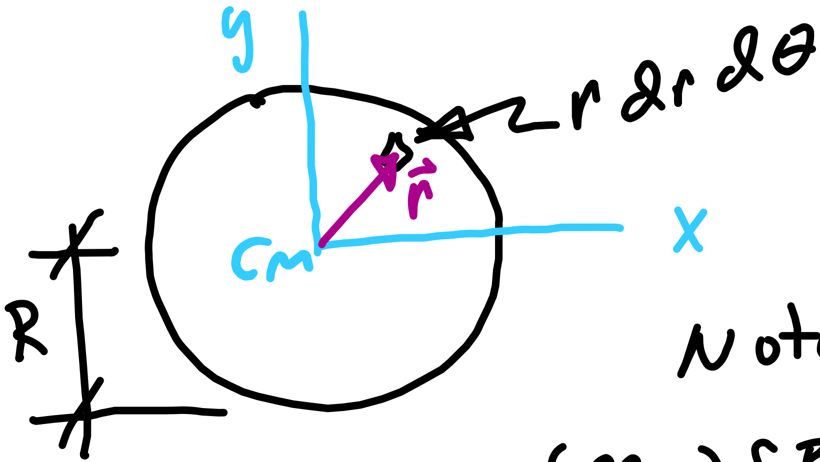

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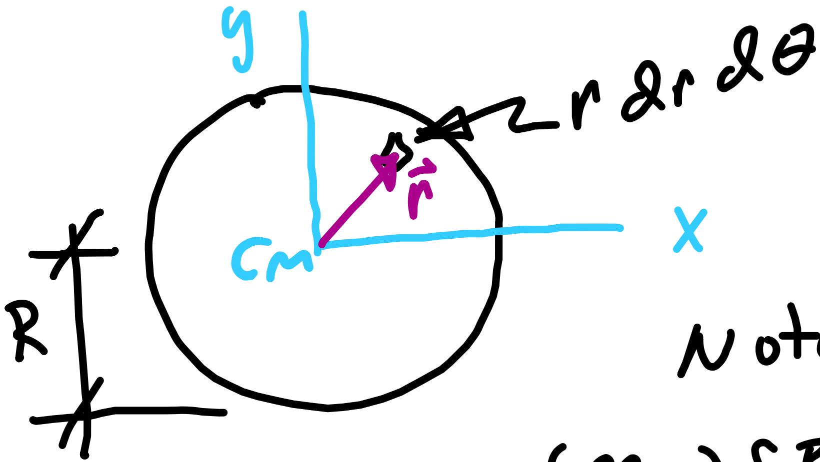

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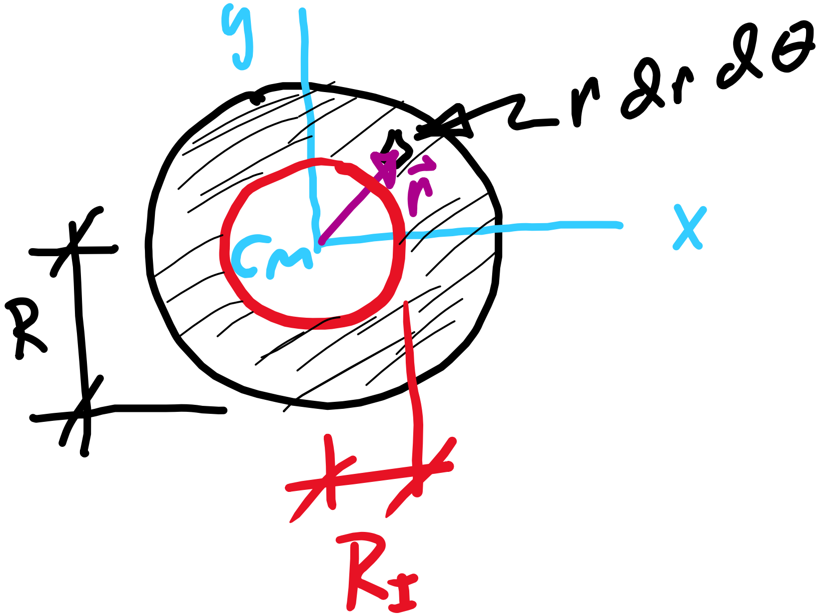
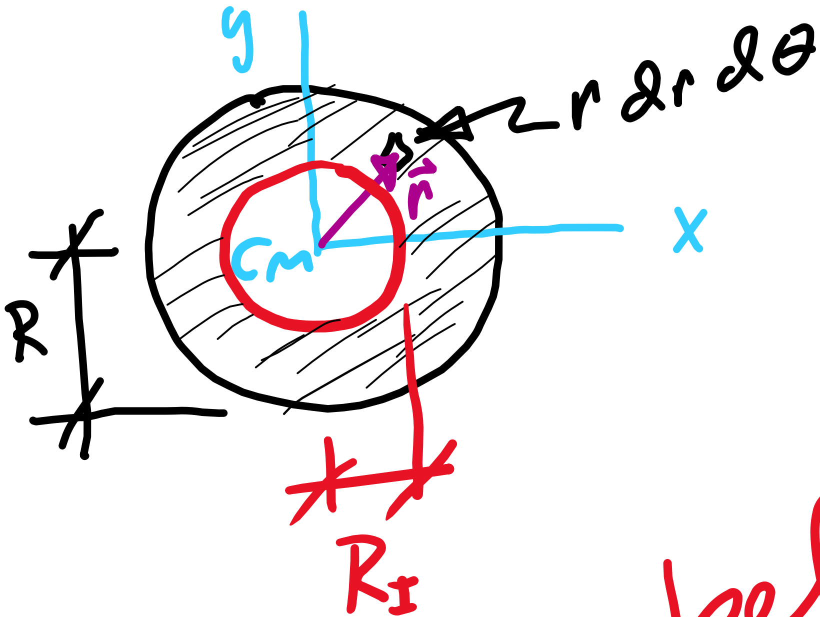
Uniform disk WITH HOLE

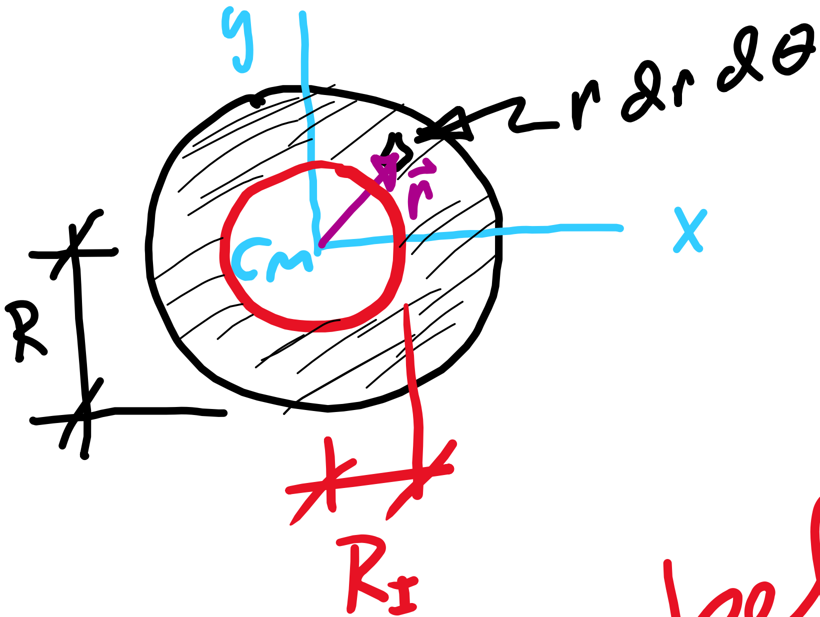


Uniform disk WITH HOLE



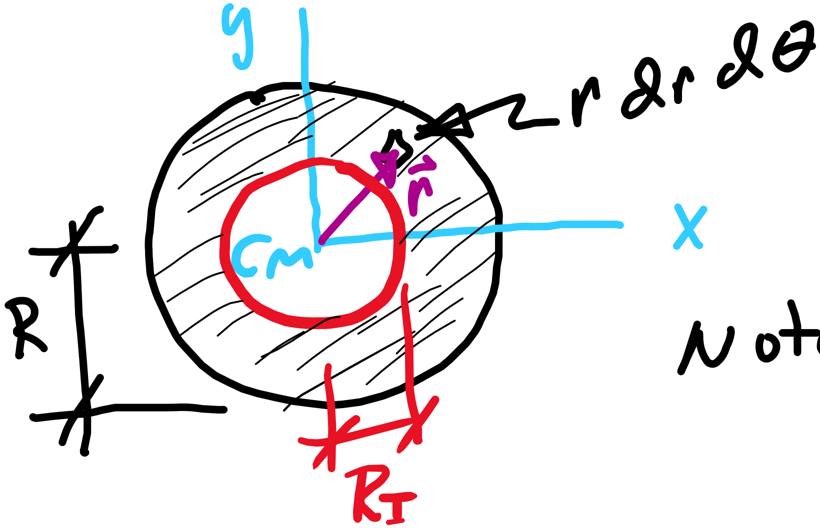
Same as before but put lower limit in r as R_I

Uniform disk WITH HOLE



Same as before but put lower limit in r as R_I & $A=\pi(R^2-R_I^2)$

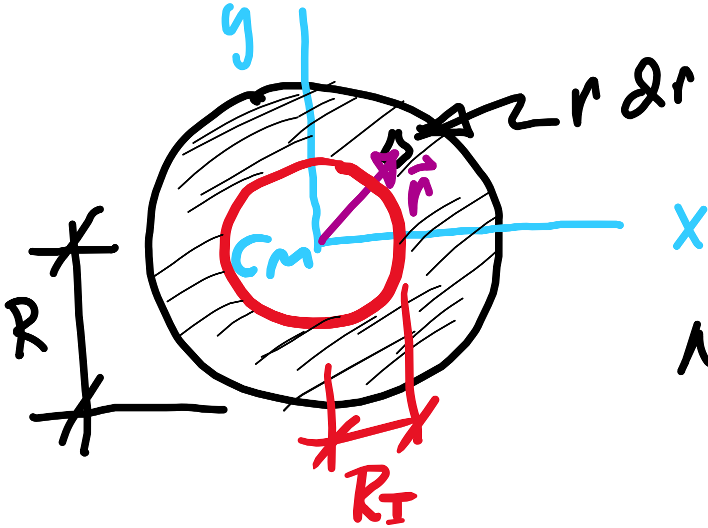
Uniform disk WITH HOLE



$$I_{cm} = \left(\frac{M}{A}\right)\int_{R_I}^{R}\int_0^{2\pi}(r)^2\,r\,dr\,d\theta$$

Note: $A = \pi(R^2 - R_I^2)$

Uniform disk WITH HOLE



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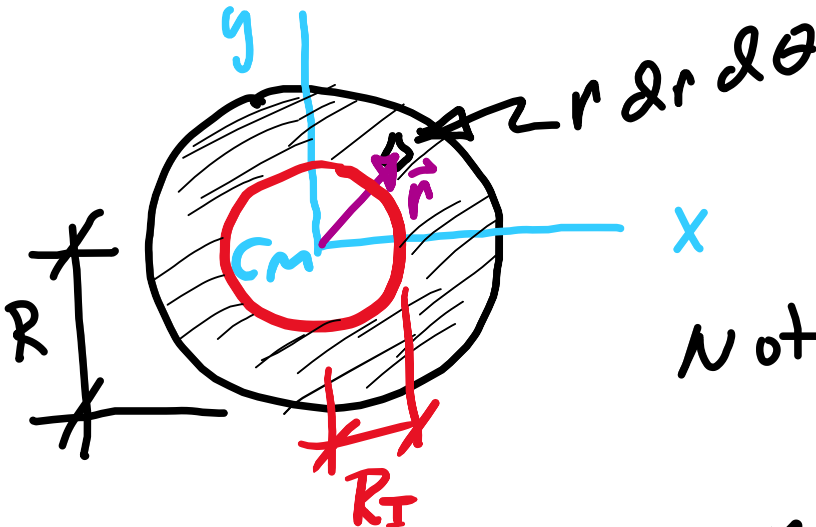

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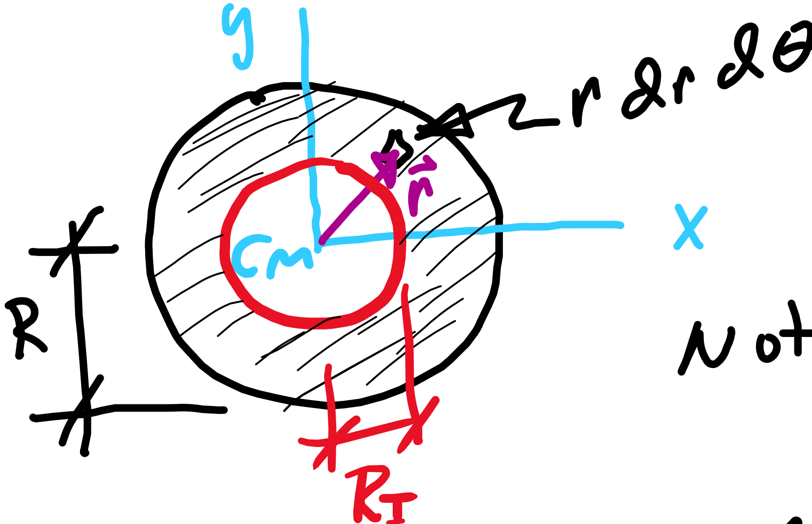
Uniform disk WITH HOLE



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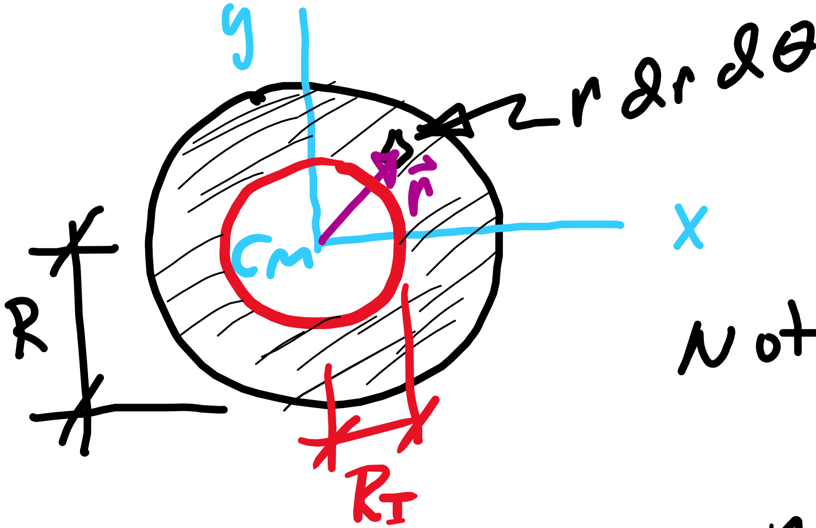
$$\Rightarrow \quad I_{cm} = \left(\frac{2\pi M}{A}\right)\frac{1}{4}r^4\Big|_{R_I}^{R}$$

