

Today 9.3, 9.4

L29



Today 9.3, 9.4  
└

L29

Relating  
linear &  
angular kinematics

Today 9.3, 9.4

L29

Relating  
linear &  
angular kinematics

Energy in  
rotational  
motion

Today 9.3, 9.4

L29

Monday 9.5, 9.6



Today 9.3, 9.4

L29

Monday 9.5, 9.6

Parallel  
axis theorem

Today 9.3, 9.4

L28

Monday 9.5, 9.6

Parallel  
axis theorem

Moment  
of inertia  
calculations

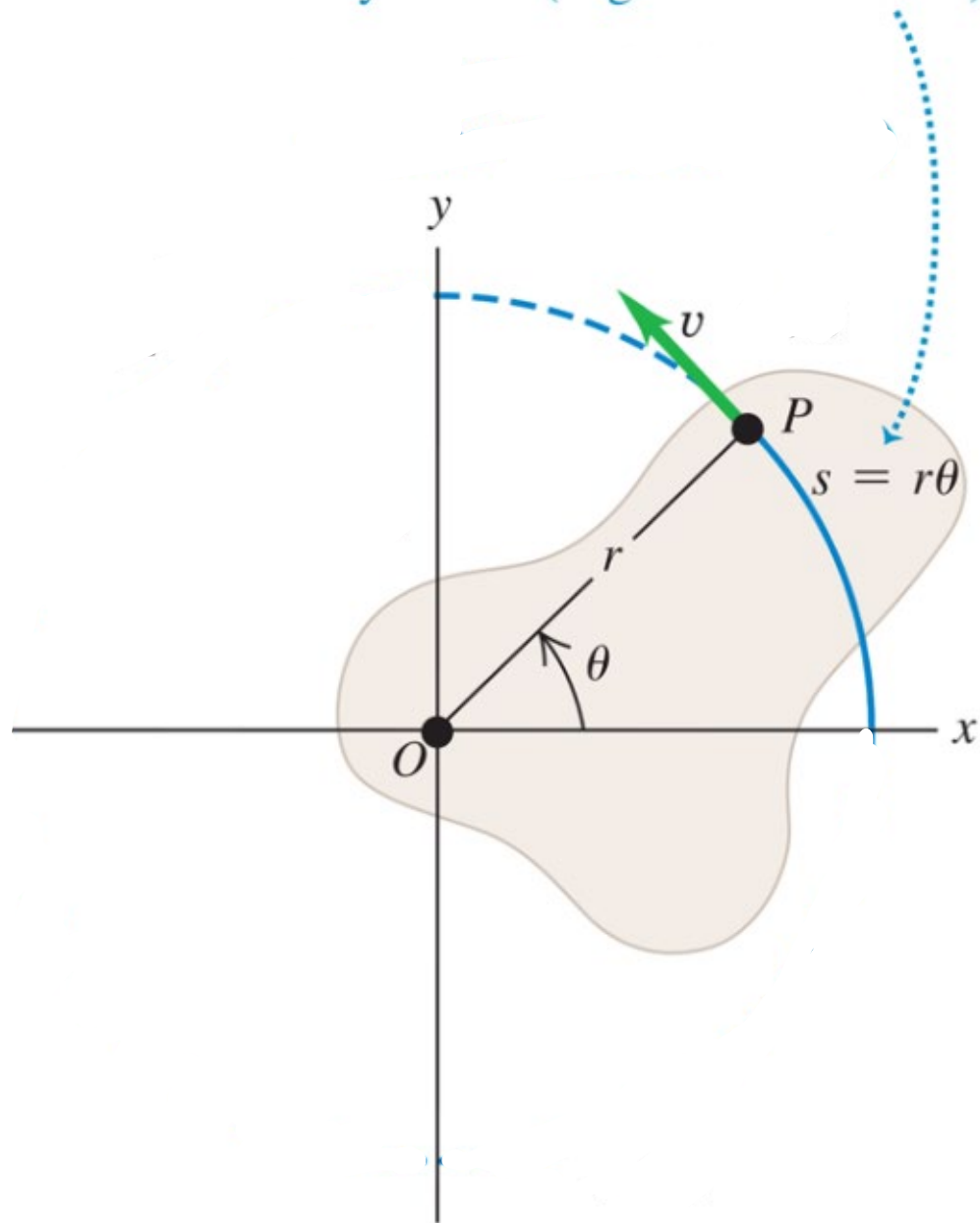
Today 9.3, 9.4

L28

Monday 9.5, 9.6

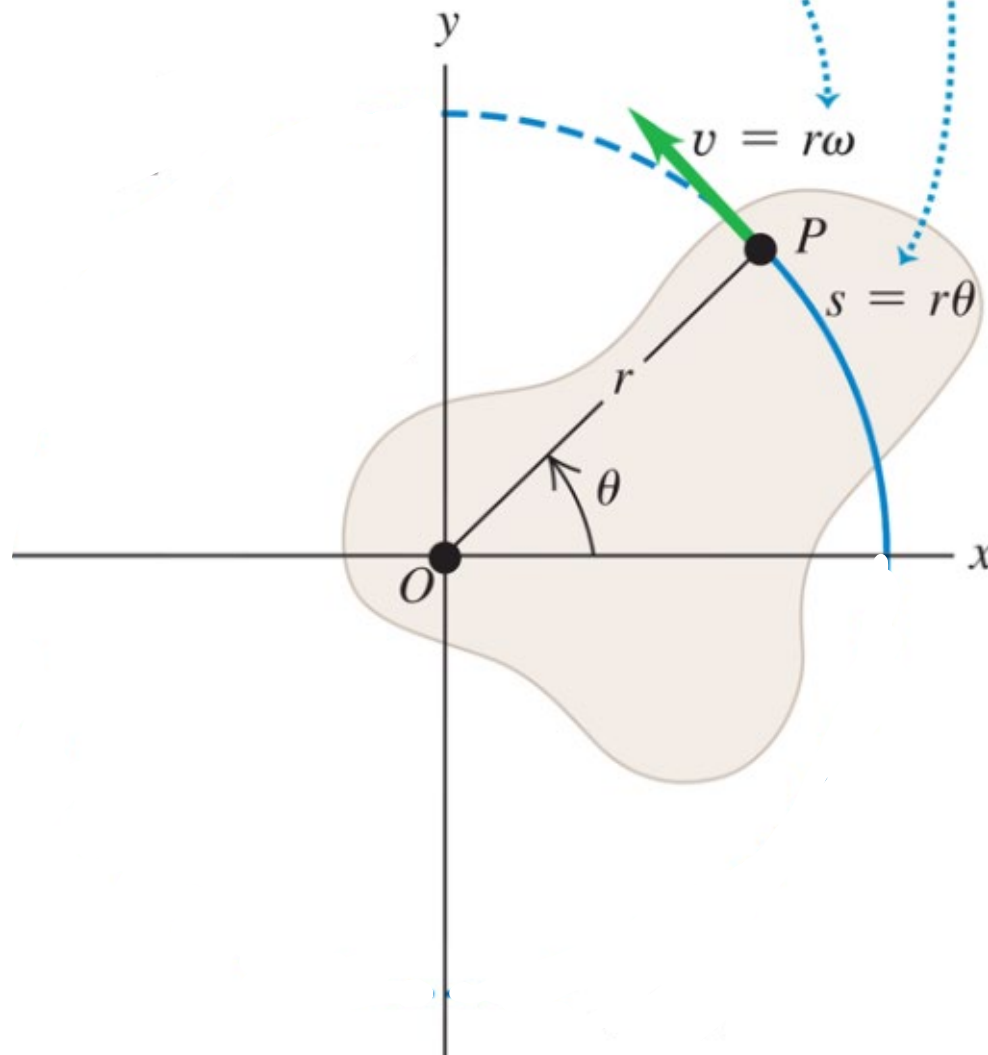
Wednesday Exam #3

Distance through which point  $P$  on the body moves (angle  $\theta$  is in radians)



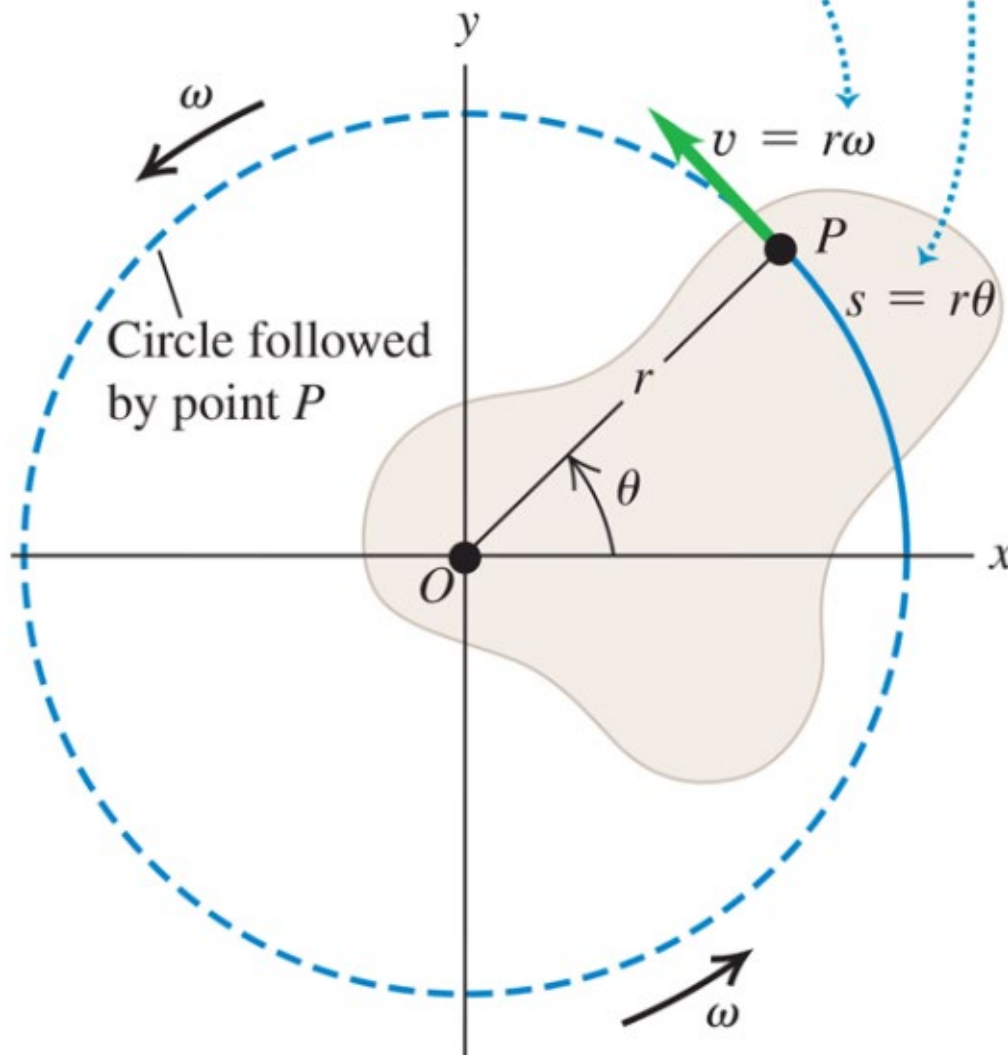
Distance through which point  $P$  on the body moves (angle  $\theta$  is in radians)

Linear speed of point  $P$   
(angular speed  $\omega$  is in rad/s)