Uniform disk WITH HOLE



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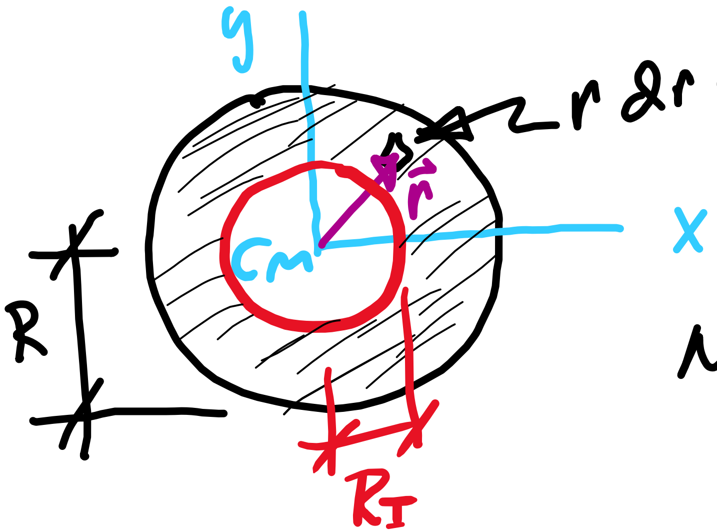
Uniform disk WITH HOLE



$$I_{cm} = \left(\frac{M}{A}\right)\int_{R_I}^{R}\int_0^{2\pi} (r)^2\, r\, dr\, d\theta$$

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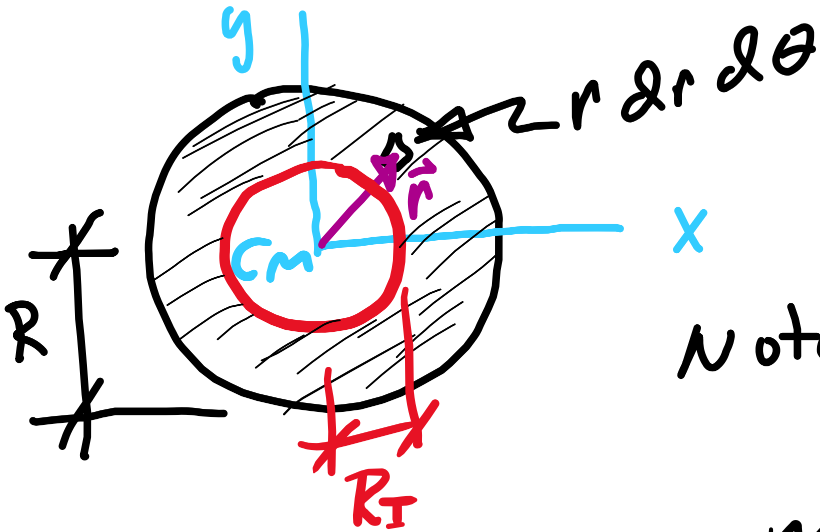
$$\Rightarrow \quad I_{cm} = \left(\frac{2\pi M}{A}\right)\frac{1}{4}r^4\Big|_{R_I}^{R} = \left(\frac{\pi M}{2A}\right)(R^4 - R_I^4)$$

$$\Rightarrow \quad I_{cm} = \left[\frac{M}{2(R^2 - R_I^2)}\right](R^2 + R_I^2)(R^2 - R_I^2)$$

Uniform disk WITH HOLE



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$$\Rightarrow \quad I_{cm} = \left[\frac{M}{2(R^2 - R_I^2)}\right](R^2 + R_I^2)(R^2 - R_I^2) \Rightarrow$$

$$I_{cm} = \frac{M(R^2 + R_I^2)}{2}$$

Note to extend geometry in z-direction [xy-cross section stays same with respect to z]

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Now

3-D

$$I_{cm} = \left(\frac{M}{V}\right) \int r^2 \, dx \, dy \, dz$$

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Now

3-d

$$I_{cm} = \left(\frac{M}{V}\right)\int r^2 dx\,dy\,dz = \left(\frac{M}{AL}\right)\int r^2 dx\,dy\,dz$$

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3-d

$$I_{cm} = \left(\frac{M}{V}\right)\int r^2 \, dx\,dy\,dz = \left(\frac{M}{AL}\right)\int r^2 \, dx\,dy\,dz \Rightarrow$$

$$I_{cm} = \frac{M}{AL}\left(\int r^2 \, dx\,dy\right)L$$

<u>Note</u> to extend geometry in z-direction [xy-cross section stays same with respect to z] Volume $V = AL$, where $A \equiv xy$ cross sectional area & $L \equiv$ length in z-direction

Now

3-d

$$I_{cm} = \left(\frac{M}{V}\right)\int r^2\,dx\,dy\,dz = \left(\frac{M}{AL}\right)\int r^2\,dx\,dy\,dz \Rightarrow$$

$$I_{cm} = \frac{M}{AL}\left(\int r^2\,dx\,dy\right)L \Rightarrow I_{cm} = \frac{M}{A}\int r^2\,dx\,dy$$

Same as 2-d case

So, <u>IF</u> xy cross section remains constant over z-direction for some fixed length

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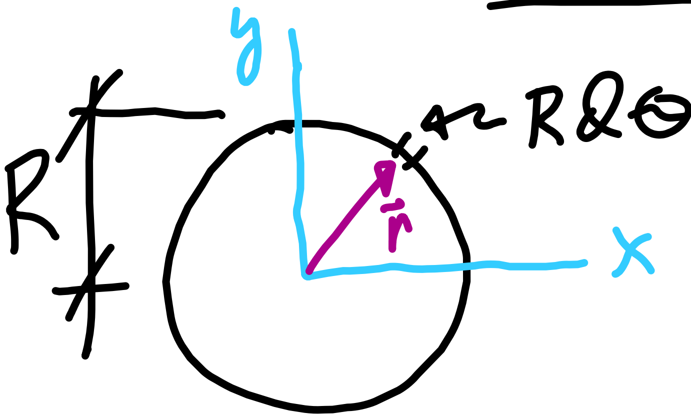
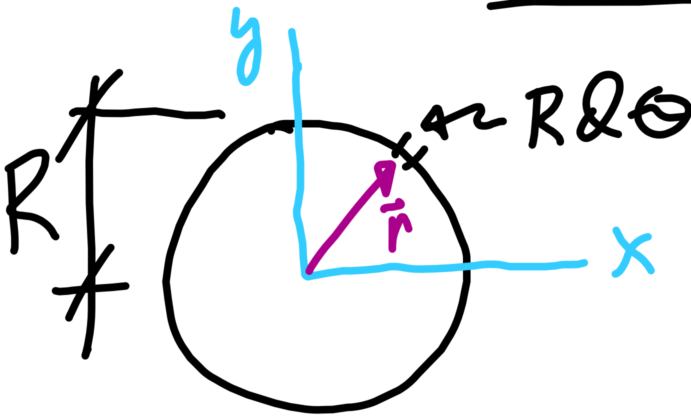
So, IF xy cross section remains constant over z-direction for some fixed length, with the axis of rotation in the z-direction, then the moment of inertia is the same for 2D as for 3D

So disk & cylinder have the same $I_{cm} = MR^2/2$ But Sphere will have different I_{cm}

Thin pipe or hoop

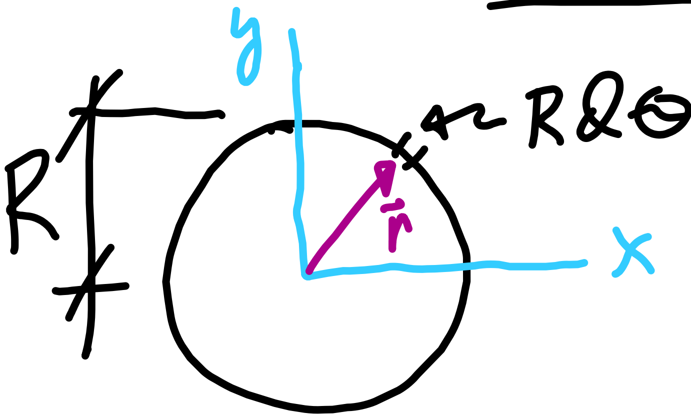


Thin pipe or hoop



Just need 1d

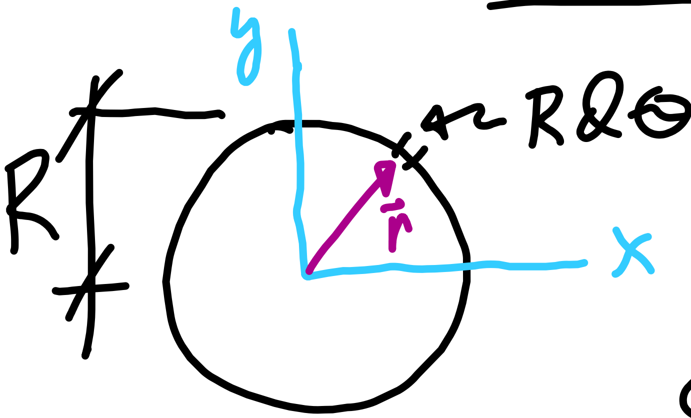
Thin pipe or hoop



Just need 1d

$$\xi \quad \frac{M}{A} \to \frac{M}{C}$$

Thin pipe or hoop

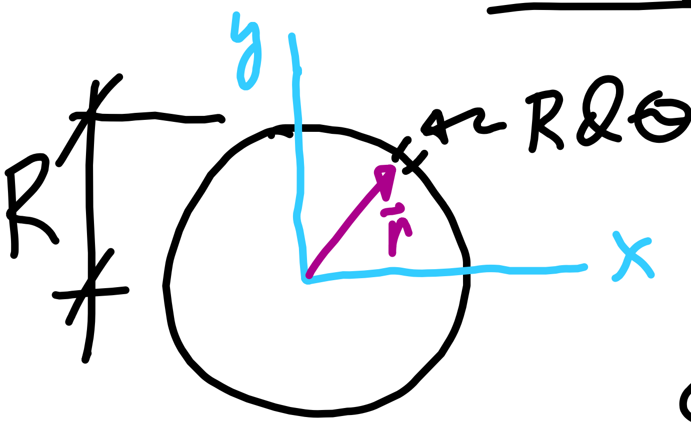


Just need 1d

$\xi \quad \frac{M}{A} \rightarrow \frac{M}{C}$, where

$C = 2\pi R$

Thin pipe or hoop

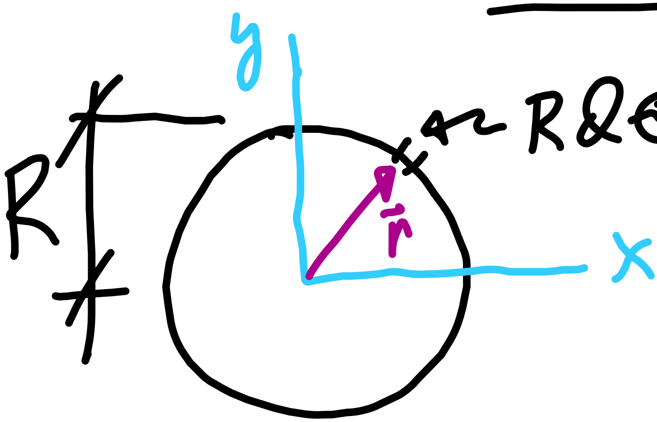


Just need 1d

$\cancel{\frac{M}{A}} \rightarrow \frac{M}{C}$, where

$C = 2\pi R$ [circumference of circle]

Thin pipe or hoop



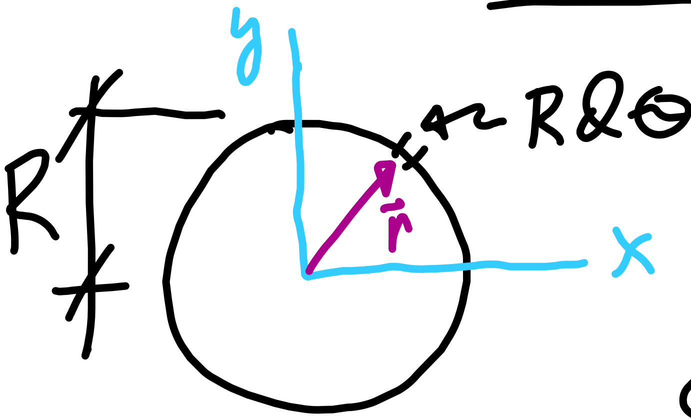
Just need 1d

$$\oint \quad \frac{M}{A} \to \frac{M}{C}, \text{ where}$$

$C = 2\pi R$ [circumference of circle]

Now $$I_{cm} = \left(\frac{M}{C}\right)\int_0^{2\pi} r'^2 R\,d\theta$$

Thin pipe or hoop



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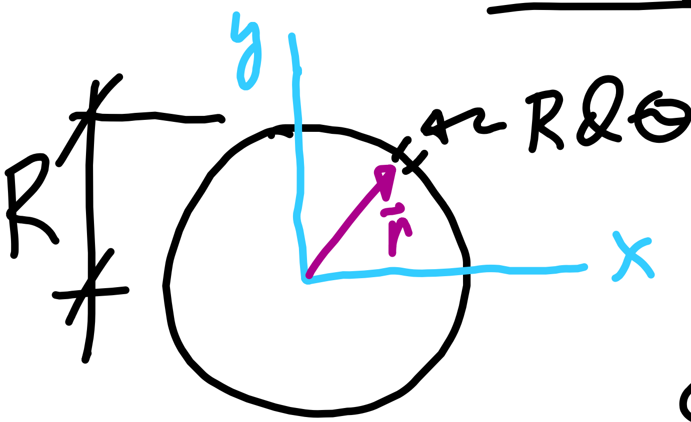
Thin pipe or hoop



Just need 1d

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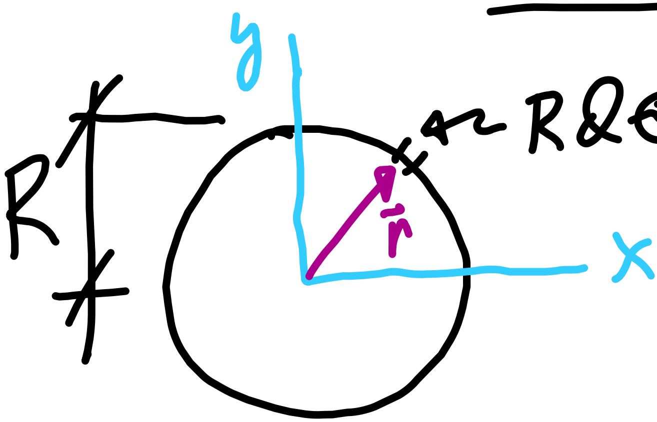
So $\quad I_{cm} = \frac{M}{C} R^2 \int_0^{2\pi} R\,d\Theta$

Thin pipe or hoop



Just need 1d & $\frac{M}{A} \to \frac{M}{C}$, where

$C = 2\pi R$ [circumference of circle]

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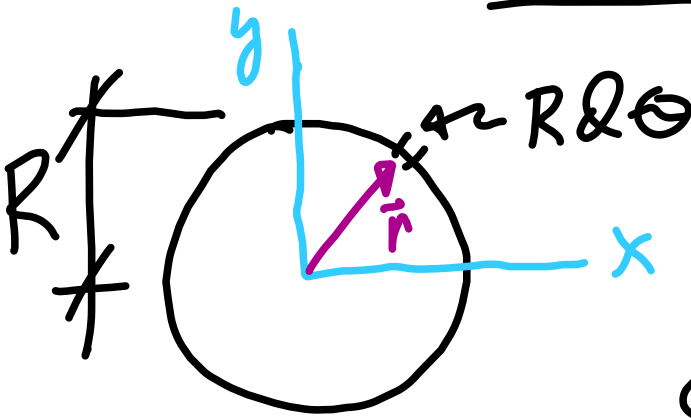
Thin pipe or hoop



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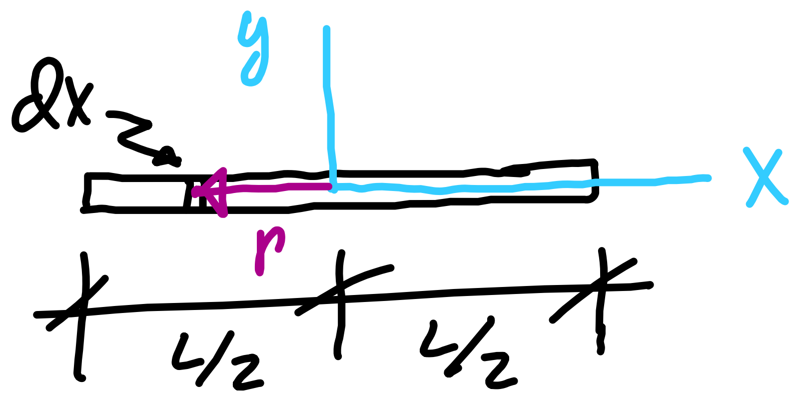
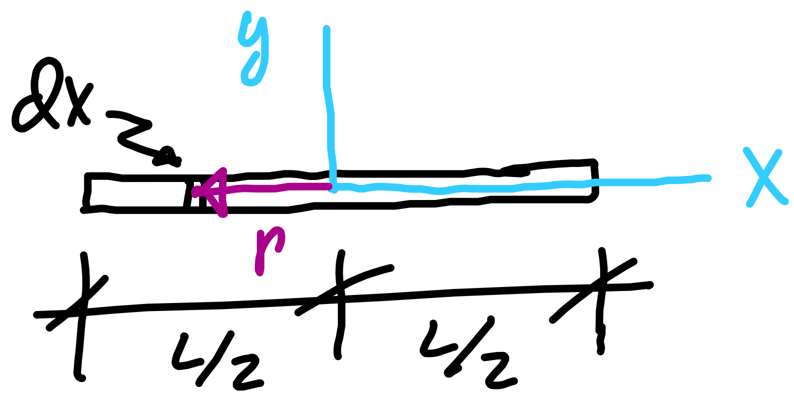
So $I_{cm} = \frac{M}{C} R^2 \underbrace{\int_0^{2\pi} R\,d\theta}_{\text{circumference of circle } C}$

So $\boxed{I_{cm} = MR^2}$

Slender rod

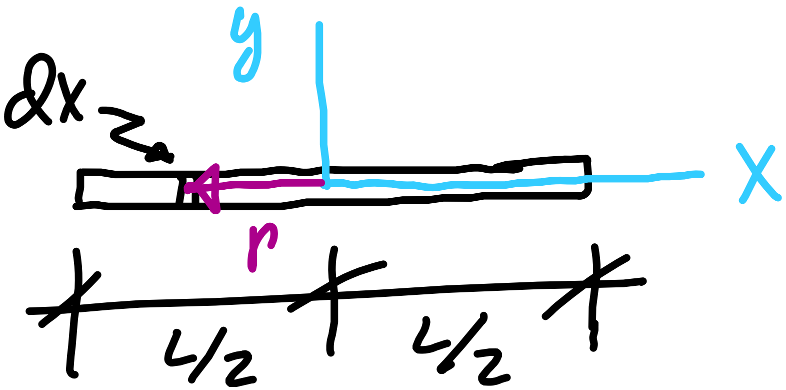


Slender rod



Only need 1D in this case

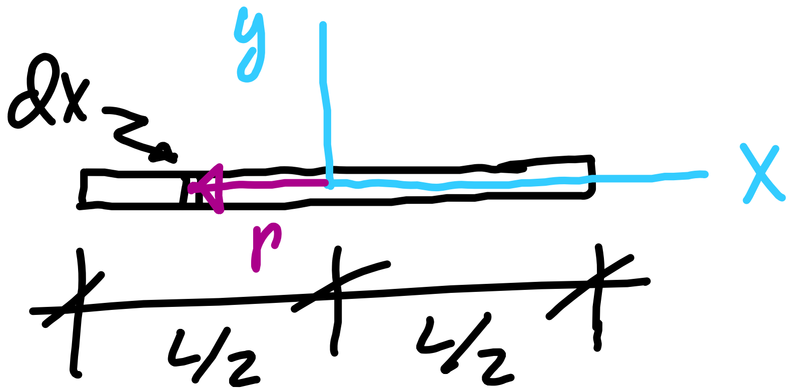
Slender rod



Only need 1d in this case

Here $\frac{M}{A} \to \frac{M}{L}$

Slender rod

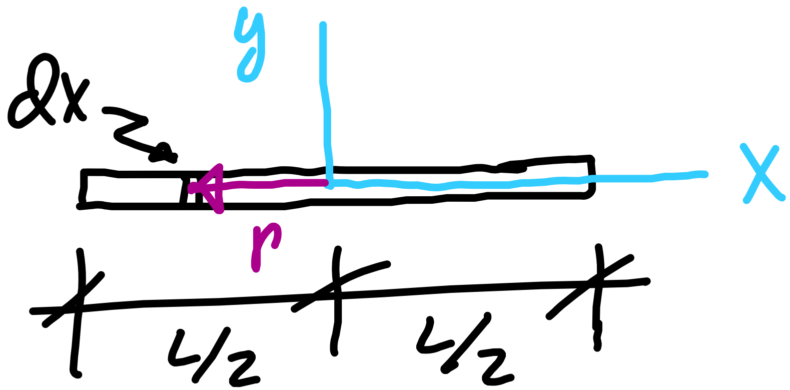


Only need 1d in this case

Here $\frac{M}{A} \rightarrow \frac{m}{L}$

Now $I_{cm} = \left(\frac{m}{L}\right)\int_{-L/2}^{L/2} x^2\, dx'$

Slender rod

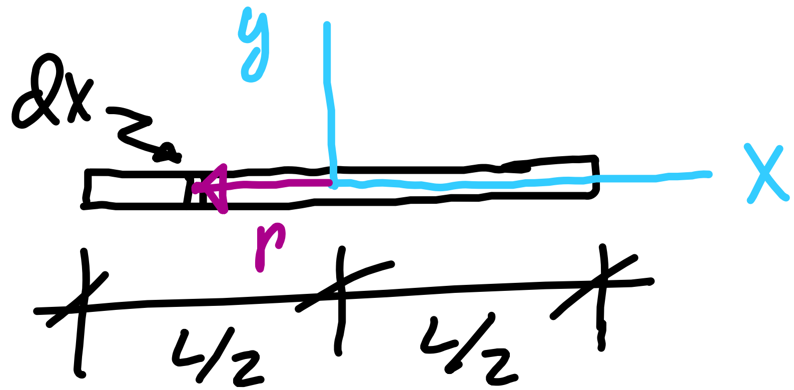


Only need 1d in this case

Here $\frac{M}{A} \to \frac{M}{L}$

Now $I_{cm} = \left(\frac{M}{L}\right) \int_{-L/2}^{L/2} x'^2 \, dx' = \left(\frac{M}{L}\right) \left[\frac{x^3}{3}\right] \Big|_{-L/2}^{L/2}$

Slender rod



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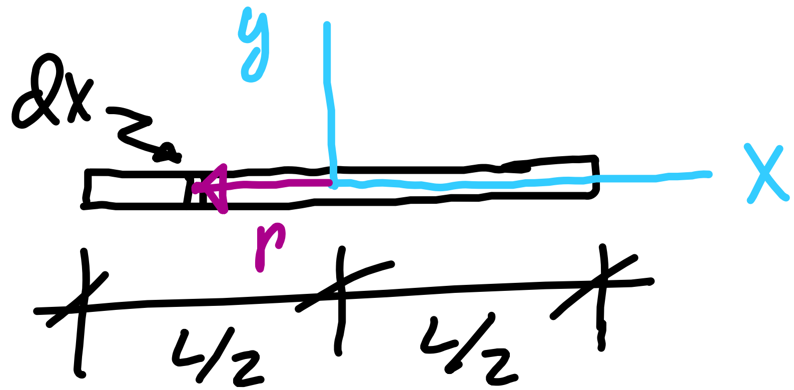
$\Rightarrow \quad I_{cm} = \left(\frac{M}{3L}\right)\left[\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right]$

Slender rod



Only need 1d in this case

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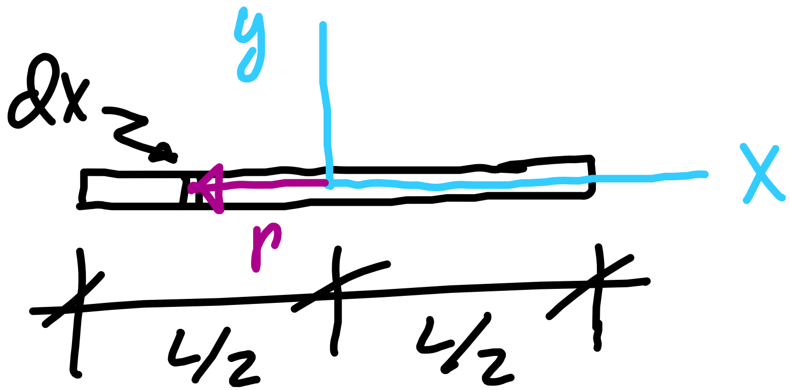
$\Rightarrow \quad I_{cm} = \left(\frac{M}{3L}\right) \left[\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right] \Rightarrow \boxed{I_{cm} = \frac{ML^2}{12}}$

Slender rod



Only need 1d in this case

Here $\frac{M}{A} \rightarrow \frac{M}{L}$

Now $I_{cm} = \left(\frac{M}{L}\right)\int_{-L/2}^{L/2} x'^2 \, dx' = \left(\frac{M}{L}\right)\left[\frac{x^3}{3}\right]\Big|_{-L/2}^{L/2}$

$\Rightarrow \quad I_{cm} = \left(\frac{M}{3L}\right)\left[\frac{L^3}{8} - \left(-\frac{L^3}{8}\right)\right] \Rightarrow \boxed{I_{cm} = \frac{ML^2}{12}}$

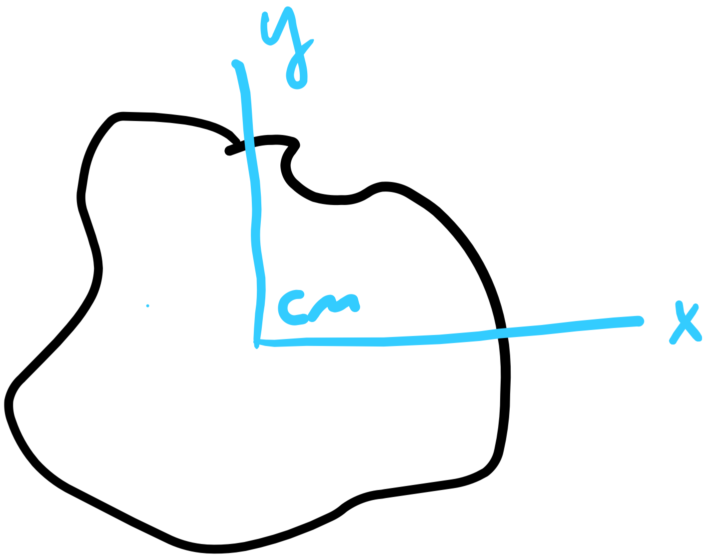
Compare to plate

$I_{plate} = \left(\frac{M}{12}\right)\left[L^2 + w^2\right]$

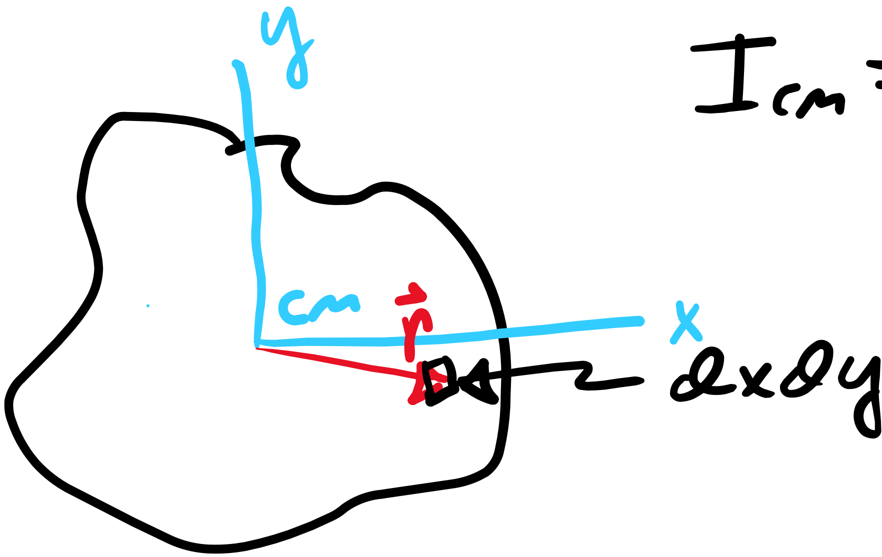
Slender rod is just a plate with $w \rightarrow 0$

Parallel axis theorem

Parallel axis theorem

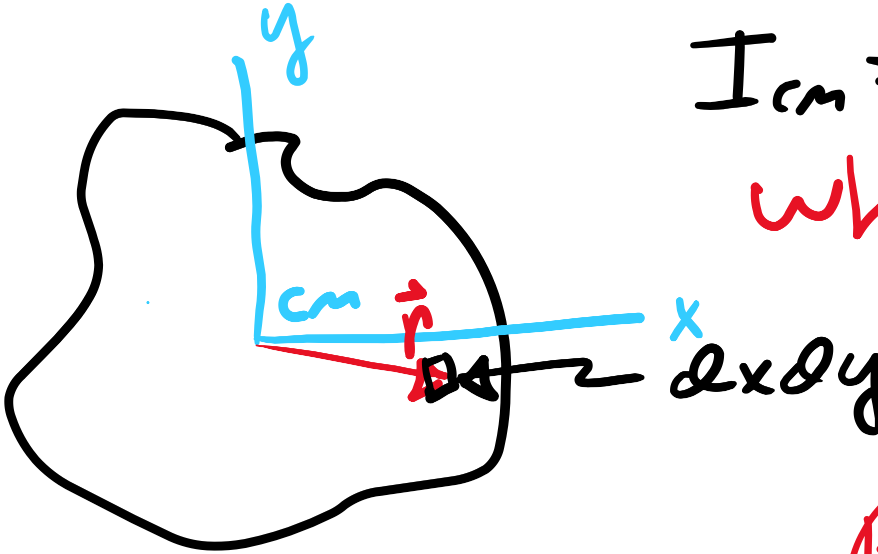


Parallel axis theorem



$$I_{cm} = \frac{M}{A}\int r^2\,dx\,dy$$

Parallel axis theorem

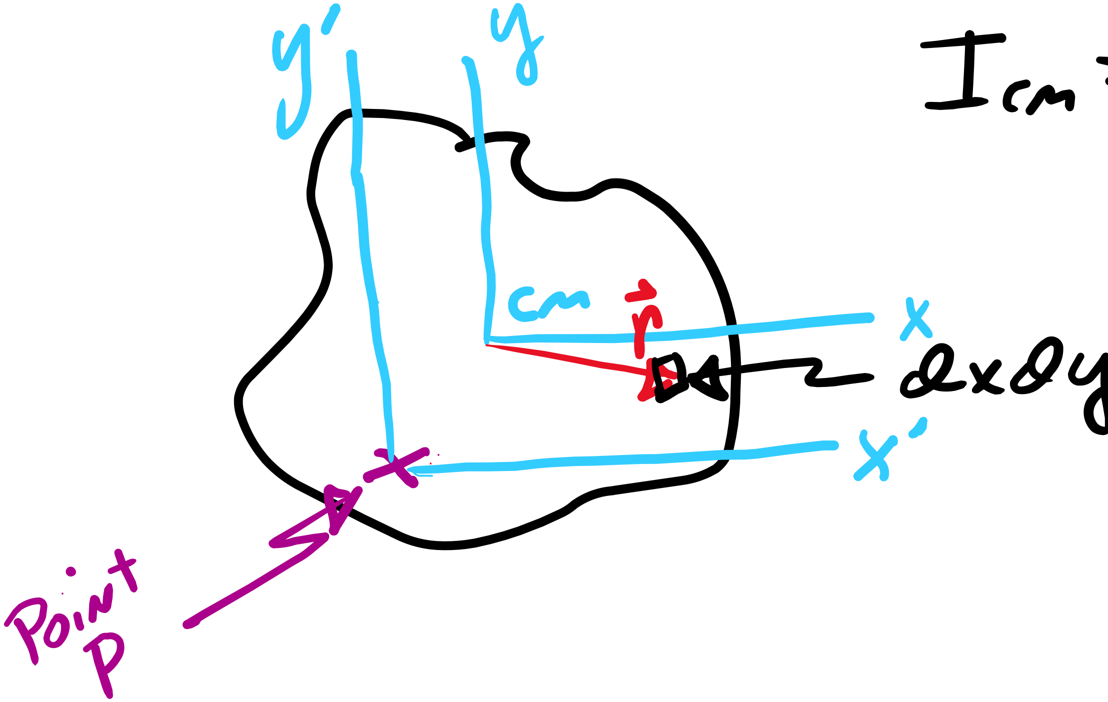


$$I_{cm} = \frac{M}{A}\int r^2\,dx\,dy$$

what about I_p for parallel axis through some other point?