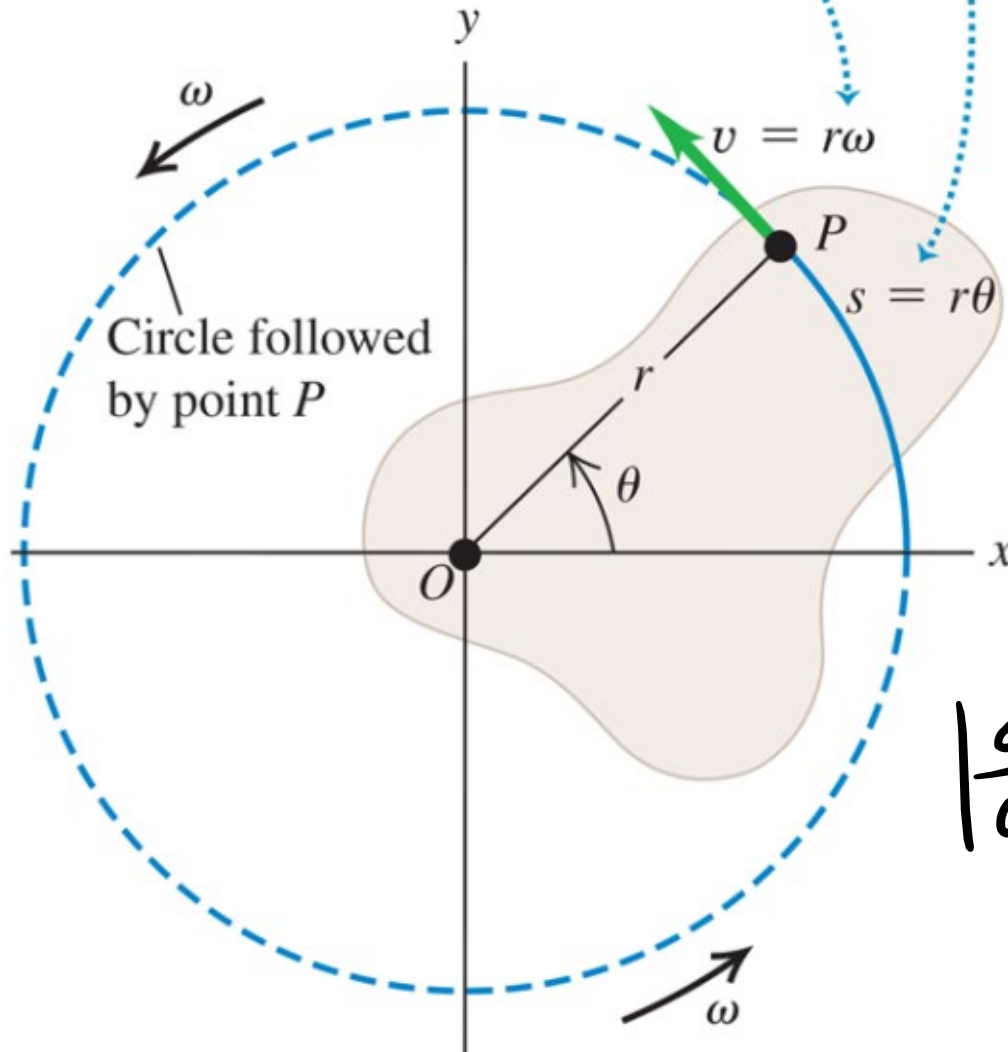
Distance through which point  $P$  on the body moves (angle  $\theta$  is in radians)

Linear speed of point  $P$   
(angular speed  $\omega$  is in rad/s)



Distance through which point  $P$  on the body moves (angle  $\theta$  is in radians)

Linear speed of point  $P$   
(angular speed  $\omega$  is in rad/s)



$$\left| \frac{ds}{dt} \right| = r \left| \frac{d\theta}{dt} \right|$$

$$V = r_{\text{cell}}$$

$$v = r\omega \quad \text{so}$$

$$a_{\text{tan}} = \frac{dv}{dt}$$

$$v = r\omega \quad \text{so}$$

$$a_{\text{tan}} = \frac{dv}{dt}$$

Note: v is not  
vector

$$v = r\omega \quad \text{so}$$

$$a_{\text{tan}} = \frac{dv}{dt} = \frac{d}{dt}(r\omega)$$

$$v = r\omega \quad \text{so}$$

$$a_{\text{tan}} = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r \frac{d\omega}{dt}$$

$$v = r\omega \quad \text{so}$$

$$a_{\text{tan}} = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r \frac{d\omega}{dt}$$

$$\Rightarrow a_{\text{tan}} = r\alpha$$

$$v = r\omega \quad \text{so}$$

$$a_{\text{tan}} = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r \frac{d\omega}{dt}$$

$$\Rightarrow a_{\text{tan}} = r\alpha \quad \&$$

$$a_{\text{rad}} = \frac{v^2}{r}$$

$$v = r\omega \quad \text{so}$$

$$a_{\text{tan}} = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r \frac{d\omega}{dt}$$

$$\Rightarrow a_{\text{tan}} = r\alpha \quad \&$$

$$a_{\text{rad}} = \frac{v^2}{r} = \frac{r^2\omega^2}{r}$$

$$v = r\omega$$

so

$$a_{\text{tan}} = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r \frac{d\omega}{dt}$$

$$\Rightarrow a_{\text{tan}} = r\alpha$$

$$a_{\text{rad}} = \frac{v^2}{r} = \frac{r^2\omega^2}{r} \Rightarrow$$

$$a_{\text{rad}} = r\omega^2$$

$$v = r\omega$$

so

$$a_{\text{tan}} = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r \frac{d\omega}{dt}$$

$\Rightarrow$

$$a_{\text{tan}} = r\alpha$$

$\dagger$

$$a_{\text{rad}} = \frac{v^2}{r} = \frac{r^2\omega^2}{r} \Rightarrow$$

$$a_{\text{rad}} = r\omega^2$$

$$v = r\omega$$

so

$$a_{\text{tan}} = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r \frac{d\omega}{dt}$$

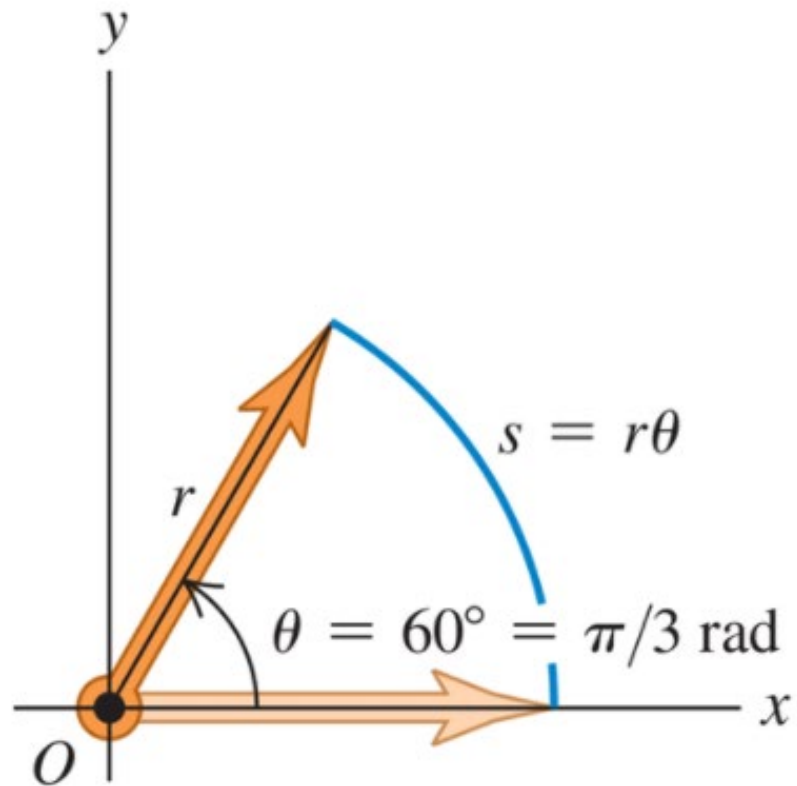
$\Rightarrow$

$$a_{\text{tan}} = r\alpha$$

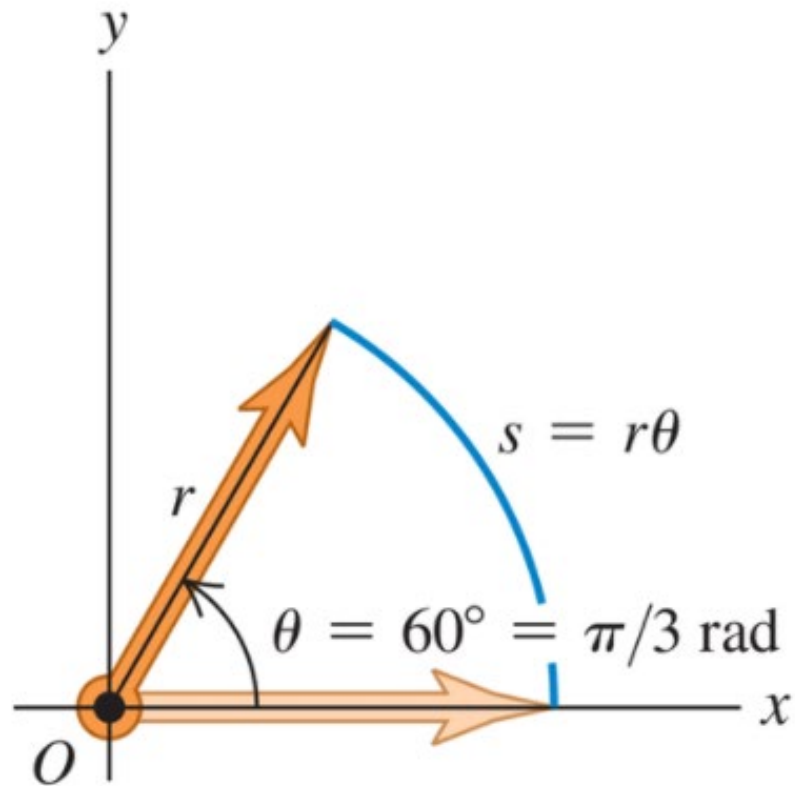
$\dagger$

$$a_{\text{rad}} = \frac{v^2}{r} = \frac{r^2\omega^2}{r} \Rightarrow$$

$$a_{\text{rad}} = r\omega^2$$

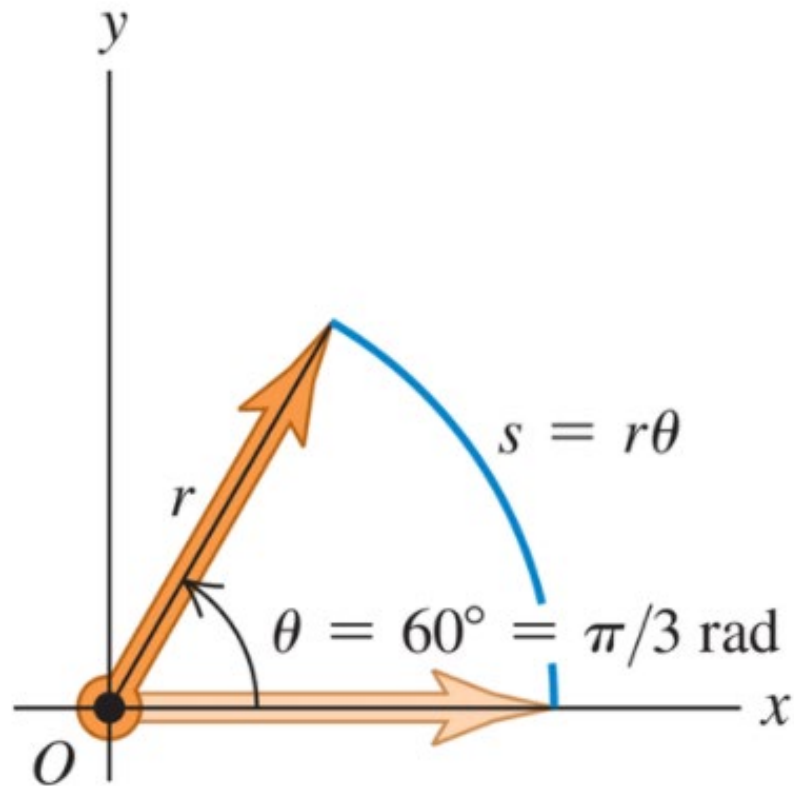


In any equation that relates linear quantities to angular quantities, the angles **MUST** be expressed in radians ...



In any equation that relates linear quantities to angular quantities, the angles **MUST** be expressed in radians ...

**RIGHT!** ►  $s = (\pi/3)r$



In any equation that relates linear quantities to angular quantities, the angles **MUST** be expressed in radians ...

**RIGHT!** ►  $s = (\pi/3)r$

... never in degrees or revolutions.