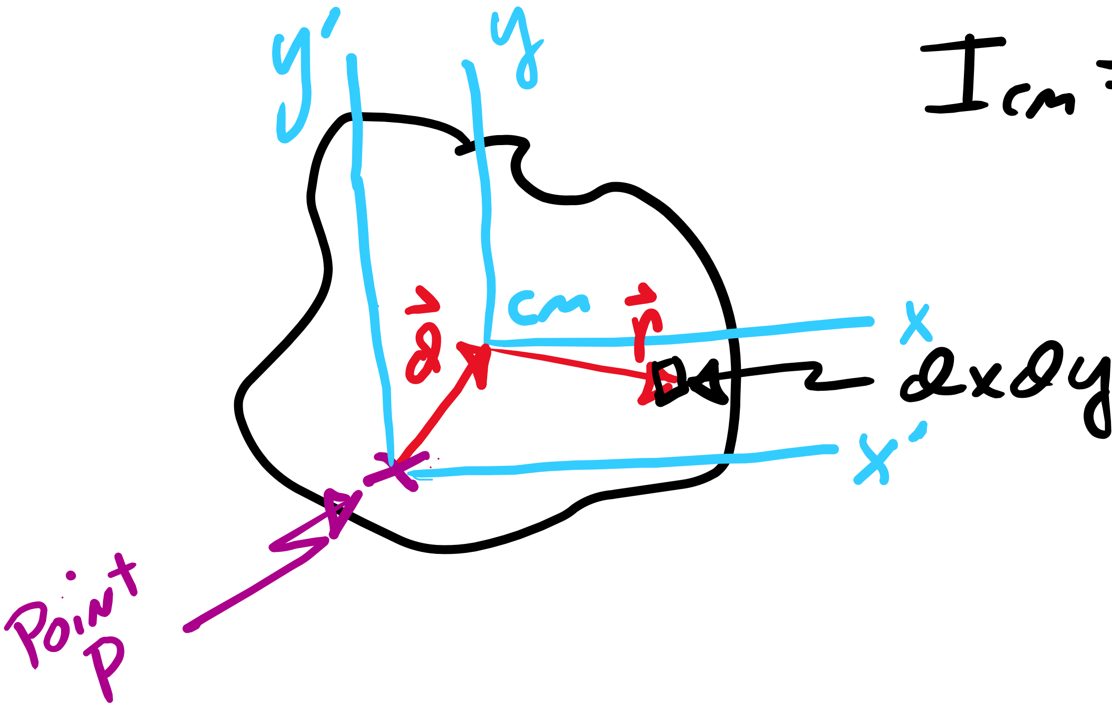
Parallel axis theorem

$$I_{cm} = \frac{M}{A}\int r^2\,dx\,dy$$



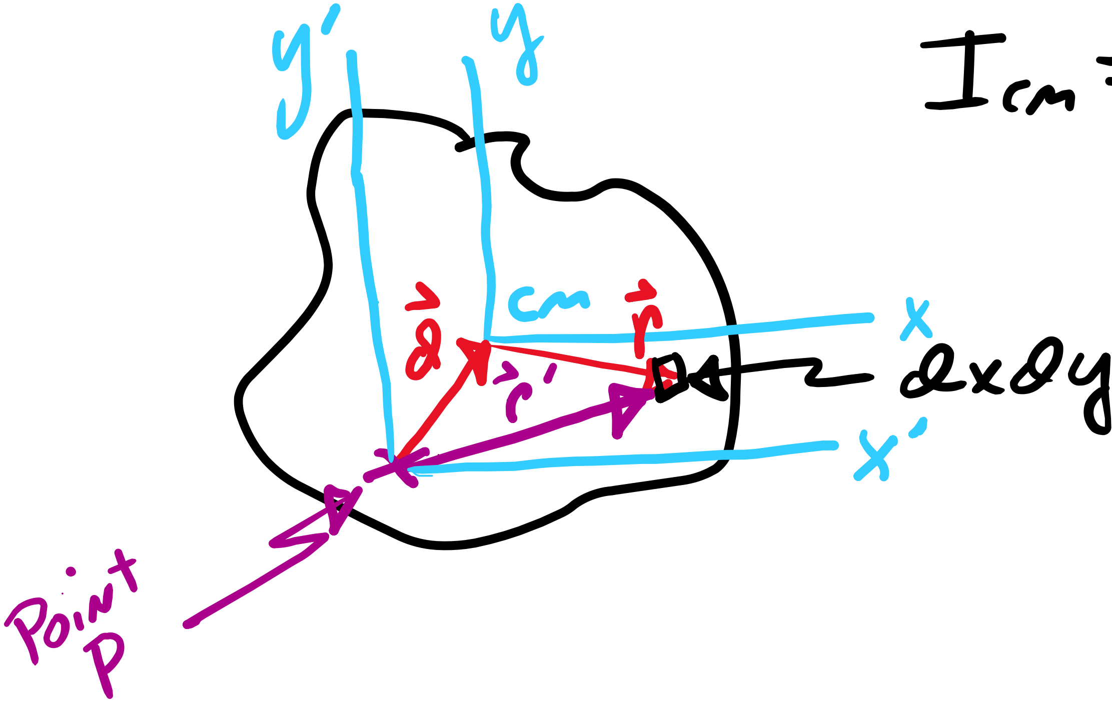
Parallel axis theorem

$$I_{cm} = \frac{M}{A}\int r^2\,dx\,dy$$

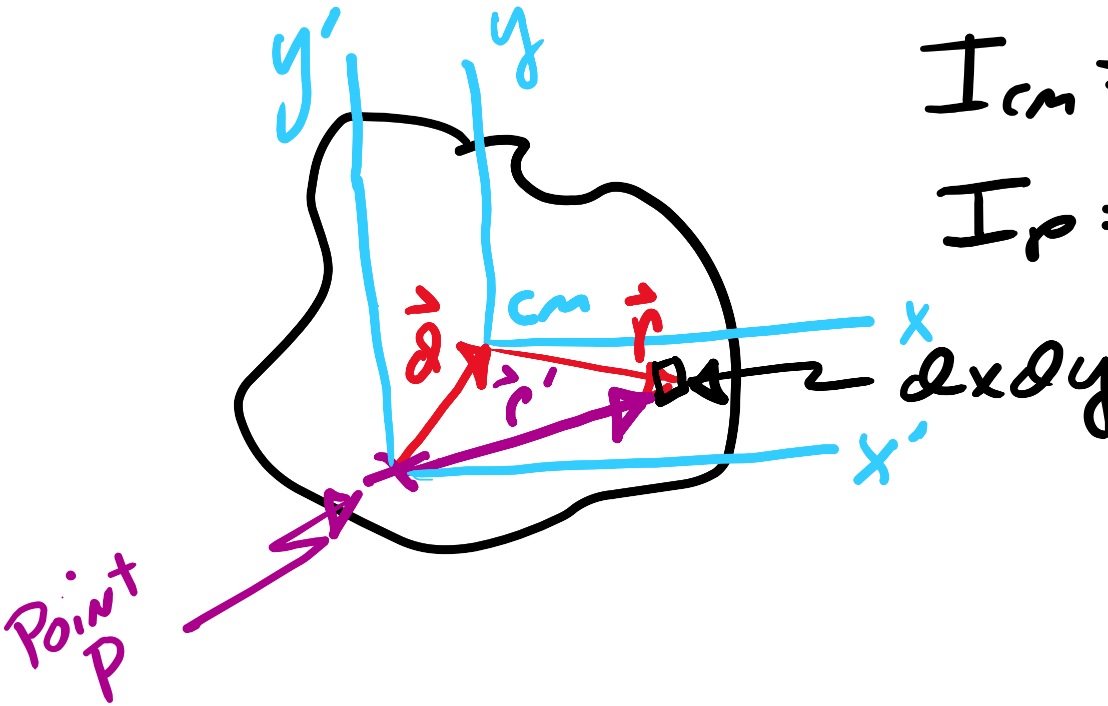


Parallel axis theorem

$$I_{cm} = \frac{M}{A} \int r^2 \, dx \, dy$$



Parallel axis theorem



$$I_{cm} = \frac{M}{A}\int r^2\,dx\,dy$$

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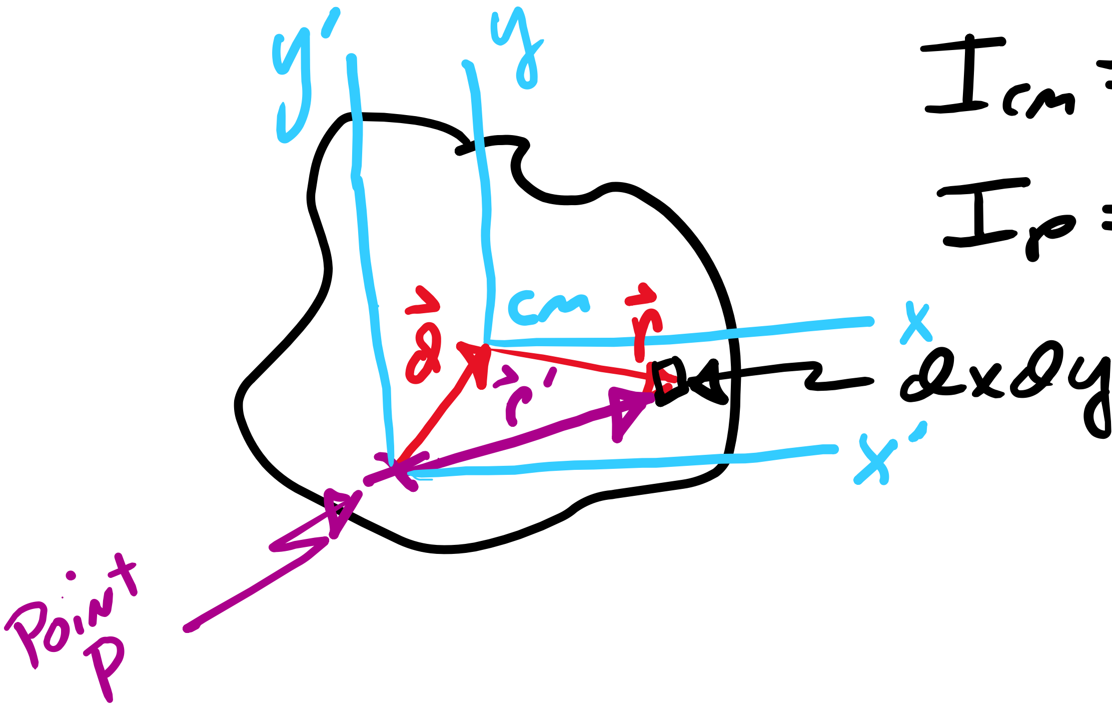
Parallel axis theorem



$$I_{cm} = \frac{M}{A}\int r^2\,dx\,dy$$

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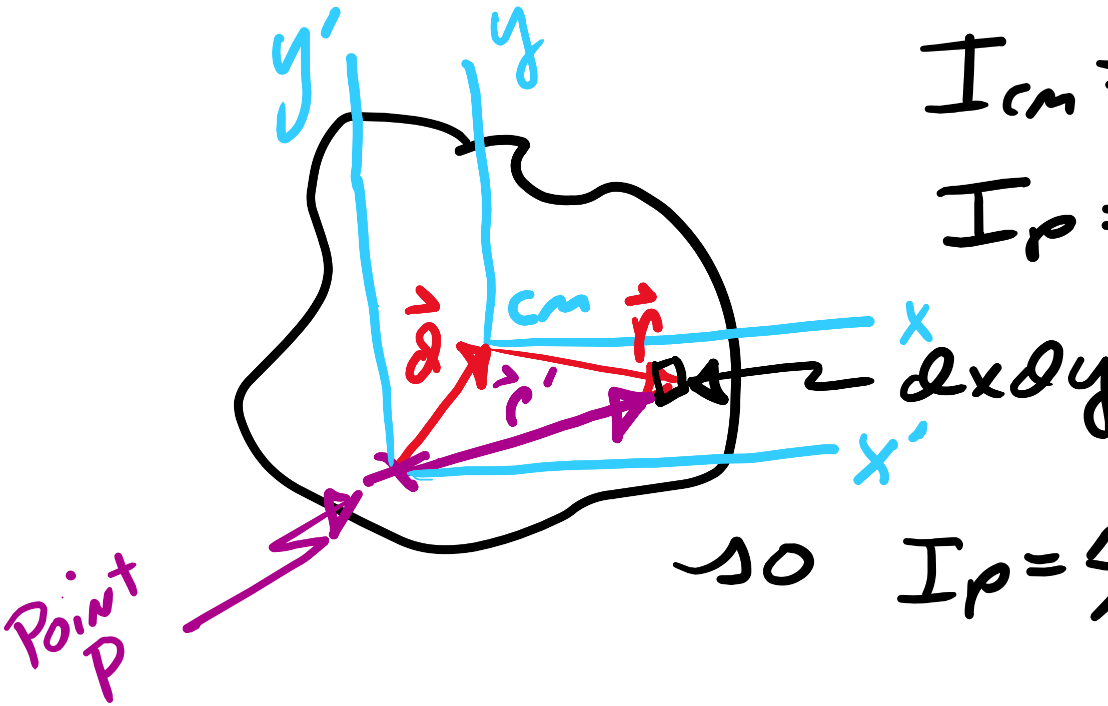
But $\vec{r}\,' = \vec{d} + \vec{r}$

Parallel axis theorem



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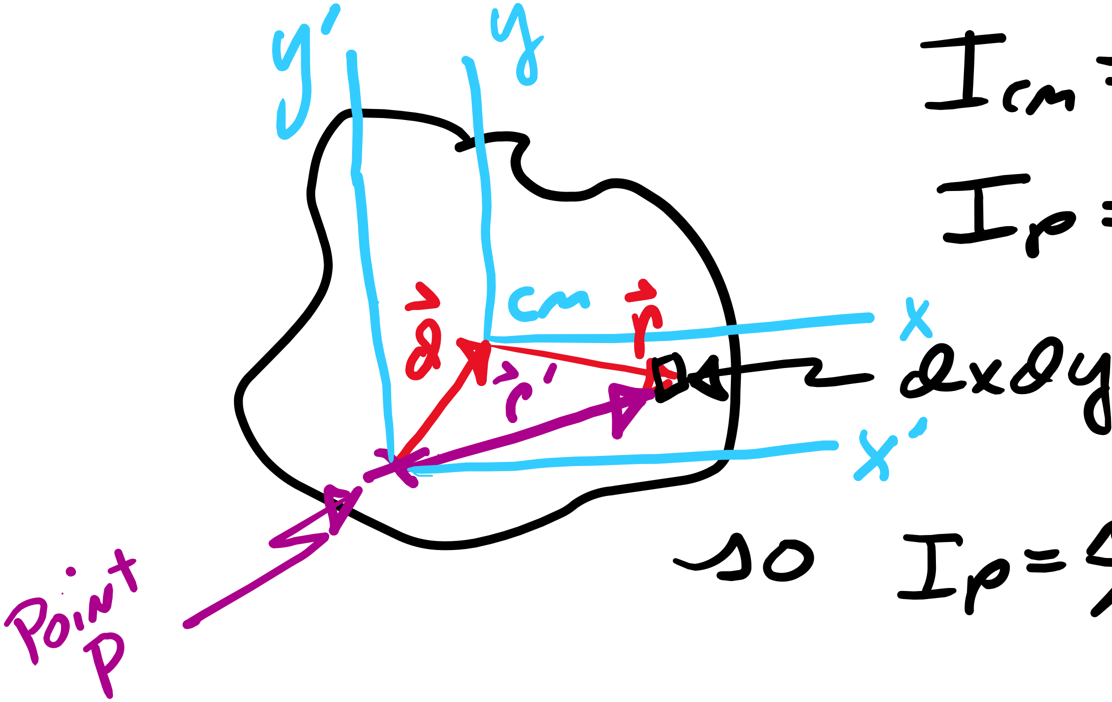
Parallel axis theorem



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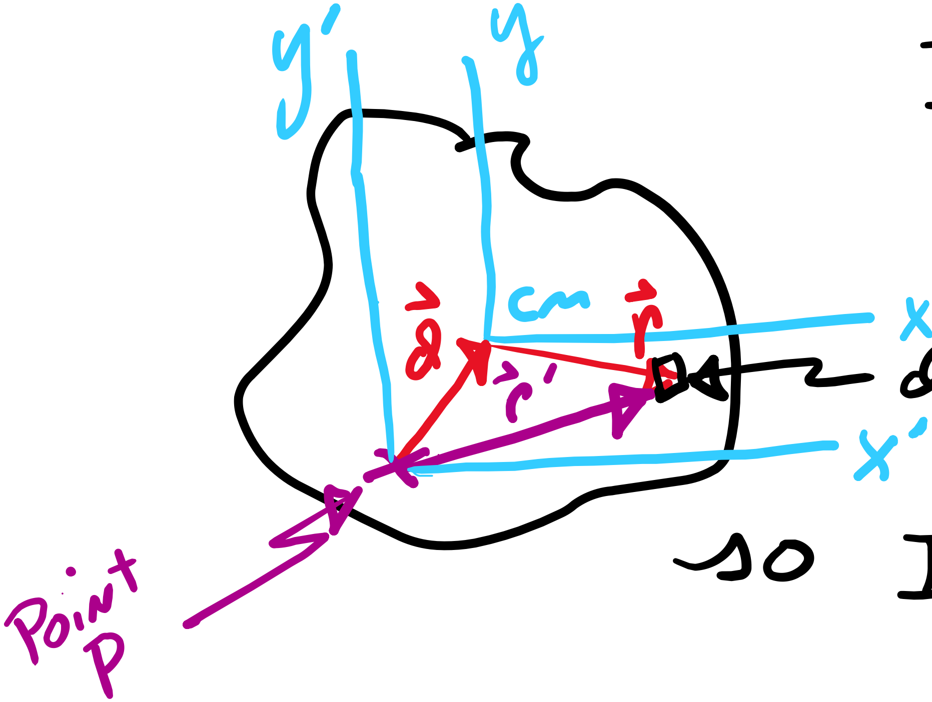
Note: $r'^2 = \vec{r}'\cdot\vec{r}'$

Parallel axis theorem



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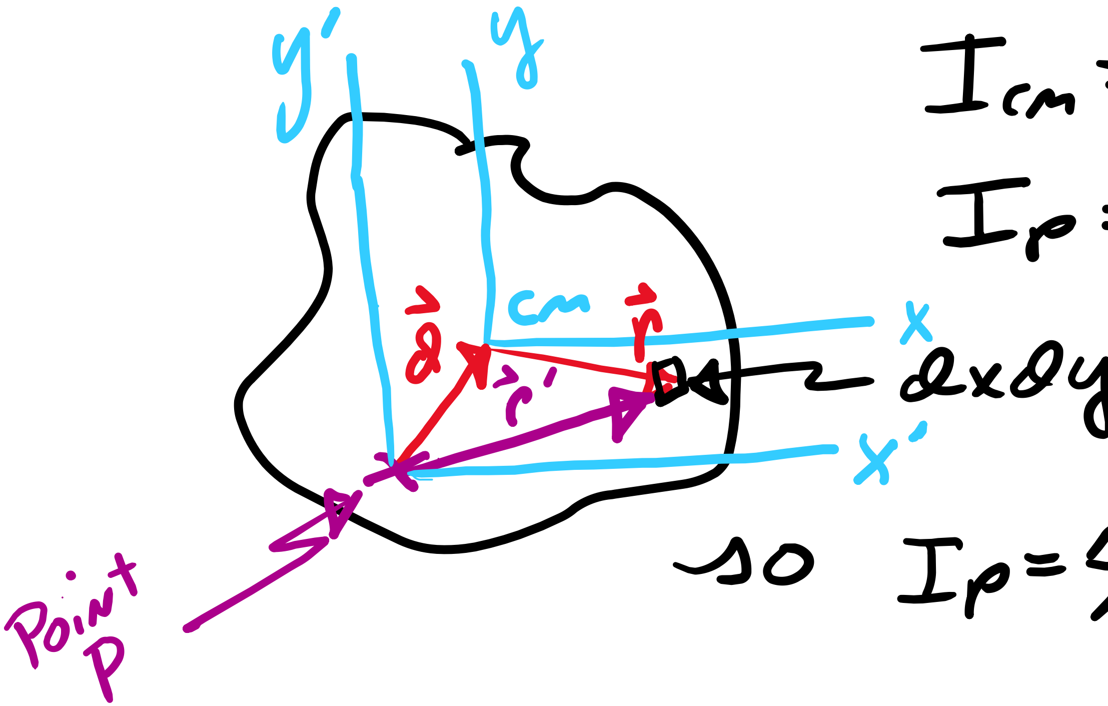
Parallel axis theorem



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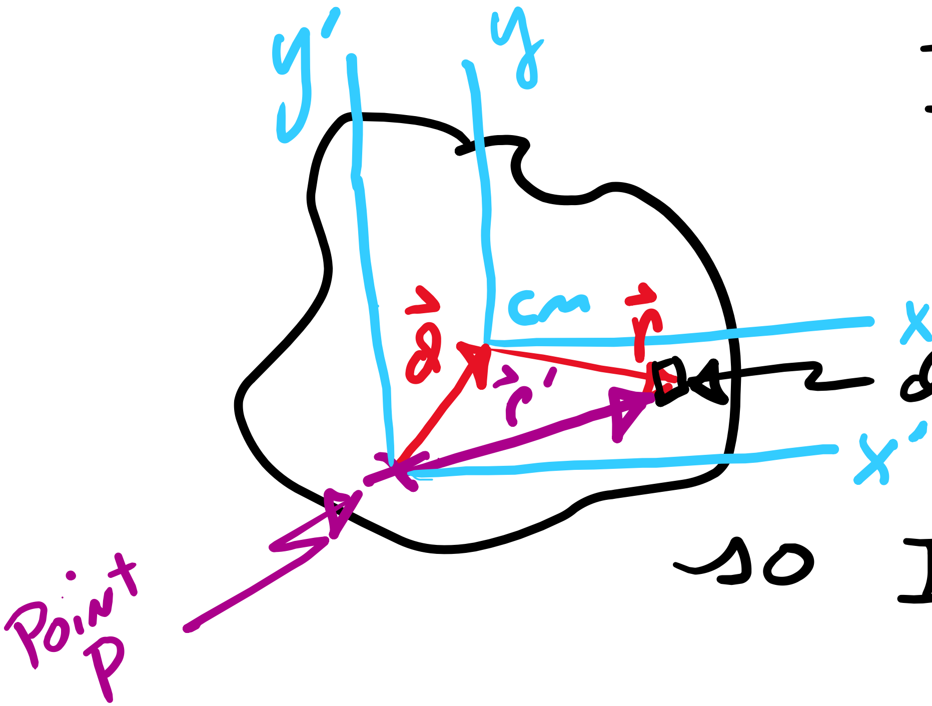
Note: $r'^2 = \vec{r}'\cdot\vec{r}' = (\vec{d}+\vec{r})\cdot(\vec{d}+\vec{r})$

$= d^2 + 2\vec{d}\cdot\vec{r} + r^2$

Parallel axis theorem



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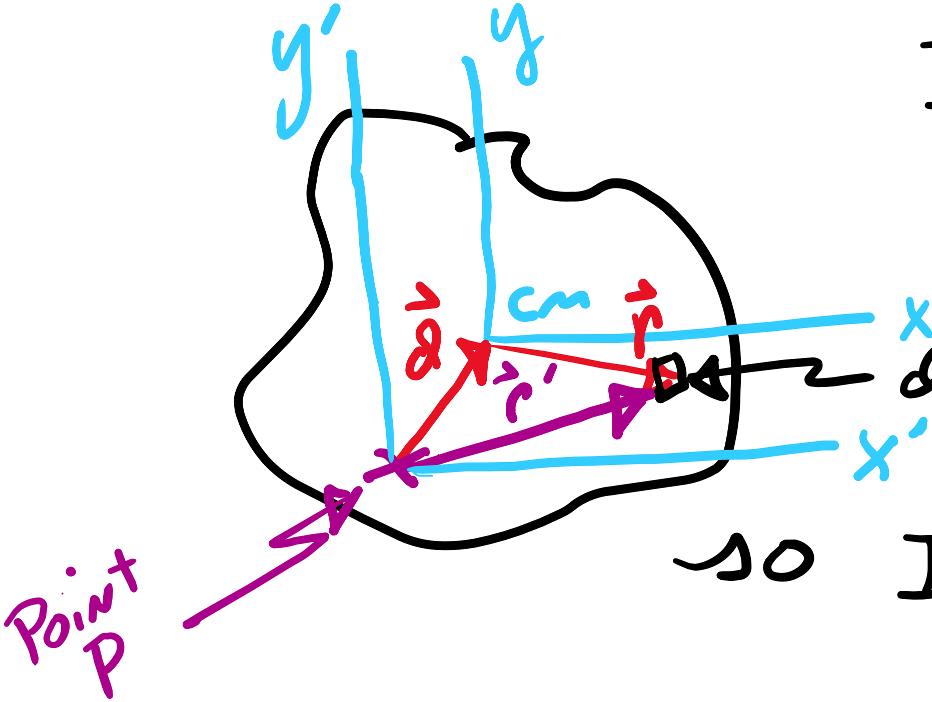
Also $dx'\,dy' = dx\,dy$

Parallel axis theorem



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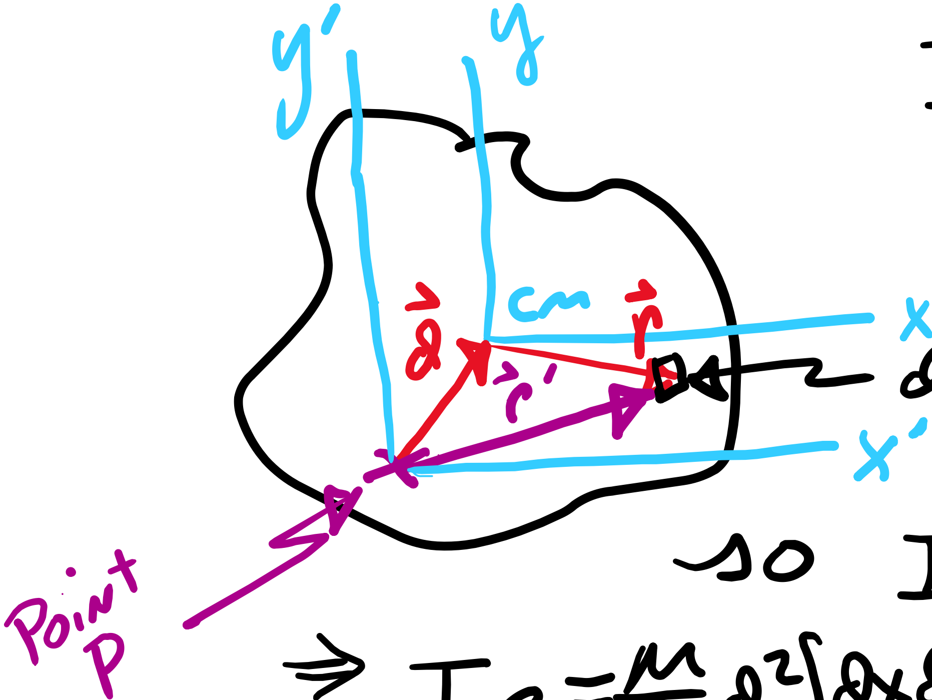
Parallel axis theorem



$$I_{cm} = \frac{M}{A}\int r^2 \, dx\, dy$$

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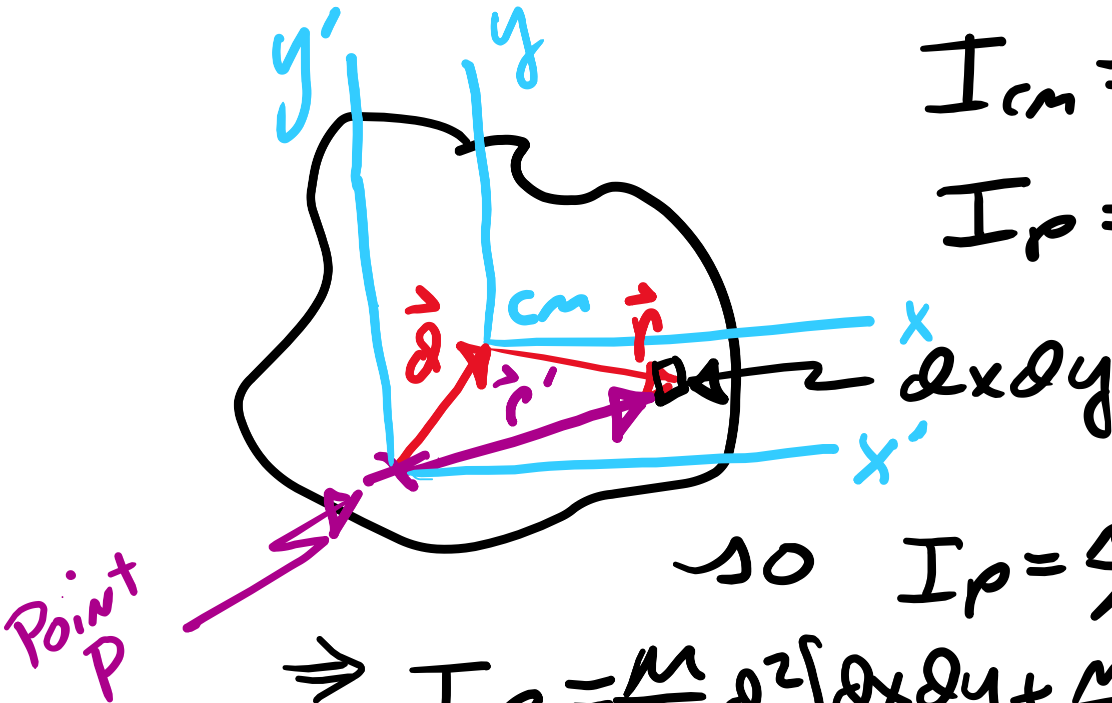
$\Rightarrow I_P = \frac{M}{A} d^2 \int dx\, dy +$