**WRONG** ►  $s = \cancel{60}r$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.



An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at  $10.0 \text{ rad/s}$  and the angular speed is increasing at  $50.0 \text{ rad/s}^2$ . For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}$$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}$$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

Find  $a_{\text{tan}}$  &  $a_{\text{rad}}$ :

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

Find  $a_{\text{tan}}$  &  $a_{\text{rad}}$ :  $a_{\text{tan}} = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = r\alpha$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

Find  $a_{\text{tan}}$  &  $a_{\text{rad}}$ :  $a_{\text{tan}} = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = r\alpha$

$$\Rightarrow a_{\text{tan}} = (0.8 \text{ m} * 50 / \text{s}^2)$$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

Find  $a_{\text{tan}}$  &  $a_{\text{rad}}$ :  $a_{\text{tan}} = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = r\alpha$

$$\Rightarrow a_{\text{tan}} = (0.8 \text{ m} \times 50 / \text{s}^2) = 40 \text{ m/s}^2$$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

Find  $a_{\text{tan}}$  &  $a_{\text{rad}}$ :  $a_{\text{tan}} = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = r\alpha$

$$\Rightarrow a_{\text{tan}} = (0.8 \text{ m} * 50 / \text{s}^2) = 40 \text{ m/s}^2$$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

Find  $a_{\text{tan}}$  &  $a_{\text{rad}}$ :  $a_{\text{tan}} = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = r\alpha$

$$\Rightarrow a_{\text{tan}} = (0.8 \text{ m} \times 50 / \text{s}^2) = 40 \text{ m/s}^2$$

$$a_{\text{rad}} = \frac{v^2}{R} = R\omega^2$$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

Find  $a_{\text{tan}}$  &  $a_{\text{rad}}$ :  $a_{\text{tan}} = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = r\alpha$

$$\Rightarrow a_{\text{tan}} = (0.8 \text{ m} \times 50 / \text{s}^2) = 40 \text{ m/s}^2$$

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{R^2 \omega^2}{R} = R\omega^2$$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

Find  $a_{\text{tan}}$  &  $a_{\text{rad}}$ :  $a_{\text{tan}} = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = r\alpha$

$$\Rightarrow a_{\text{tan}} = (0.8 \text{ m} \times 50 / \text{s}^2) = 40 \text{ m/s}^2$$

$$a_{\text{rad}} = \frac{v^2}{R} = R\omega^2 / R = r\omega^2 = (0.8 \text{ m})(100 / \text{s}^2)$$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

Find  $a_{\text{tan}}$  &  $a_{\text{rad}}$ :  $a_{\text{tan}} = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = r\alpha$

$$\Rightarrow a_{\text{tan}} = (0.8 \text{ m} \times 50 / \text{s}^2) = 40 \text{ m/s}^2$$

$$a_{\text{rad}} = \frac{v^2}{R} = R\omega^2 / R = r\omega^2 = (0.8 \text{ m})(100 / \text{s}^2)$$

$$\Rightarrow a_{\text{rad}} = 80 \text{ m/s}^2$$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

Find  $a_{\text{tan}}$  &  $a_{\text{rad}}$ :  $a_{\text{tan}} = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = r\alpha$

$$\Rightarrow a_{\text{tan}} = (0.8 \text{ m} \times 50 / \text{s}^2) = 40 \text{ m/s}^2$$

$$a_{\text{rad}} = \frac{v^2}{R} = R\omega^2 / R = r\omega^2 = (0.8 \text{ m})(100 / \text{s}^2)$$

$$\Rightarrow a_{\text{rad}} = 80 \text{ m/s}^2$$

Find  $|\vec{a}|$ :

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

Find  $a_{\text{tan}}$  &  $a_{\text{rad}}$ :  $a_{\text{tan}} = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = r\alpha$

$$\Rightarrow a_{\text{tan}} = (0.8 \text{ m} \times 50 / \text{s}^2) = 40 \text{ m/s}^2$$

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{R^2 \omega^2}{R} = r\omega^2 = (0.8 \text{ m})(100 / \text{s}^2)$$

$$\Rightarrow a_{\text{rad}} = 80 \text{ m/s}^2 \quad \text{Find } |\vec{a}|:$$

$$a = \sqrt{a_{\text{tan}}^2 + a_{\text{rad}}^2} = \sqrt{40^2 + 80^2} \text{ (m/s}^2\text{)}$$

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s<sup>2</sup>. For this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

$$R = 80 \text{ cm}, \omega = 10 \text{ rad/s}, \alpha = 50 \frac{\text{rad}}{\text{s}^2}$$

Find  $a_{\text{tan}}$  &  $a_{\text{rad}}$ :  $a_{\text{tan}} = \frac{dv}{dt} = \frac{d(R\omega)}{dt} = r\alpha$

$$\Rightarrow a_{\text{tan}} = (0.8 \text{ m} \times 50 / \text{s}^2) = 40 \text{ m/s}^2$$

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{R^2 \omega^2}{R} = r\omega^2 = (0.8 \text{ m})(100 / \text{s}^2)$$

$$\Rightarrow a_{\text{rad}} = 80 \text{ m/s}^2 \quad \text{Find } |\vec{a}|:$$

$$a = \sqrt{a_{\text{tan}}^2 + a_{\text{rad}}^2} = \sqrt{40^2 + 80^2} \text{ (m/s}^2\text{)}$$

$$\Rightarrow a = 89.4 \text{ m/s}^2$$

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

$$\omega = 2400 \text{ rpm}$$

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

$$\omega = 2400 \text{ rpm} \left( \frac{2\pi}{\text{rev}} \right)$$

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

$$\omega = 2400 \text{ rpm} \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right)$$

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

$$\omega = 2400 \text{ rpm} \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{\text{min}}{60\text{s}} \right) = 251 \text{ rad/s}$$

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

$$\omega = 2400 \text{ rpm} \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{\text{min}}{60\text{s}} \right) = 251 \text{ rad/s}, \quad v_p = 75 \text{ m/s}$$

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

$$\omega = 2400 \text{ rpm} \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{\text{min}}{60\text{s}} \right) = 251 \text{ rad/s}, \quad v_p = 75 \text{ m/s}$$
$$v_{\text{tip}} = \left[ v_{\text{tan}}^2 + v_p^2 \right]^{1/2} \leq 270 \text{ m/s}$$

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

$$\omega = 2400 \text{ rpm} \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{\text{min}}{60\text{s}} \right) = 251 \text{ rad/s}, \quad v_p = 75 \text{ m/s}$$
$$v_{\text{tip}} = \left[ v_{\text{tan}}^2 + v_p^2 \right]^{1/2} \leq 270 \text{ m/s} \Rightarrow v_{\text{tip}}^2 = R^2 \omega^2 + v_p^2$$

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

$$\omega = 2400 \text{ rpm} \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{\text{min}}{60 \text{ s}} \right) = 251 \text{ rad/s}, \quad v_p = 75 \text{ m/s}$$

$$v_{\text{tip}} = \left[ v_{\text{tan}}^2 + v_p^2 \right]^{1/2} \leq 270 \text{ m/s} \Rightarrow v_{\text{tip}}^2 = R^2 \omega^2 + v_p^2$$

$$\Rightarrow R = \sqrt{v_{\text{tip}}^2 - v_p^2} / \omega$$

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

$$\omega = 2400 \text{ rpm} \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{\text{min}}{60 \text{ s}} \right) = 251 \text{ rad/s}, \quad v_p = 75 \text{ m/s}$$

$$v_{\text{tip}} = \left[ v_{\text{tan}}^2 + v_p^2 \right]^{1/2} \leq 270 \text{ m/s} \Rightarrow v_{\text{tip}}^2 = R^2 \omega^2 + v_p^2$$

$$\Rightarrow R = \sqrt{v_{\text{tip}}^2 - v_p^2} / \omega = \left[ (270^2 - 75^2)^{1/2} / 251 \right] \text{ m}$$

$$\Rightarrow R = 1.03 \text{ m}$$

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

$$\omega = 2400 \text{ rpm} \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{\text{min}}{60 \text{ s}} \right) = 251 \text{ rad/s}, \quad v_p = 75 \text{ m/s}$$

$$v_{\text{tip}} = \left[ v_{\text{tan}}^2 + v_p^2 \right]^{1/2} \leq 270 \text{ m/s} \Rightarrow v_{\text{tip}}^2 = R^2 \omega^2 + v_p^2$$

$$\Rightarrow R = \sqrt{v_{\text{tip}}^2 - v_p^2} / \omega = \left[ (270^2 - 75^2)^{1/2} / 251 \right] \text{ m}$$

$$\Rightarrow \boxed{R = 1.03 \text{ m}} \quad a = R \omega^2$$

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

$$\omega = 2400 \text{ rpm} \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{\text{min}}{60 \text{ s}} \right) = 251 \text{ rad/s}, \quad v_p = 75 \text{ m/s}$$

$$v_{\text{tip}} = \left[ v_{\text{tan}}^2 + v_p^2 \right]^{1/2} \leq 270 \text{ m/s} \Rightarrow v_{\text{tip}}^2 = R^2 \omega^2 + v_p^2$$

$$\Rightarrow R = \sqrt{v_{\text{tip}}^2 - v_p^2} / \omega = \left[ (270^2 - 75^2)^{1/2} / 251 \right] \text{ m}$$

$$\Rightarrow \boxed{R = 1.03 \text{ m}} \quad a = R \omega^2 = (1.03)(251)^2 \text{ m/s}^2$$

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

$$\omega = 2400 \text{ rpm} \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{\text{min}}{60 \text{ s}} \right) = 251 \text{ rad/s}, \quad v_p = 75 \text{ m/s}$$

$$v_{\text{tip}} = \left[ v_{\text{tan}}^2 + v_p^2 \right]^{1/2} \leq 270 \text{ m/s} \Rightarrow v_{\text{tip}}^2 = R^2 \omega^2 + v_p^2$$

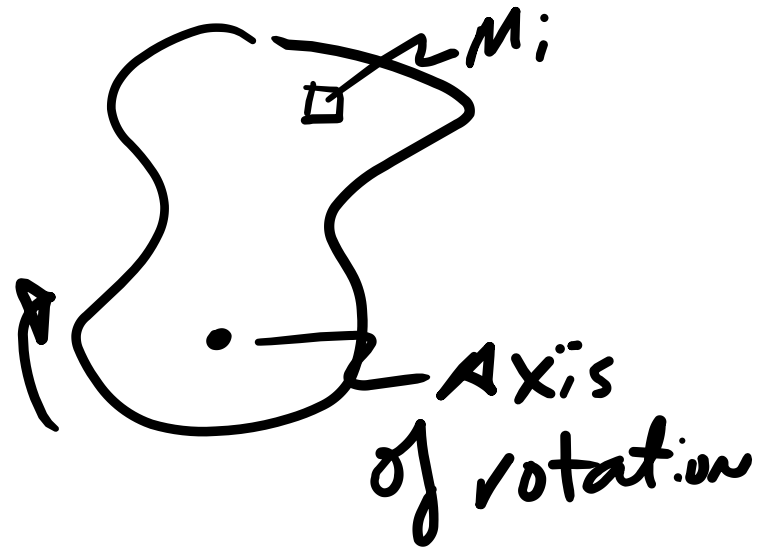
$$\Rightarrow R = \sqrt{v_{\text{tip}}^2 - v_p^2} / \omega = \left[ (270^2 - 75^2)^{1/2} / 251 \right] \text{ m}$$

$$\Rightarrow R = 1.03 \text{ m} \quad a = R \omega^2 = (1.03)(251)^2 \text{ m/s}^2$$

$\Rightarrow$

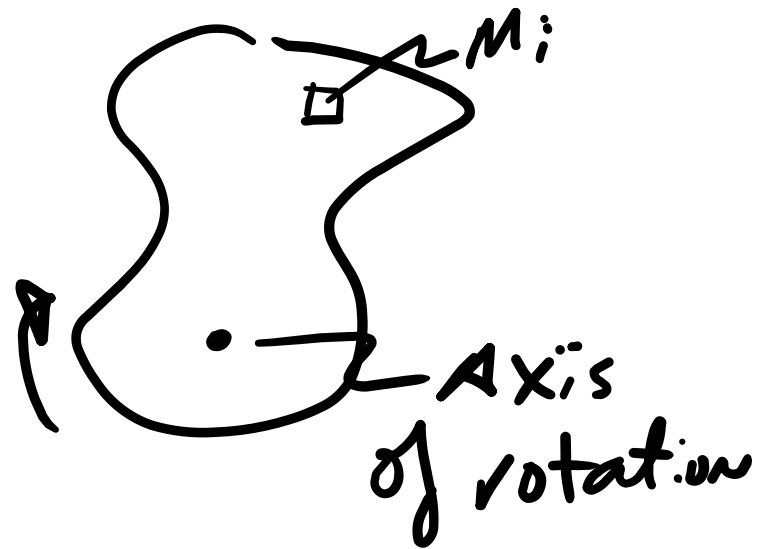
$$a = 6.49 \times 10^4 \text{ m/s}^2$$

Rigid body made  
of many small  
parts rotating  
about fixed axis



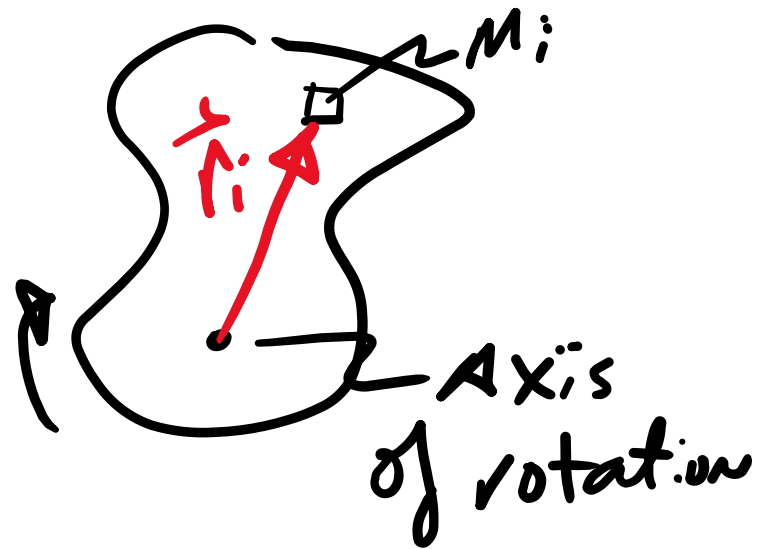
Rigid body made  
of many small  
parts rotating  
about fixed axis