Parallel axis theorem



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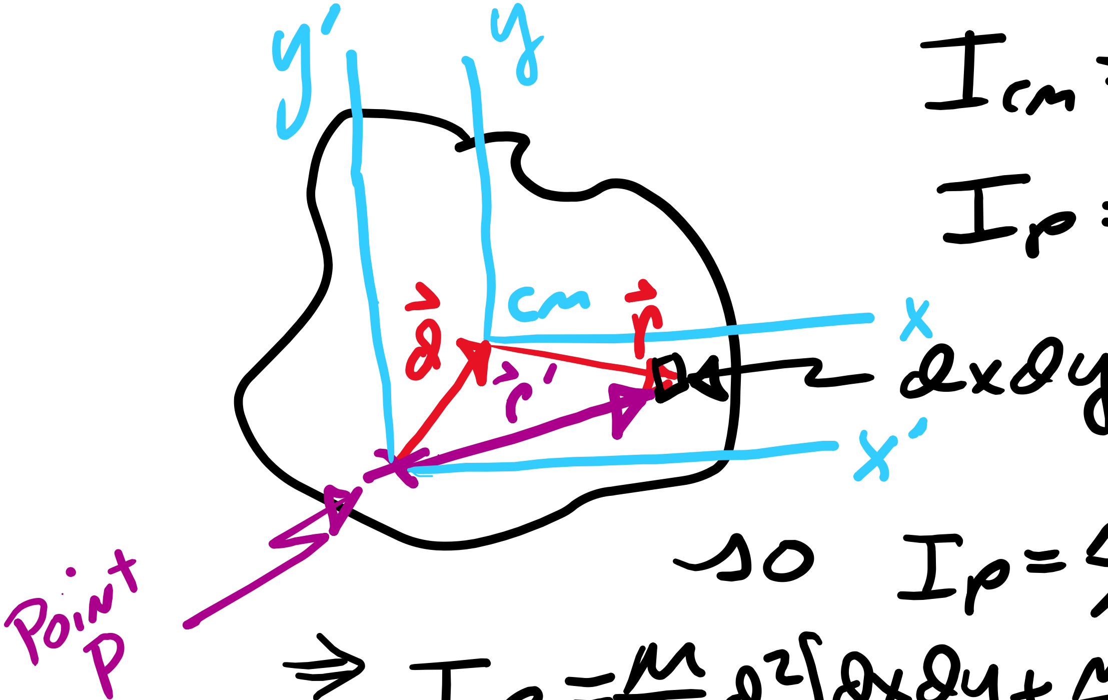
But $\vec{r}' = \vec{d} + \vec{r}$

so $I_P = \frac{M}{A}\int (d^2 + r^2 + 2\vec{d}\cdot\vec{r})\, dx\, dy$

$$\Rightarrow I_P = \frac{M}{A} d^2 \int dx\, dy + \frac{M}{A}\int r^2 \, dx\, dy$$

Parallel axis theorem



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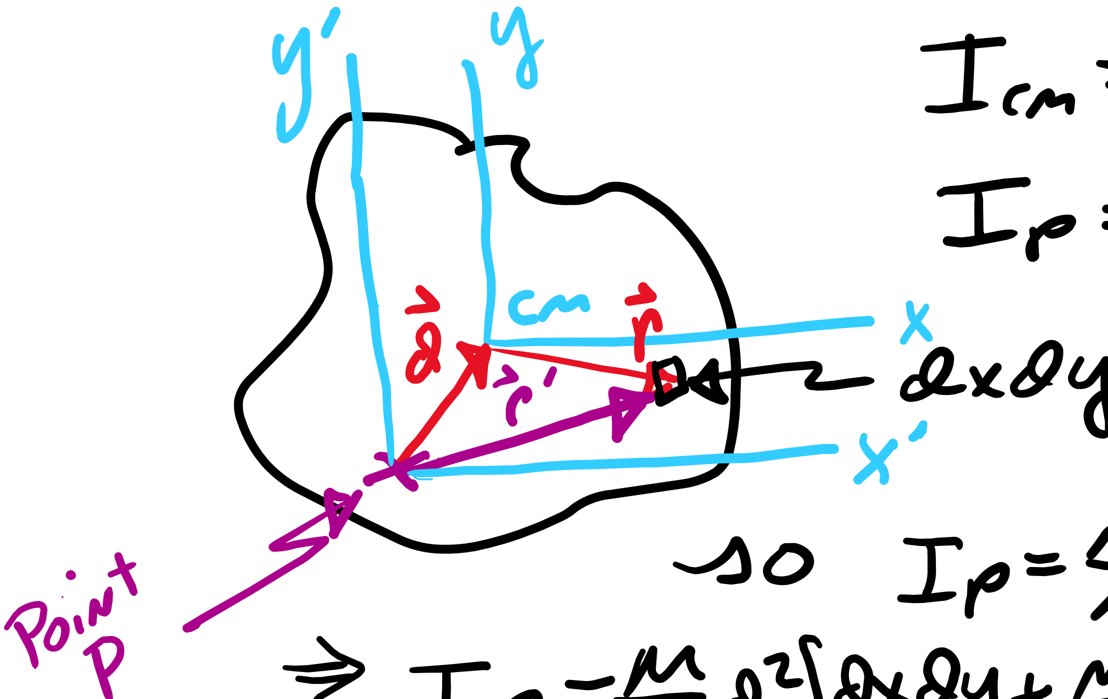
Parallel axis theorem



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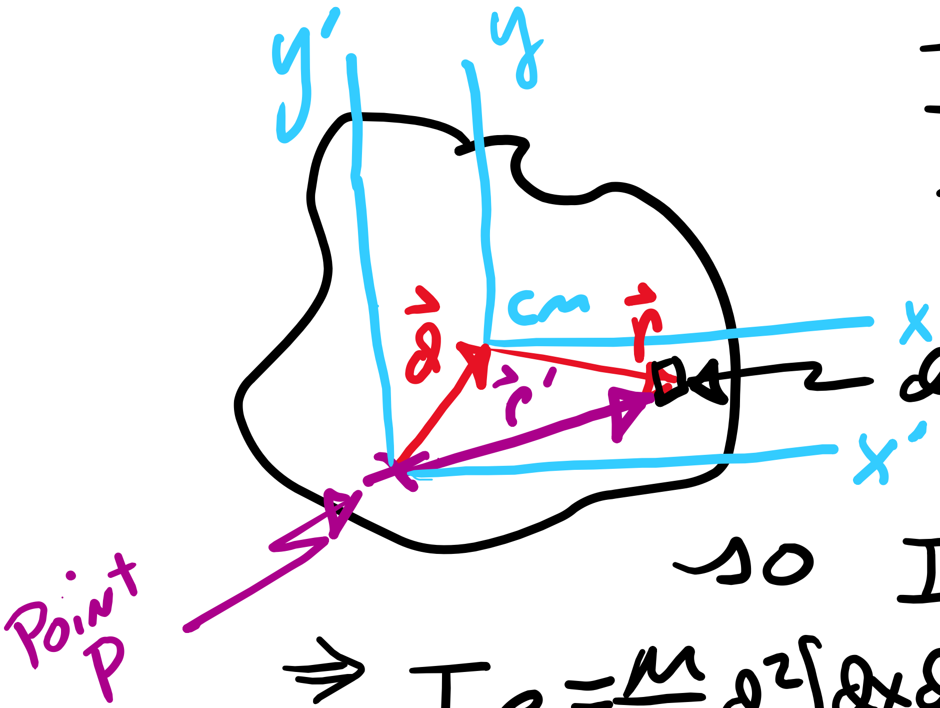
Parallel axis theorem



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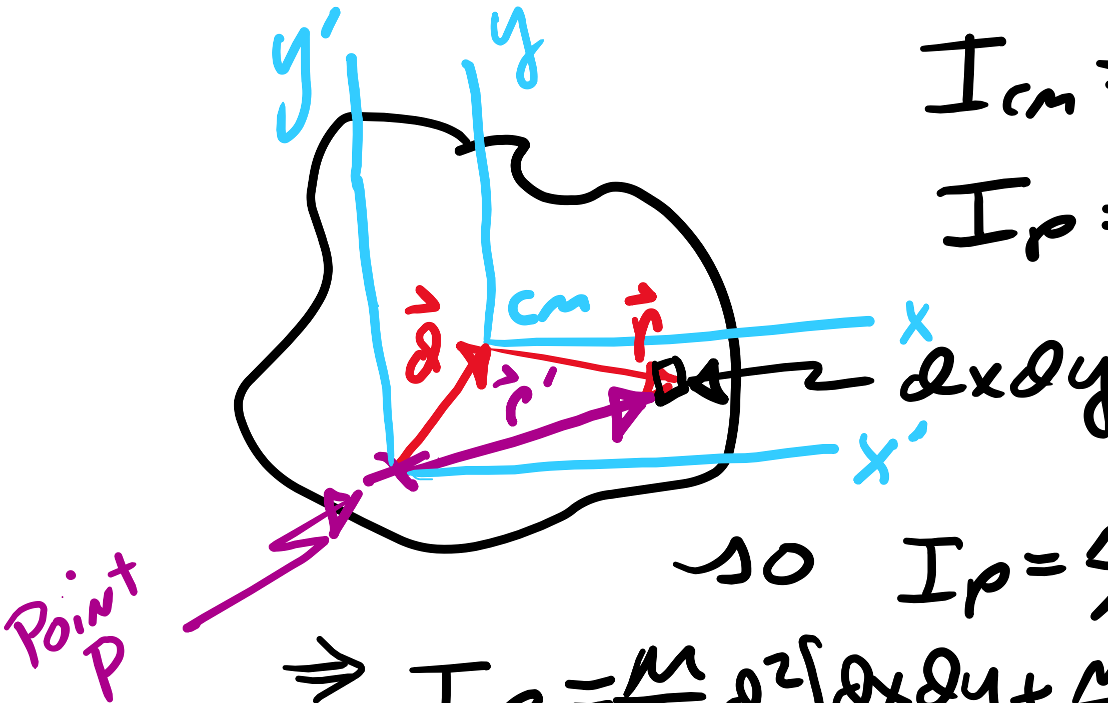
Parallel axis theorem



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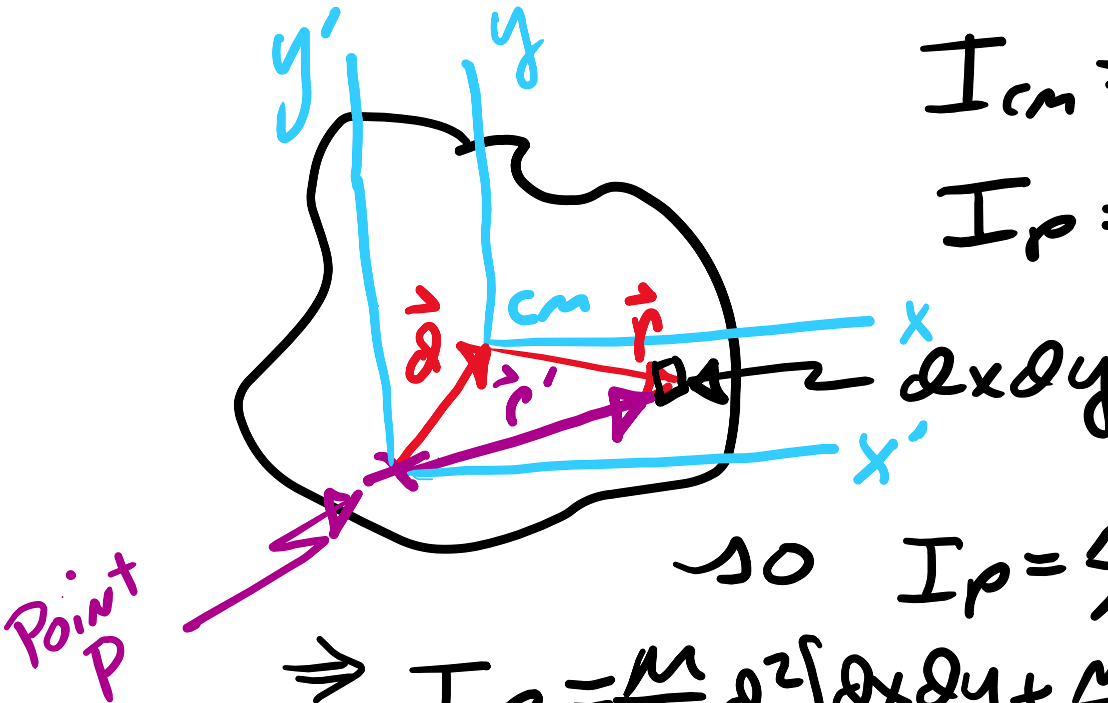
Parallel axis theorem



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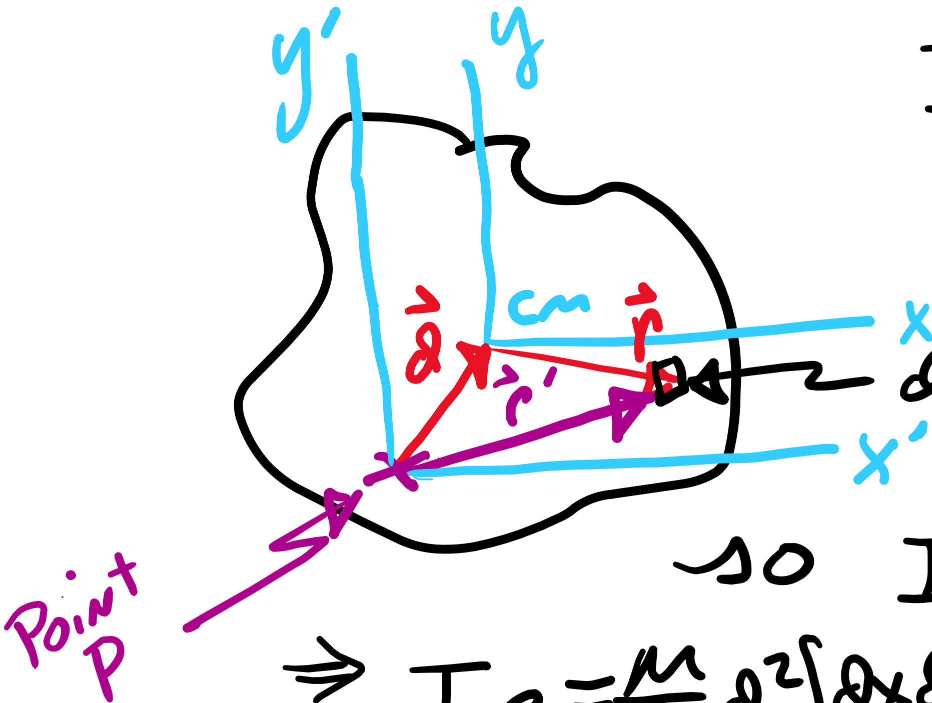
Note $$\vec{r}_{cm} = \frac{M}{A}\int \vec{r}\,dx\,dy$$

Parallel axis theorem



$$I_{cm} = \frac{M}{A}\int r^2\,dx\,dy$$

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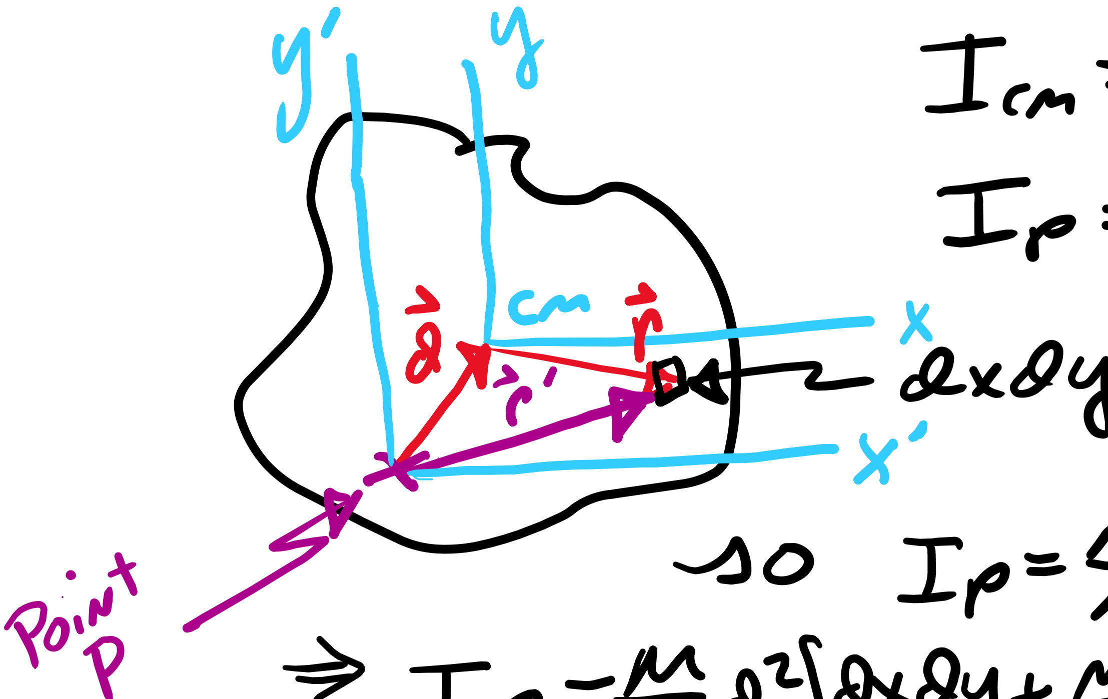
$$\Rightarrow I_P = \frac{M}{A}d^2\underbrace{\int dx\,dy}_{A} + \underbrace{\frac{M}{A}\int r^2\,dx\,dy}_{I_{cm}} + \frac{2M}{A}\vec{d}\cdot\underbrace{\int \vec{r}\,dx\,dy}_{0}$$

Note $\vec{r}_{cm} = \frac{M}{A}\int \vec{r}\,dx\,dy$, but $\vec{r}_{cm} = 0$ for coordinate system with origin at cm

Parallel axis theorem



$$I_{cm} = \frac{M}{A}\int r^2\,dx\,dy$$

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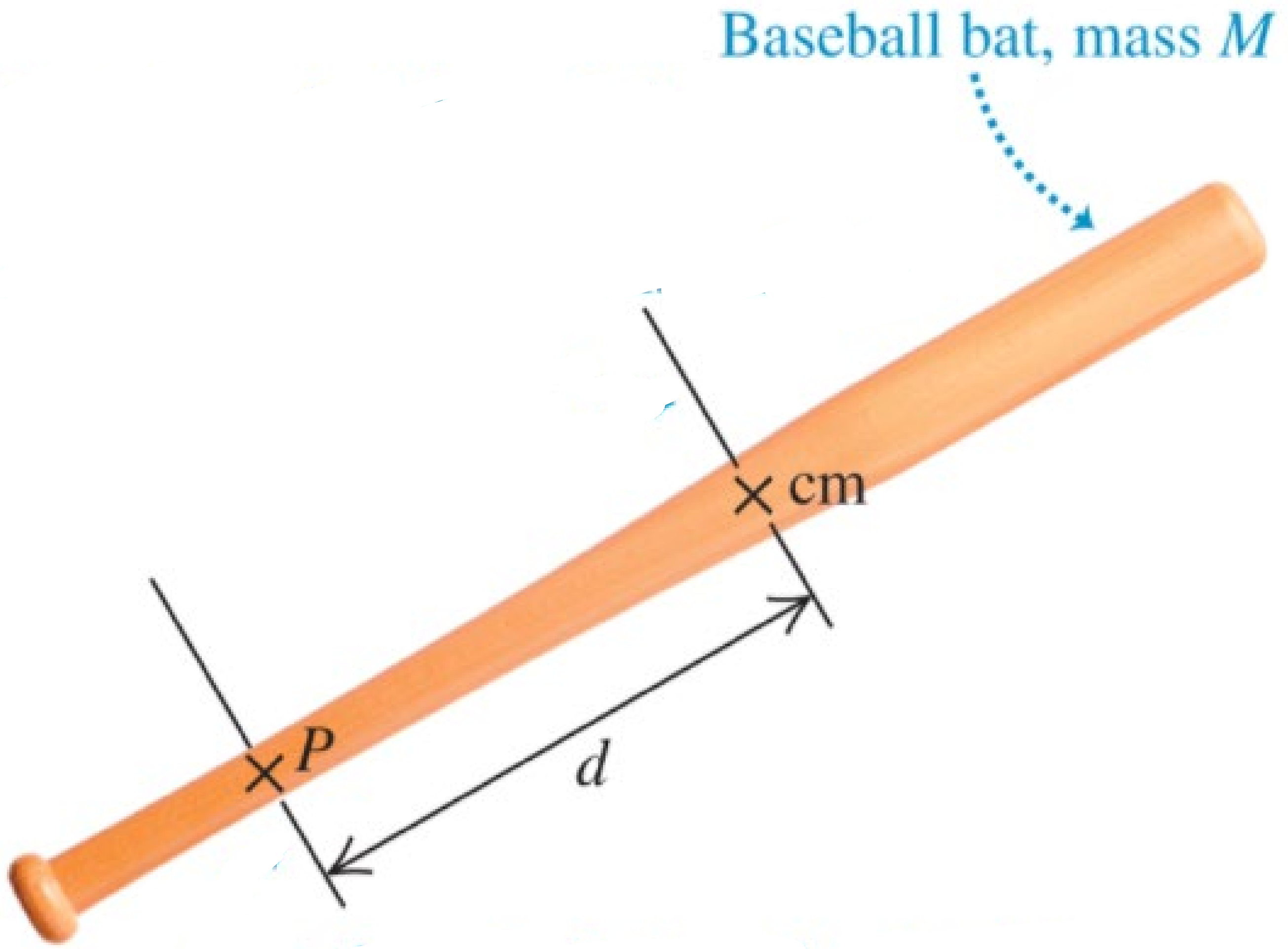
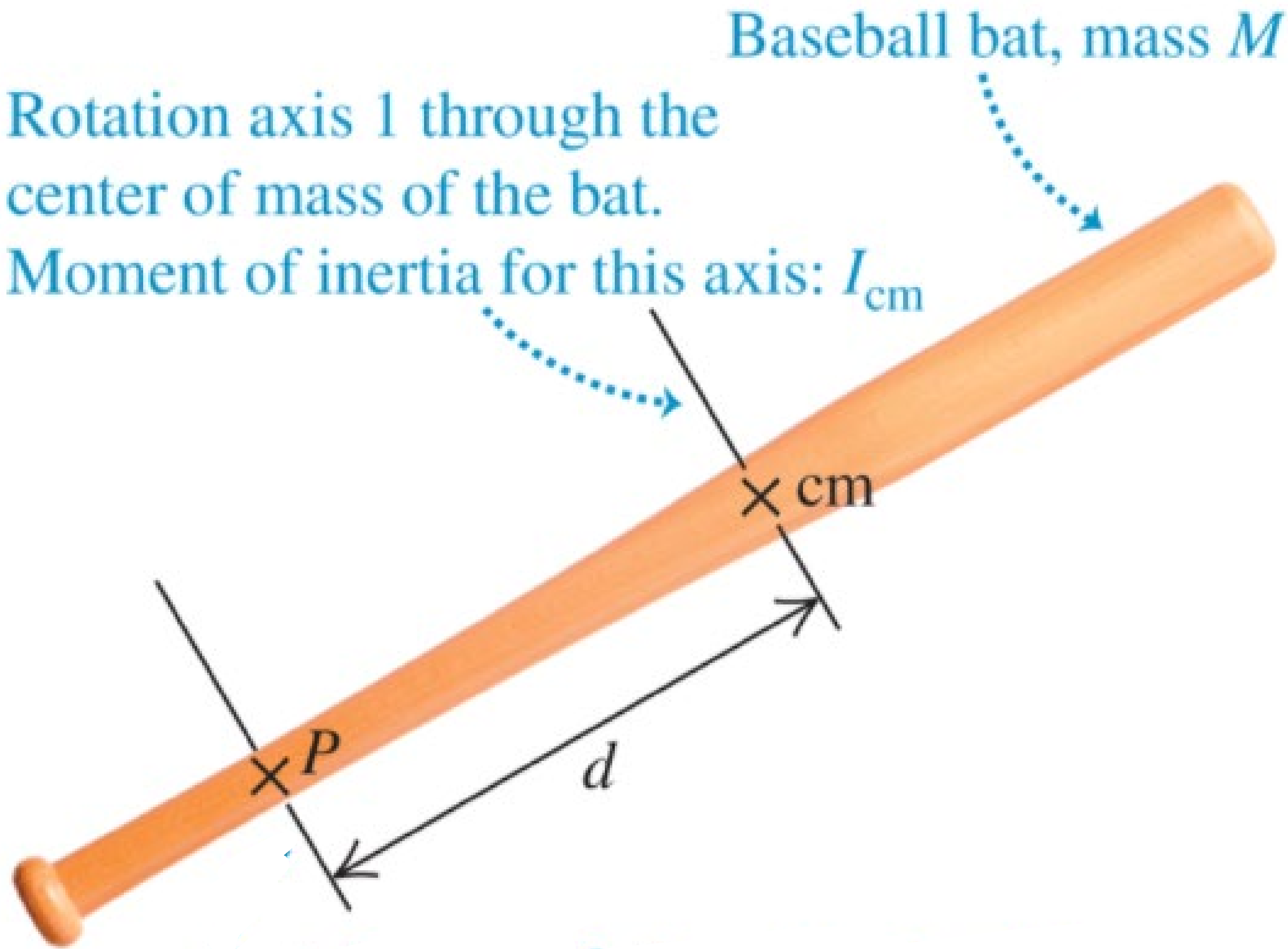
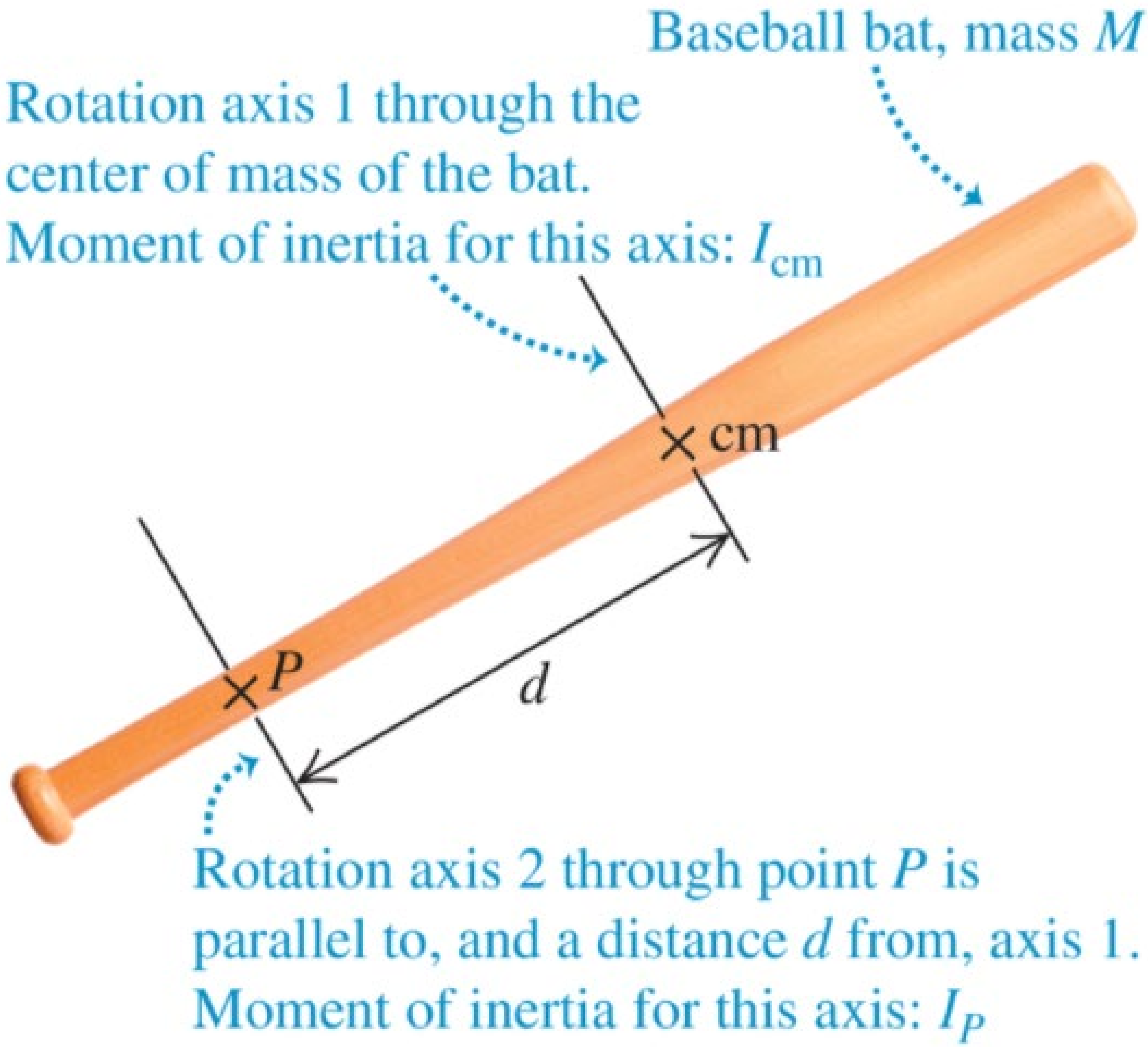
Now