$$K_i = \frac{1}{2} m v_i^2$$



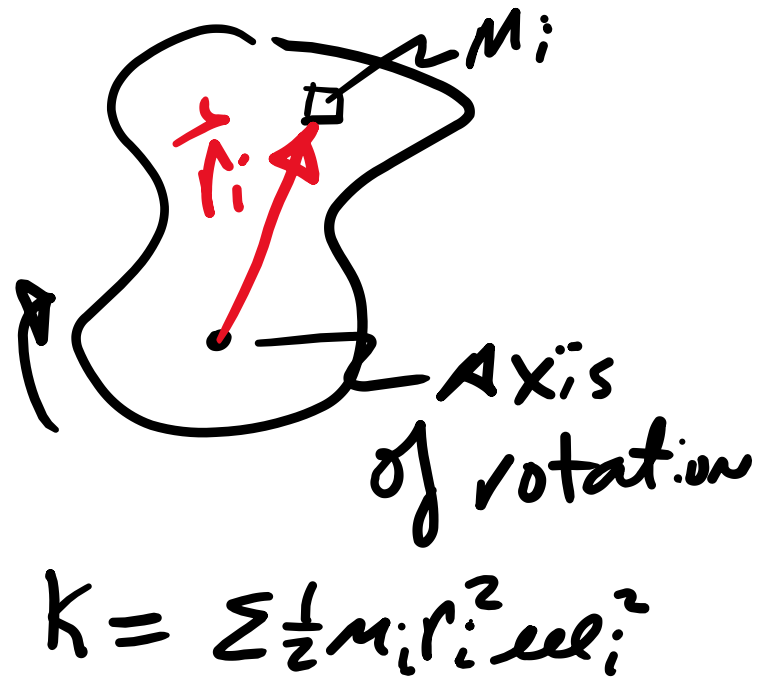
Rigid body made  
of many small  
parts rotating  
about fixed axis

$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2} m r_i^2 \omega^2$$



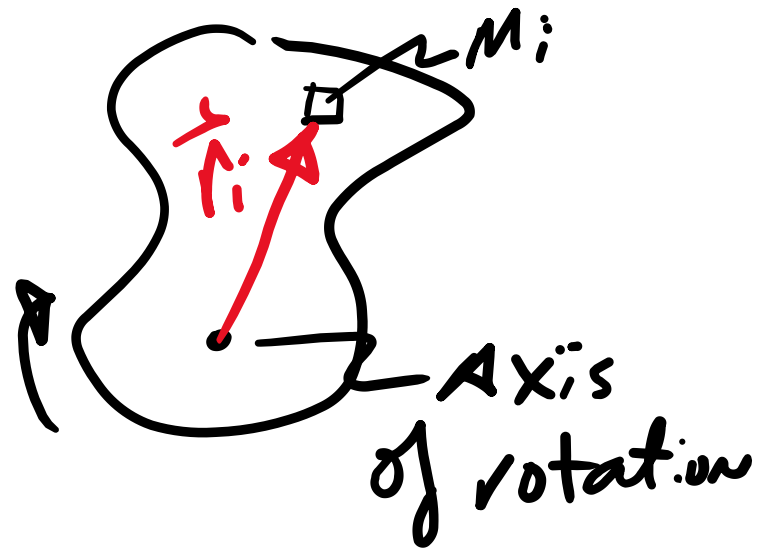
Rigid body made  
of many small  
parts rotating  
about fixed axis

$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2} m r_i^2 \omega_i^2 \Rightarrow$$



$$K = \sum \frac{1}{2} m_i r_i^2 \omega_i^2$$

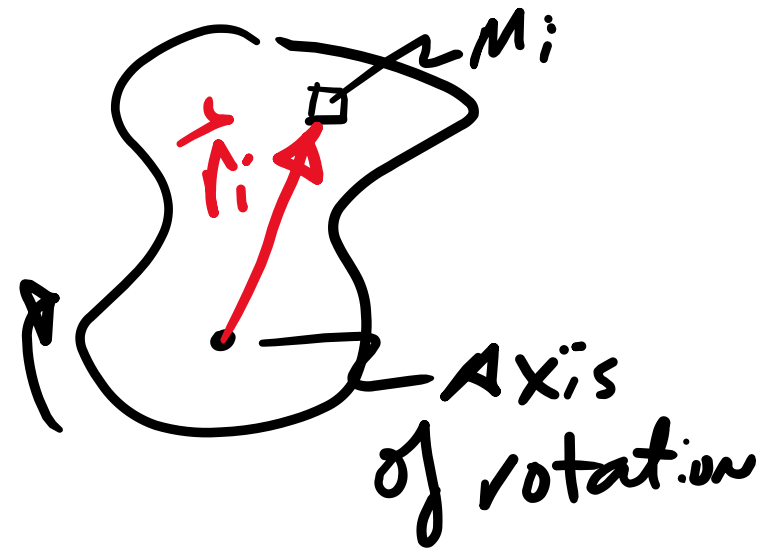
Rigid body made  
of many small  
parts rotating  
about fixed axis



$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2} m r_i^2 \omega^2 \Rightarrow K = \sum \frac{1}{2} m_i r_i^2 \omega^2$$

$$\Rightarrow K = \frac{1}{2} [\sum m_i r_i^2] \omega^2$$

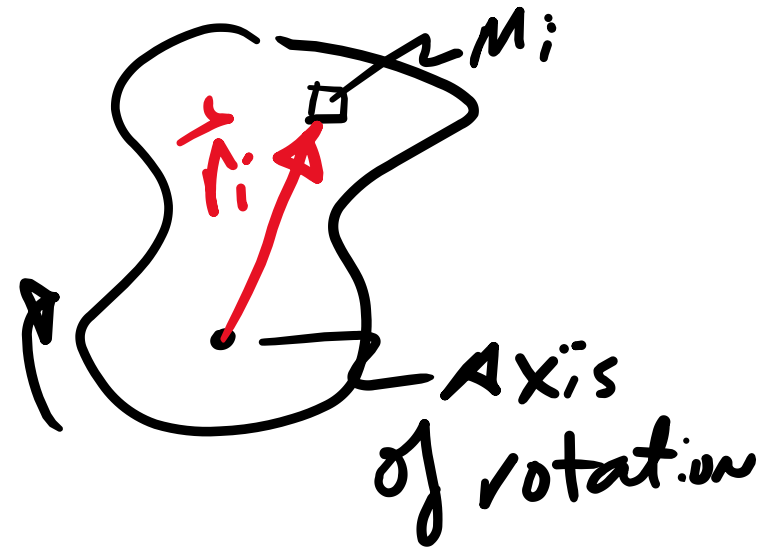
Rigid body made  
of many small  
parts rotating  
about fixed axis



$$K_i = \frac{1}{2} M v_i^2 = \frac{1}{2} M r_i^2 \omega_i^2 \Rightarrow K = \sum \frac{1}{2} M_i r_i^2 \omega_i^2$$

$$\Rightarrow K = \frac{1}{2} [\sum M_i r_i^2] \omega_i^2 \Rightarrow K = \frac{1}{2} I \omega_i^2$$

Rigid body made  
of many small  
parts rotating  
about fixed axis

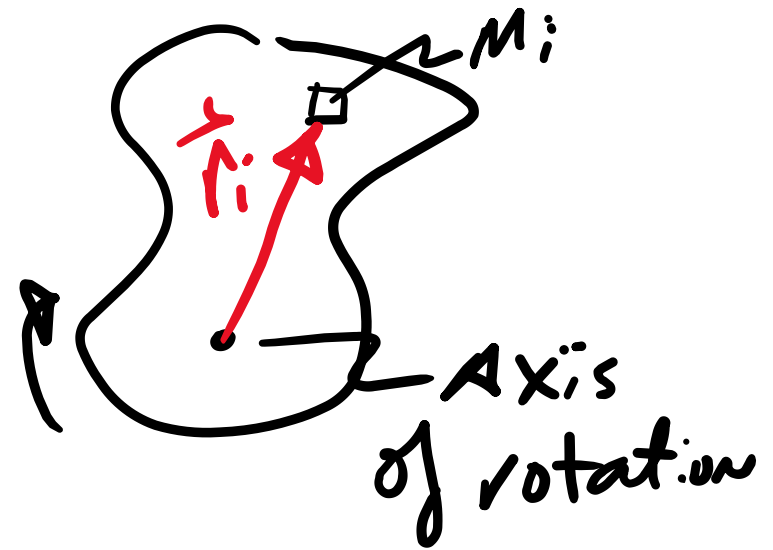


$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2} m r_i^2 \omega_i^2 \Rightarrow K = \sum \frac{1}{2} m_i r_i^2 \omega_i^2$$

$$\Rightarrow K = \frac{1}{2} [\sum m_i r_i^2] \omega_i^2 \Rightarrow K = \frac{1}{2} I \omega_i^2$$

where  $I \equiv \sum m_i r_i^2$

Rigid body made  
of many small  
parts rotating  
about fixed axis

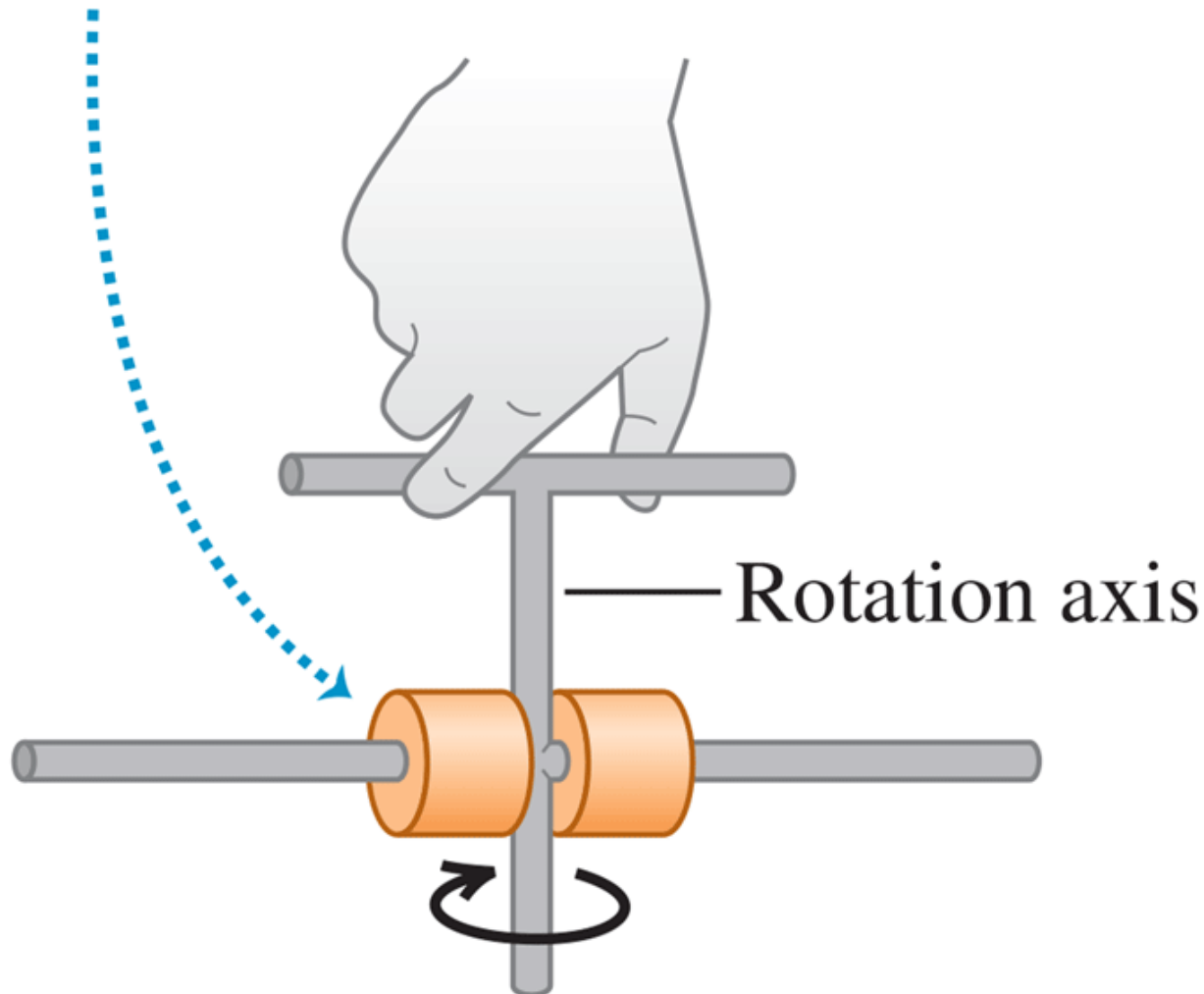


$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2} m r_i^2 \omega_i^2 \Rightarrow K = \sum \frac{1}{2} m_i r_i^2 \omega_i^2$$