$$\boxed{I_p = Md^2 + I_{cm}}$$

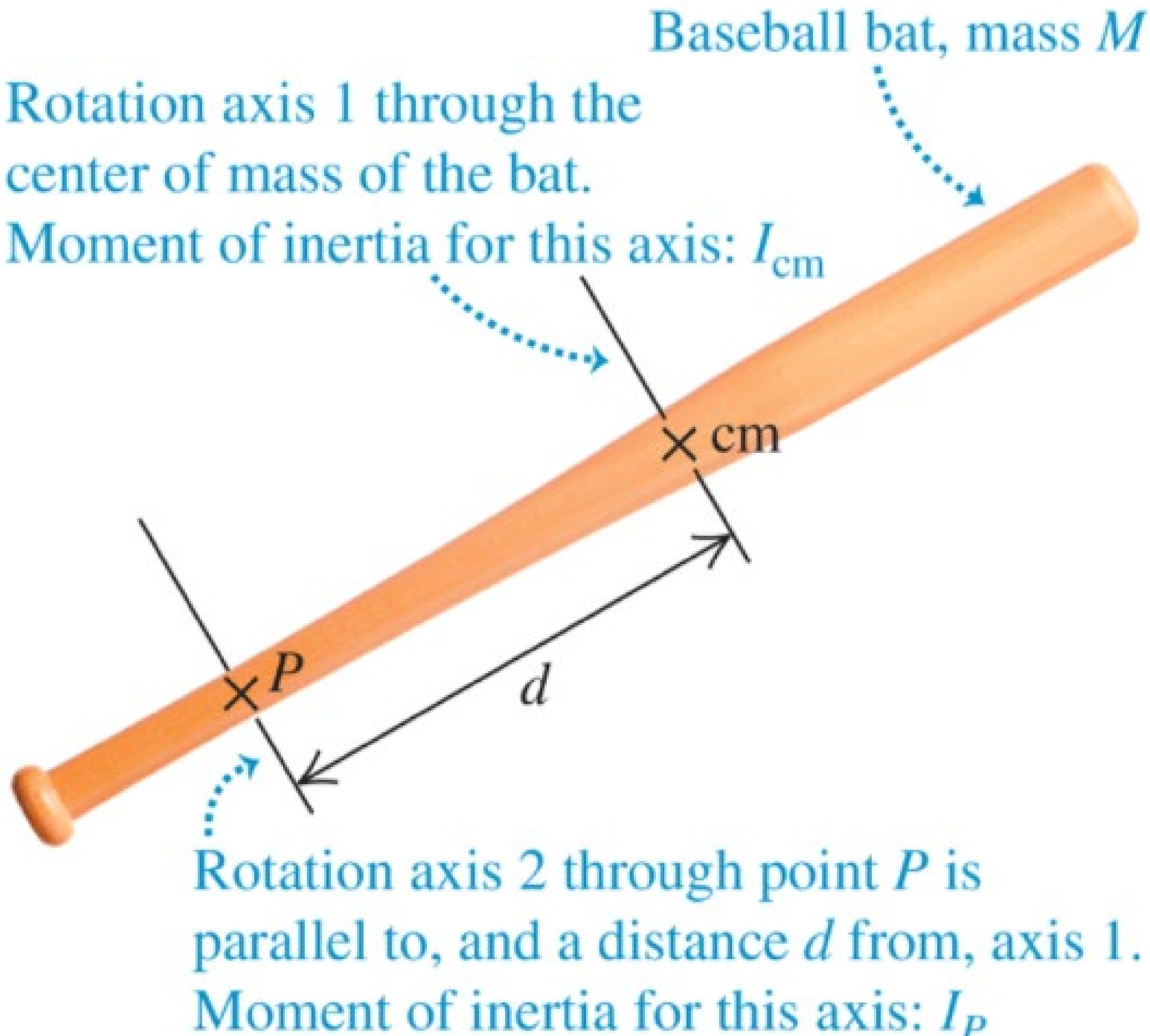


Baseball bat, mass M
cm
P
d


Baseball bat, mass M
Rotation axis 1 through the center of mass of the bat. Moment of inertia for this axis: I_{cm}
cm
P
d

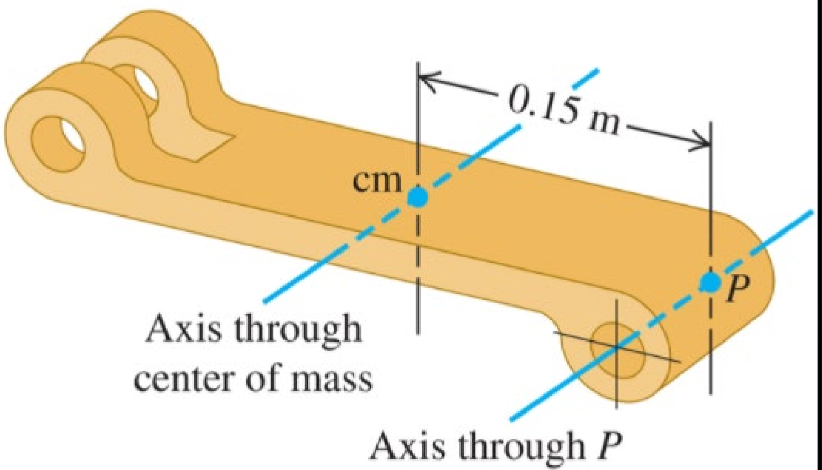


Baseball bat, mass M
Rotation axis 1 through the center of mass of the bat. Moment of inertia for this axis: I_{cm}
cm
P
d
Rotation axis 2 through point P is parallel to, and a distance d from, axis 1. Moment of inertia for this axis: I_P



Parallel-axis theorem: $I_P = I_{cm} + Md^2$

A part of a mechanical linkage (Fig. 9.21) has a mass of 3.6 kg. Its moment of inertia I_P about an axis 0.15 m from its center of mass is $I_P = 0.132\ \text{kg}\cdot\text{m}^2$. What is the moment of inertia I_{cm} about a parallel axis through the center of mass?



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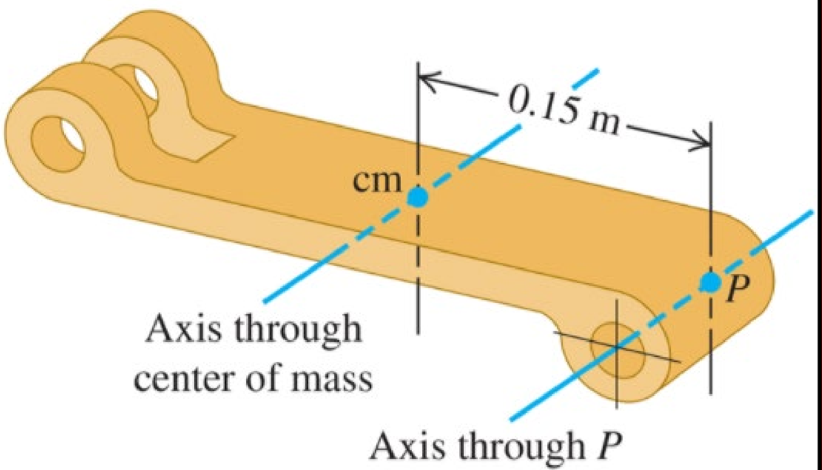
$m = 3{,}6\ kg$



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$m = 3{,}6\ \text{kg}$, $d = 0{,}15\ \text{m}$,

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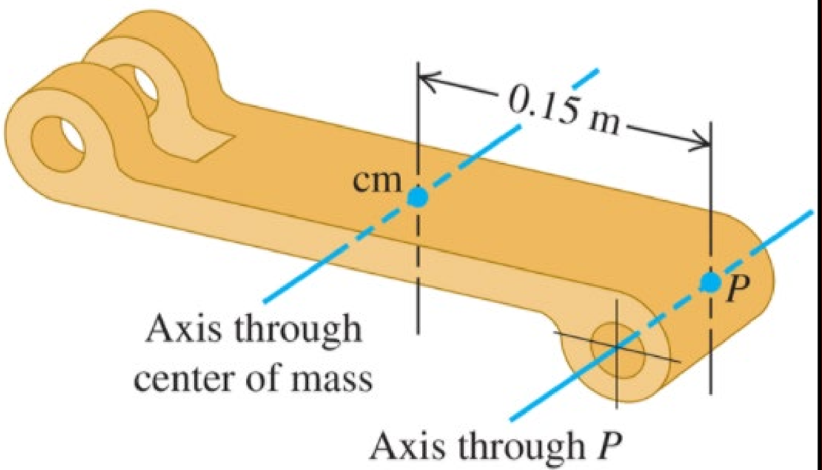

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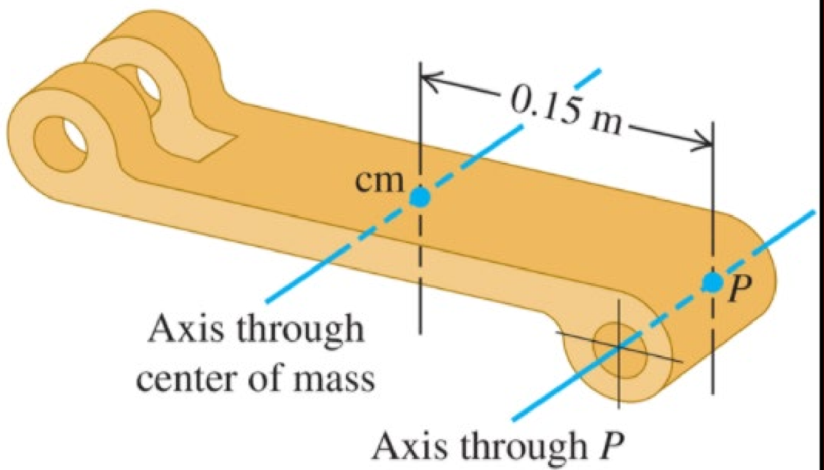
$m = 3,6\ \mathrm{kg}$, $d = 0,15\ \mathrm{m}$,

$I_P = 0.132\ \mathrm{Kg \cdot m^2}$

Find I_{cm}:



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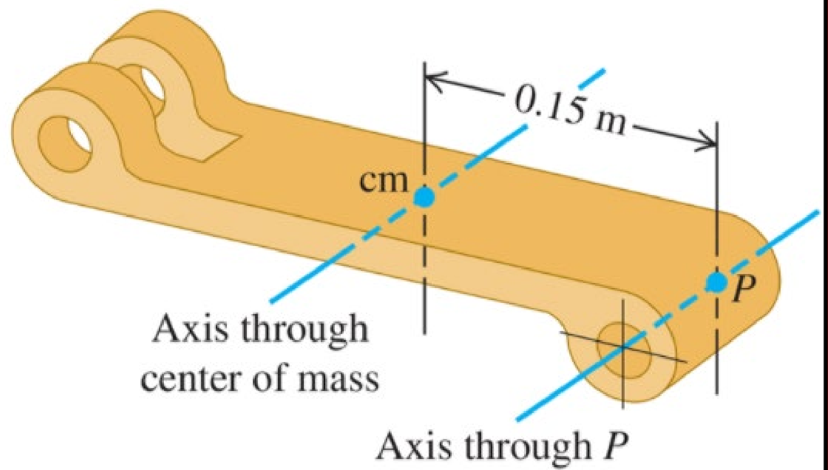
$m = 3.6\ kg$, $d = 0.15\ m$,

$I_p = 0.132\ Kg \cdot m^2$



Find I_{cm}:

$$I_p = I_{cm} + md^2$$

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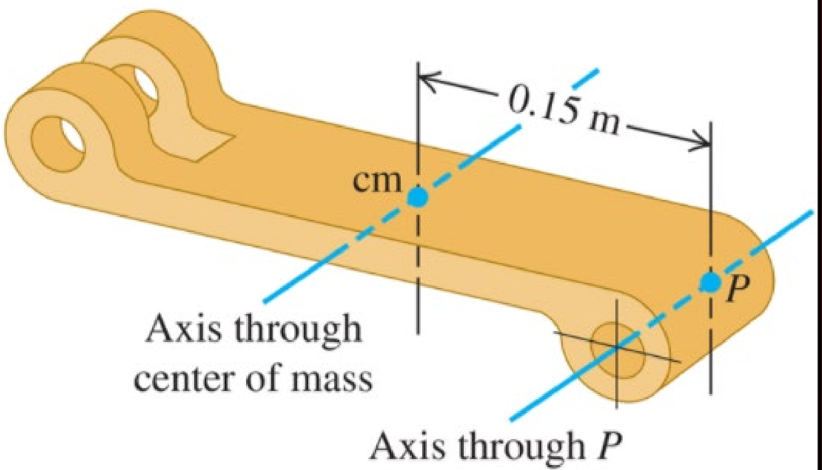
$m = 3{,}6\ \text{kg}$, $d = 0{,}15\ \text{m}$,

$I_P = 0.132\ \text{Kg}\cdot\text{m}^2$

Find I_{cm}:

$I_P = I_{cm} + md^2$ so

$I_{cm} = I_P - md^2$



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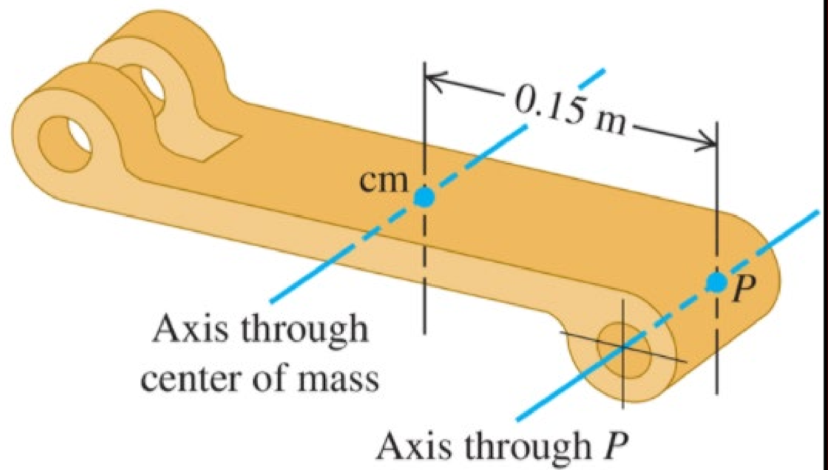


$m = 3.6\ \text{kg}$, $d = 0.15\ \text{m}$,

$I_p = 0.132\ \text{Kg}\cdot\text{m}^2$

Find I_{cm}:

$I_p = I_{cm} + md^2$ so

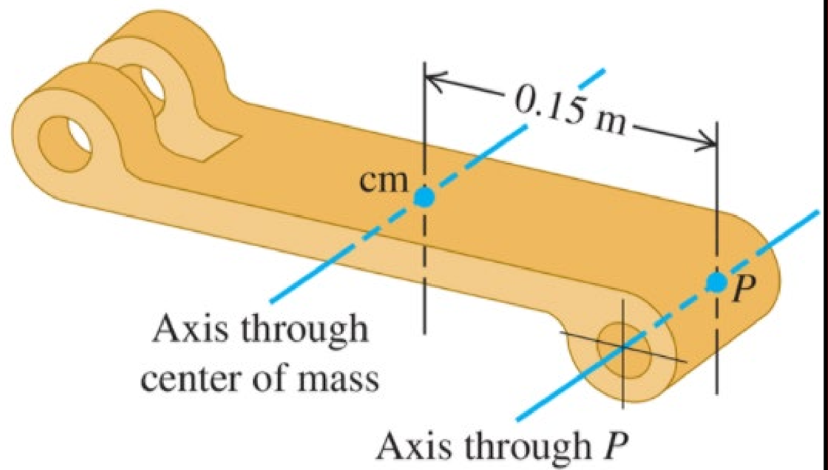
$I_{cm} = I_p - md^2 = (0.132 - 3.6 * 0.15^2)\ \text{Kg}\cdot\text{m}^2$

ASU

A part of a mechanical linkage (Fig. 9.21) has a mass of 3.6 kg. Its moment of inertia I_P about an axis 0.15 m from its center of mass is $I_P = 0.132\ \text{kg}\cdot\text{m}^2$. What is the moment of inertia I_{cm} about a parallel axis through the center of mass?

$m = 3.6\ \text{kg}$, $d = 0.15\ \text{m}$,

$I_p = 0.132\ \text{Kg}\cdot\text{m}^2$



Find I_{cm}:

$I_p = I_{cm} + md^2$ so

$I_{cm} = I_p - md^2 = (0.132 - 3.6 * 0.15^2)\ \text{Kg}\cdot\text{m}^2$

$\Rightarrow$ $\boxed{I_{cm} = 0.051\ \text{kg}\cdot\text{m}^2}$