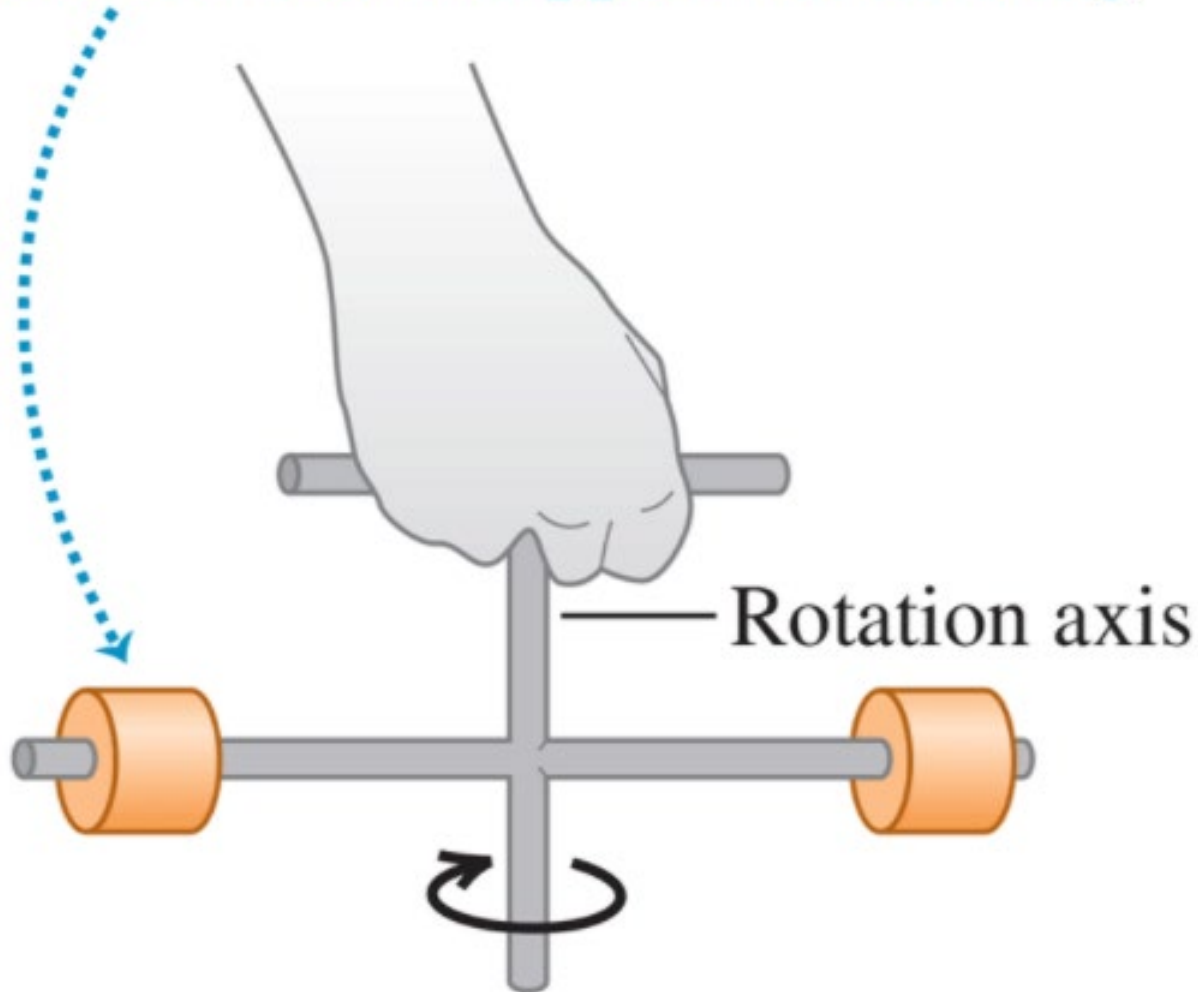
$$\Rightarrow K = \frac{1}{2} [\sum m_i r_i^2] \omega_i^2 \Rightarrow K = \frac{1}{2} I \omega_i^2$$

where  $I \equiv \sum m_i r_i^2$  &  $I \equiv$  moment of inertia

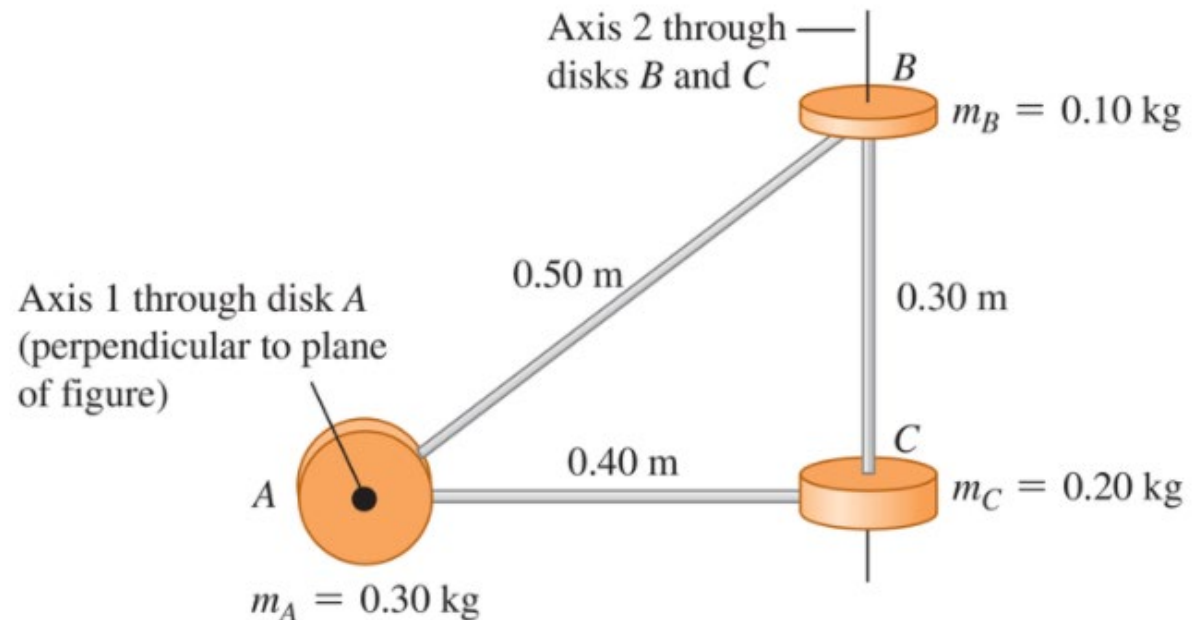
- Mass close to axis
- Small moment of inertia
- Easy to start apparatus rotating



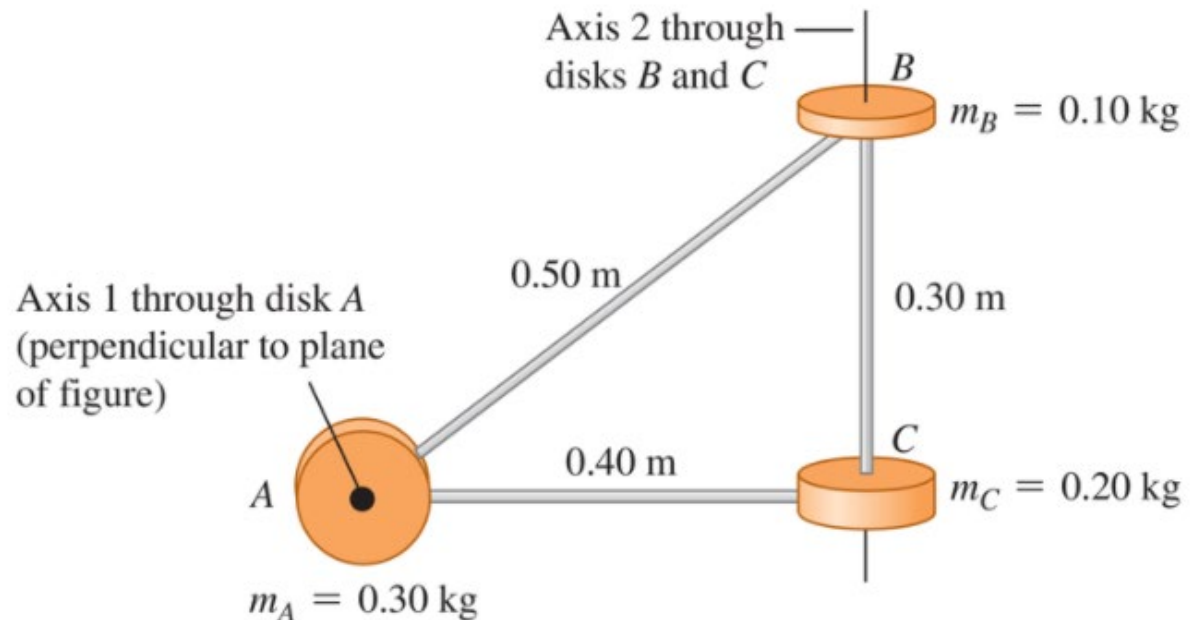
- Mass farther from axis
- Greater moment of inertia
- Harder to start apparatus rotating



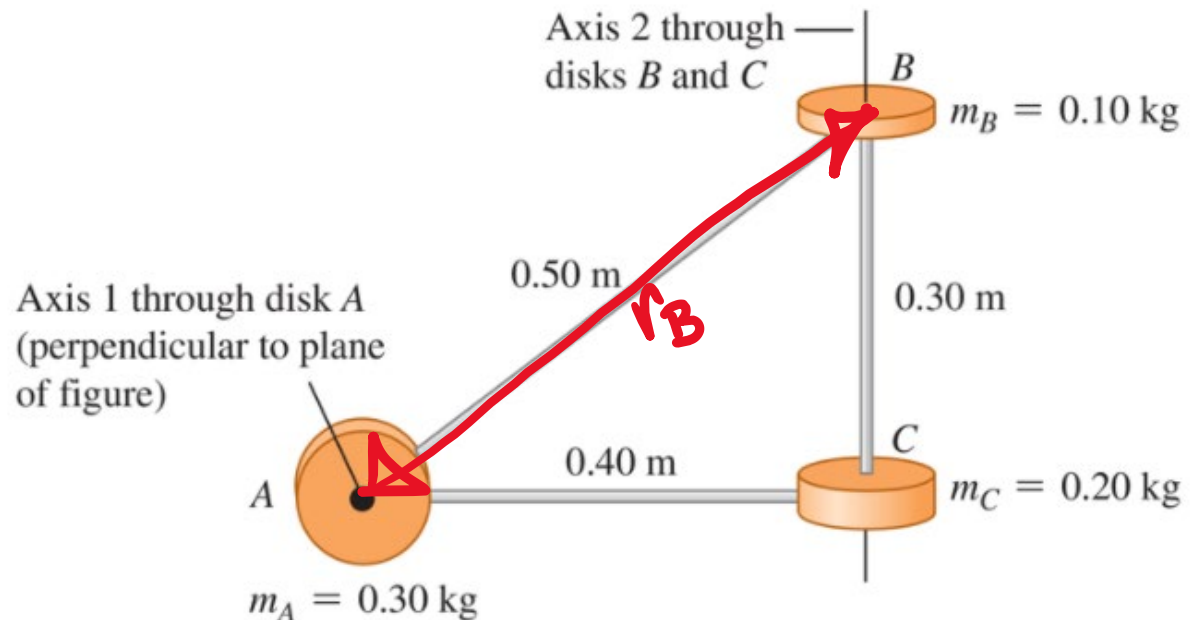
A machine part (Fig. 9.15) consists of three small disks linked by lightweight struts. (a) What is this body's moment of inertia about axis 1 through the center of disk *A*, perpendicular to the plane of the diagram? (b) What is its moment of inertia about axis 2 through the centers of disks *B* and *C*? (c) What is the body's kinetic energy if it rotates about axis 1 with angular speed  $\omega = 4.0 \text{ rad/s}$ ?



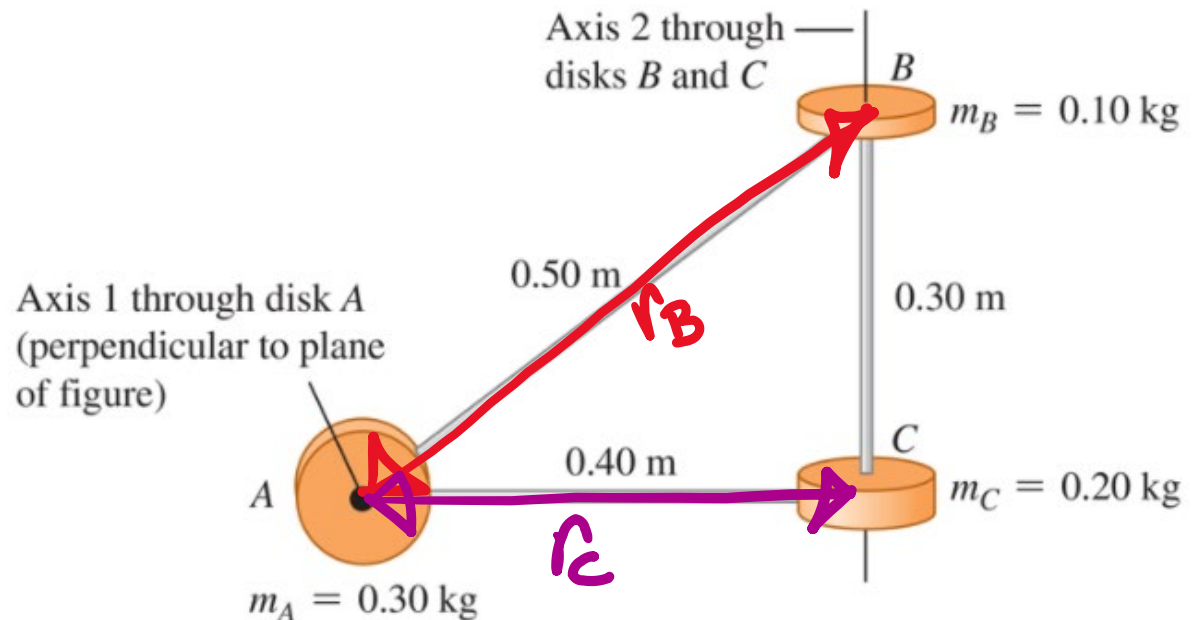
A machine part (Fig. 9.15) consists of three small disks linked by lightweight struts. (a) What is this body's moment of inertia about axis 1 through the center of disk A, perpendicular to the plane of the diagram? (b) What is its moment of inertia about axis 2 through the centers of disks B and C? (c) What is the body's kinetic energy if it rotates about axis 1 with angular speed  $\omega = 4.0 \text{ rad/s}$ ?  $I_1 = M_B r_B^2 + M_C r_C^2$



A machine part (Fig. 9.15) consists of three small disks linked by lightweight struts. (a) What is this body's moment of inertia about axis 1 through the center of disk A, perpendicular to the plane of the diagram? (b) What is its moment of inertia about axis 2 through the centers of disks B and C? (c) What is the body's kinetic energy if it rotates about axis 1 with angular speed  $\omega = 4.0 \text{ rad/s}$ ?  $I_1 = \underline{m_B r_B^2} + m_C r_C^2$



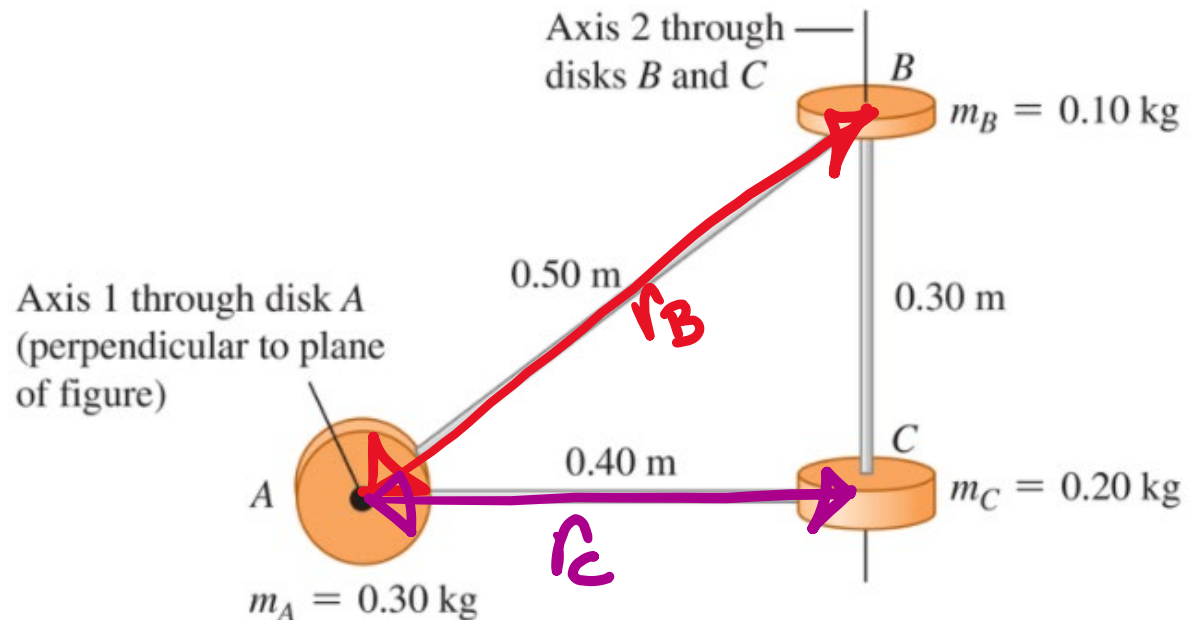
A machine part (Fig. 9.15) consists of three small disks linked by lightweight struts. (a) What is this body's moment of inertia about axis 1 through the center of disk A, perpendicular to the plane of the diagram? (b) What is its moment of inertia about axis 2 through the centers of disks B and C? (c) What is the body's kinetic energy if it rotates about axis 1 with angular speed  $\omega = 4.0 \text{ rad/s}$ ? 
$$I_1 = \underline{M_B r_B^2} + \underline{M_C r_C^2}$$



A machine part (Fig. 9.15) consists of three small disks linked by lightweight struts. (a) What is this body's moment of inertia about axis 1 through the center of disk A, perpendicular to the plane of the diagram? (b) What is its moment of inertia about axis 2 through the centers of disks B and C? (c) What is the body's kinetic energy if it rotates about axis 1 with angular speed  $\omega = 4.0 \text{ rad/s}$ ?

$$I_1 = \underbrace{M_B r_B^2} + \underbrace{M_C r_C^2} \Rightarrow$$

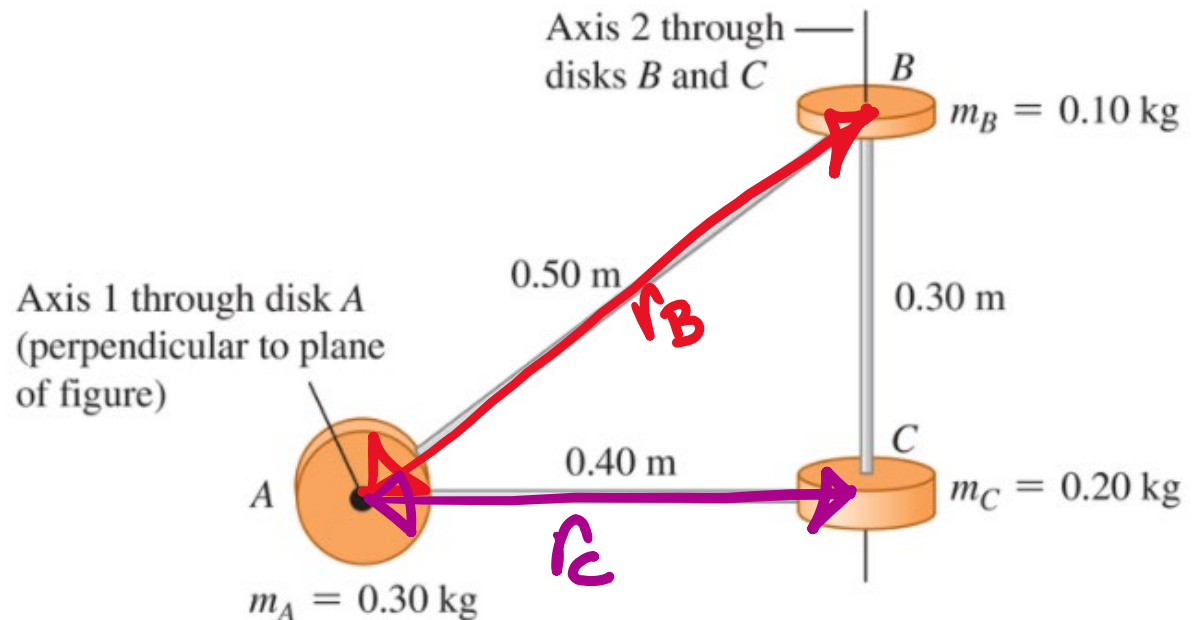
$$I_1 = [0.1 * (0.5)^2 + 0.2 * (0.4)^2] \text{ kg m}^2$$



A machine part (Fig. 9.15) consists of three small disks linked by lightweight struts. (a) What is this body's moment of inertia about axis 1 through the center of disk A, perpendicular to the plane of the diagram? (b) What is its moment of inertia about axis 2 through the centers of disks B and C? (c) What is the body's kinetic energy if it rotates about axis 1 with angular speed  $\omega = 4.0 \text{ rad/s}$ ?

$$I_1 = M_B r_B^2 + M_C r_C^2 \Rightarrow$$

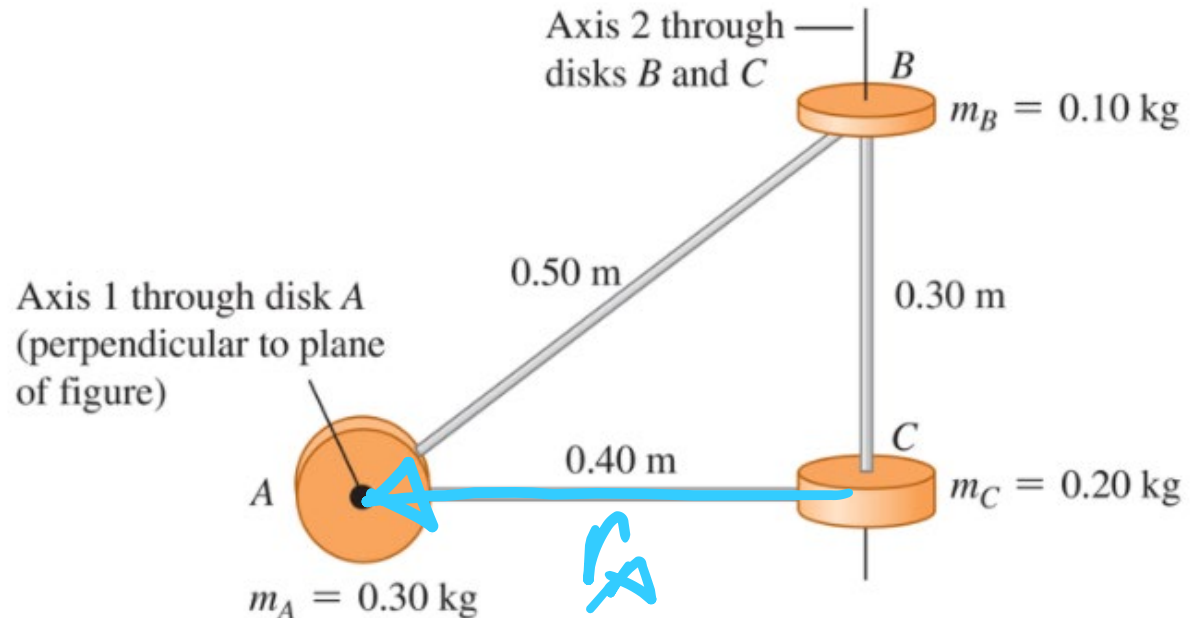
$$I_1 = [0.1 \times (0.5)^2 + 0.2 \times (0.4)^2] \text{ kg} \cdot \text{m}^2 = 0.057 \text{ kg} \cdot \text{m}^2$$



A machine part (Fig. 9.15) consists of three small disks linked by lightweight struts. (a) What is this body's moment of inertia about axis 1 through the center of disk A, perpendicular to the plane of the diagram? (b) What is its moment of inertia about axis 2 through the centers of disks B and C? (c) What is the body's kinetic energy if it rotates about axis 1 with angular speed  $\omega = 4.0 \text{ rad/s}$ ?

$$I_1 = M_B r_B^2 + M_C r_C^2 \Rightarrow I_1 = [0.1 \times (0.5)^2 + 0.2 \times (0.4)^2] \text{ kg} \cdot \text{m}^2 = 0.057 \text{ kg} \cdot \text{m}^2$$

$$I_2 = M_A r_A^2$$

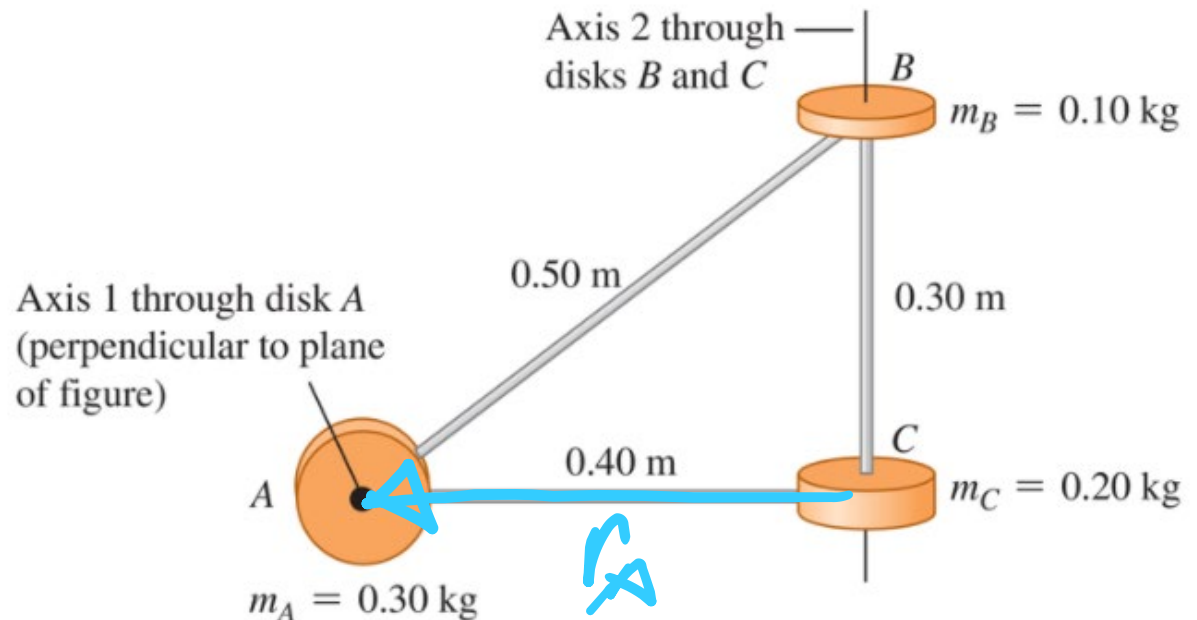


A machine part (Fig. 9.15) consists of three small disks linked by lightweight struts. (a) What is this body's moment of inertia about axis 1 through the center of disk A, perpendicular to the plane of the diagram? (b) What is its moment of inertia about axis 2 through the centers of disks B and C? (c) What is the body's kinetic energy if it rotates about axis 1 with angular speed  $\omega = 4.0 \text{ rad/s}$ ?

$$I_1 = M_B r_B^2 + M_C r_C^2 \Rightarrow$$

$$I_1 = [0.1 * (0.5)^2 + 0.2 * (0.4)^2] \text{ kg} \cdot \text{m}^2 = 0.057 \text{ kg} \cdot \text{m}^2$$

$$I_2 = M_A r_A^2 = 0.3 * (0.4)^2 \text{ kg} \cdot \text{m}^2$$

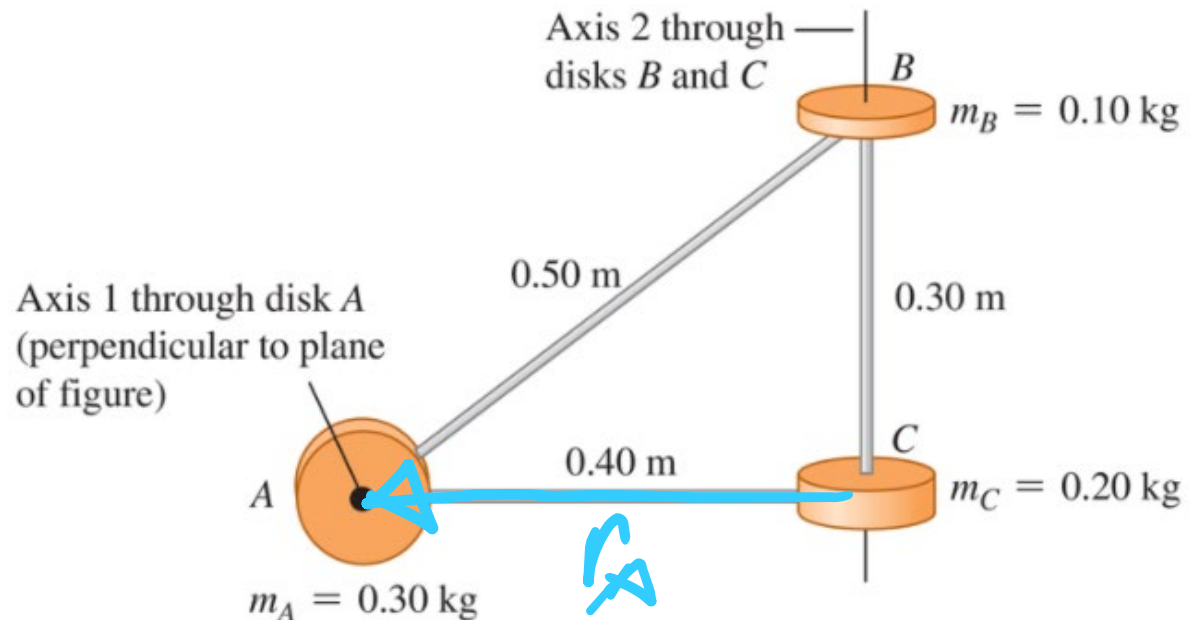


A machine part (Fig. 9.15) consists of three small disks linked by lightweight struts. (a) What is this body's moment of inertia about axis 1 through the center of disk A, perpendicular to the plane of the diagram? (b) What is its moment of inertia about axis 2 through the centers of disks B and C? (c) What is the body's kinetic energy if it rotates about axis 1 with angular speed  $\omega = 4.0 \text{ rad/s}$ ?

$$I_1 = m_B r_B^2 + m_C r_C^2 \Rightarrow I_1 = [0.1 \times (0.5)^2 + 0.2 \times (0.4)^2] \text{ kg} \cdot \text{m}^2 = 0.057 \text{ kg} \cdot \text{m}^2$$

$$I_2 = m_A r_A^2 = 0.3 \times (0.4)^2 \text{ kg} \cdot \text{m}^2 = 0.048 \text{ kg} \cdot \text{m}^2$$

$$K = \frac{1}{2} I_1 \omega^2$$

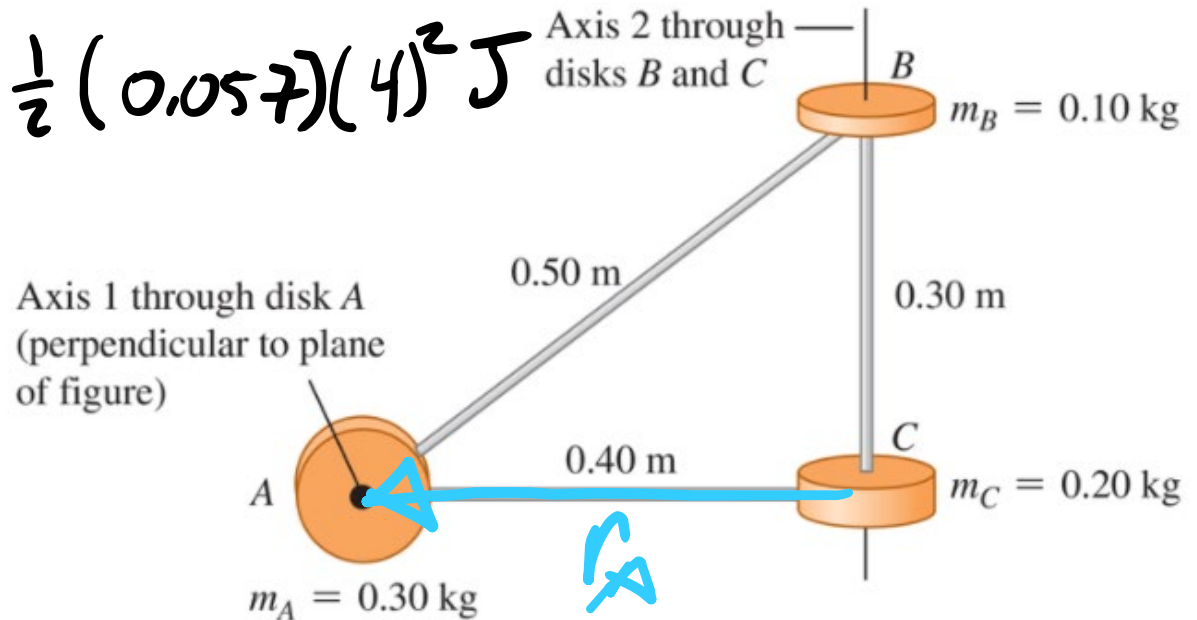


A machine part (Fig. 9.15) consists of three small disks linked by lightweight struts. (a) What is this body's moment of inertia about axis 1 through the center of disk A, perpendicular to the plane of the diagram? (b) What is its moment of inertia about axis 2 through the centers of disks B and C? (c) What is the body's kinetic energy if it rotates about axis 1 with angular speed  $\omega = 4.0 \text{ rad/s}$ ?

$$I_1 = m_B r_B^2 + m_C r_C^2 \Rightarrow I_1 = [0.1 \times (0.5)^2 + 0.2 \times (0.4)^2] \text{ kg} \cdot \text{m}^2 = 0.057 \text{ kg} \cdot \text{m}^2$$

$$I_2 = m_A r_A^2 = 0.3 \times (0.4)^2 \text{ kg} \cdot \text{m}^2 = 0.048 \text{ kg} \cdot \text{m}^2$$

$$K = \frac{1}{2} I_1 \omega^2 = \frac{1}{2} (0.057) (4)^2 \text{ J}$$



A machine part (Fig. 9.15) consists of three small disks linked by lightweight struts. (a) What is this body's moment of inertia about axis 1 through the center of disk A, perpendicular to the plane of the diagram? (b) What is its moment of inertia about axis 2 through the centers of disks B and C? (c) What is the body's kinetic energy if it rotates about axis 1 with angular speed  $\omega = 4.0 \text{ rad/s}$ ?

$$I_1 = m_B r_B^2 + m_C r_C^2 \Rightarrow I_1 = [0.1 \times (0.5)^2 + 0.2 \times (0.4)^2] \text{ kg} \cdot \text{m}^2 = 0.057 \text{ kg} \cdot \text{m}^2$$

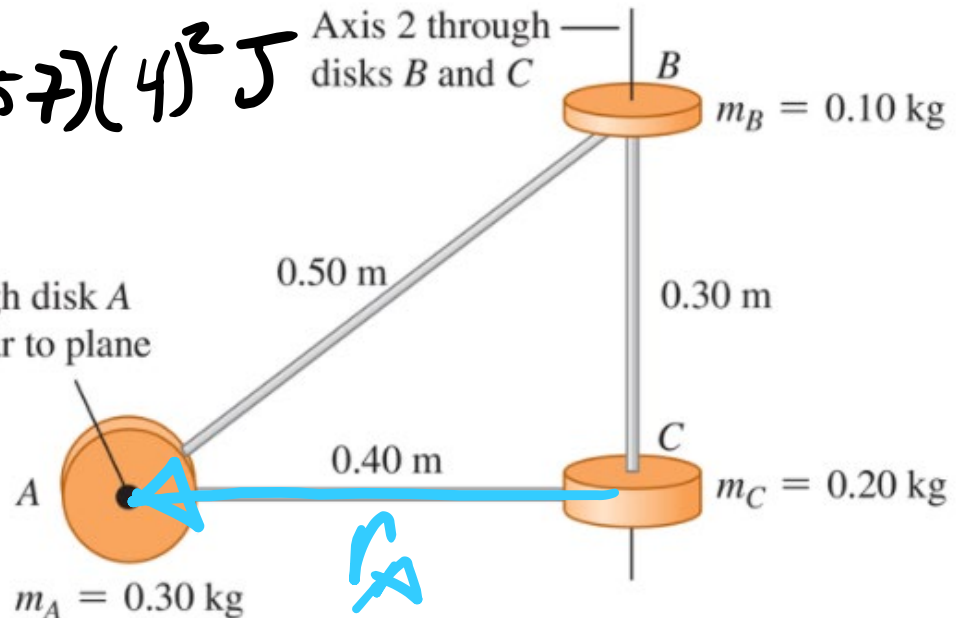
$$I_2 = m_A r_A^2 = 0.3 \times (0.4)^2 \text{ kg} \cdot \text{m}^2 = 0.048 \text{ kg} \cdot \text{m}^2$$

$$K = \frac{1}{2} I_1 \omega^2 = \frac{1}{2} (0.057) (4)^2 \text{ J}$$

$$\Rightarrow K = 0.456 \text{ J}$$

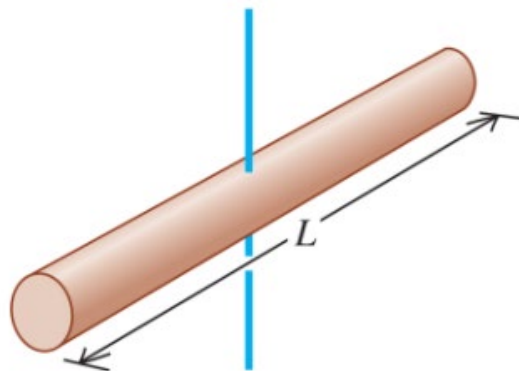
Axis 1 through disk A  
(perpendicular to plane  
of figure)

Axis 2 through  
disks B and C



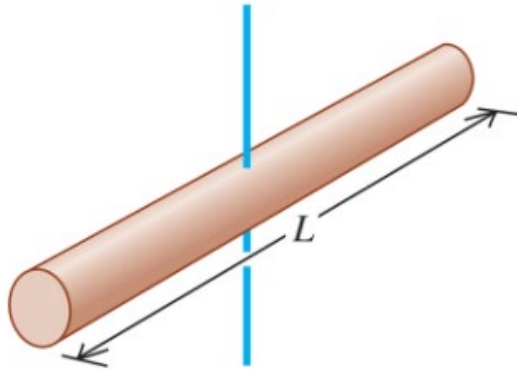
(a) Slender rod,  
axis through center

$$I = \frac{1}{12}ML^2$$



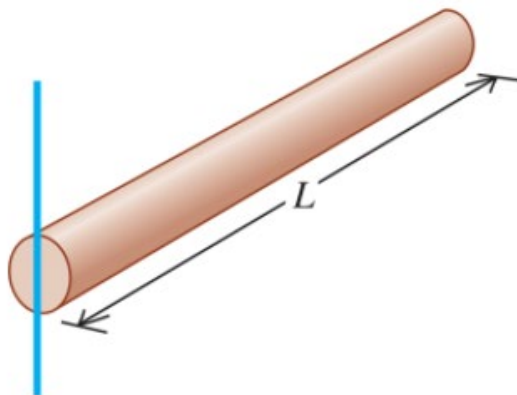
(a) Slender rod,  
axis through center

$$I = \frac{1}{12}ML^2$$



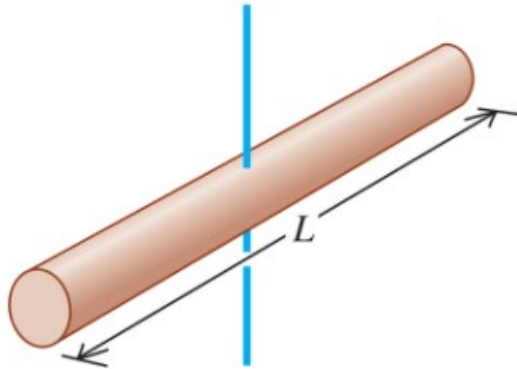
(b) Slender rod,  
axis through one end

$$I = \frac{1}{3}ML^2$$



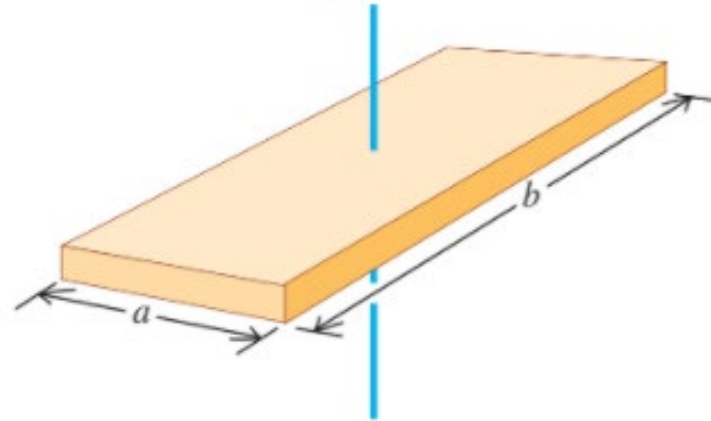
(a) Slender rod,  
axis through center

$$I = \frac{1}{12}ML^2$$



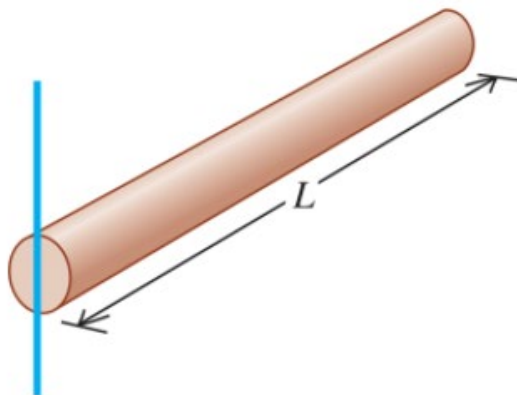
(c) Rectangular plate,  
axis through center

$$I = \frac{1}{12}M(a^2 + b^2)$$



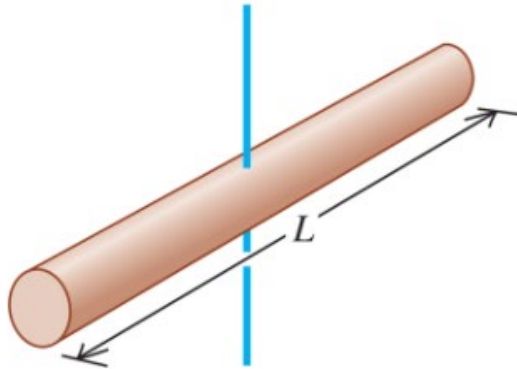
(b) Slender rod,  
axis through one end

$$I = \frac{1}{3}ML^2$$



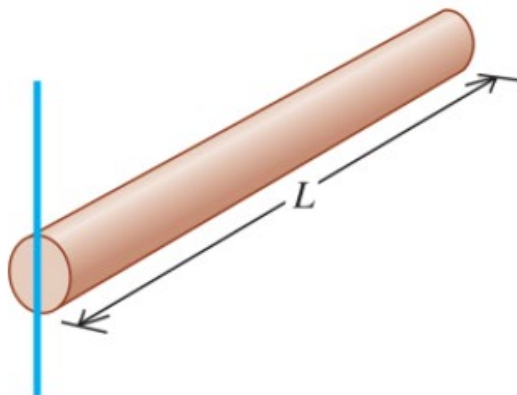
(a) Slender rod,  
axis through center

$$I = \frac{1}{12}ML^2$$



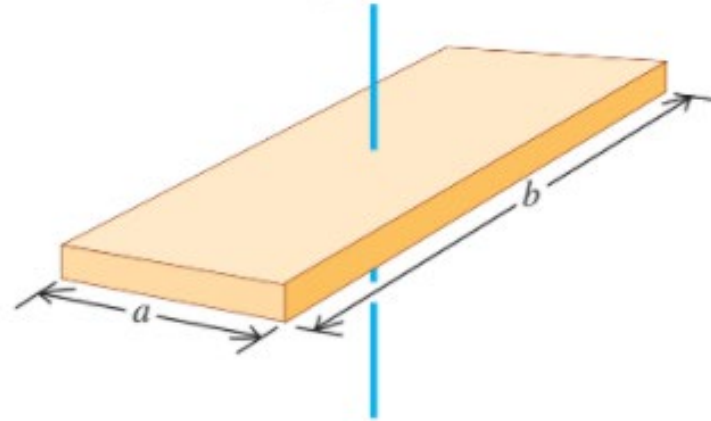
(b) Slender rod,  
axis through one end

$$I = \frac{1}{3}ML^2$$



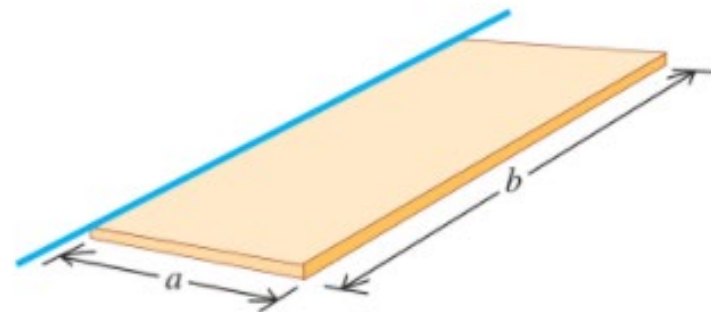
(c) Rectangular plate,  
axis through center

$$I = \frac{1}{12}M(a^2 + b^2)$$



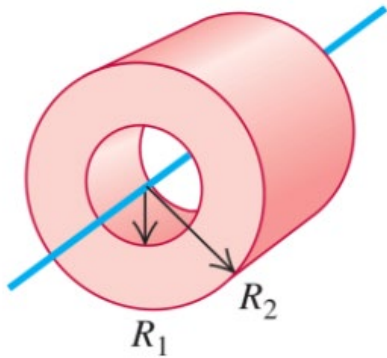
(d) Thin rectangular plate,  
axis along edge

$$I = \frac{1}{3}Ma^2$$



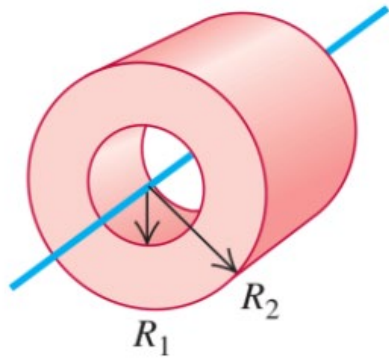
(e) Hollow cylinder

$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$



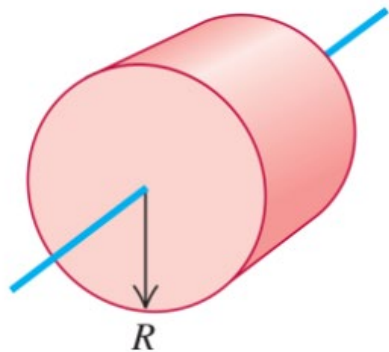
(e) Hollow cylinder

$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$



(f) Solid cylinder

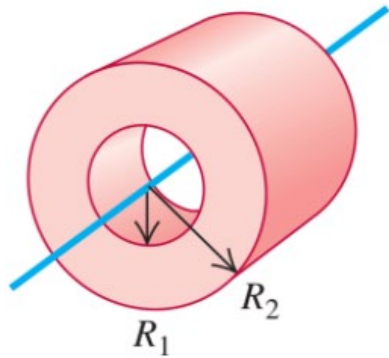
$$I = \frac{1}{2}MR^2$$



(g) Thin-walled hollow

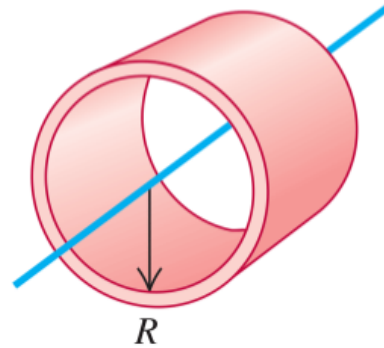
(e) Hollow cylinder

$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$



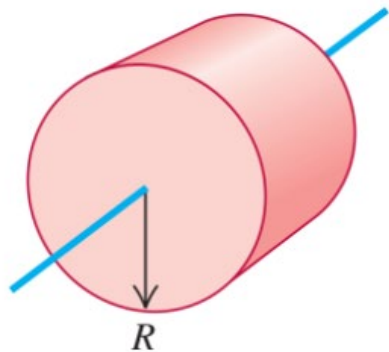
(g) Thin-walled hollow cylinder

$$I = MR^2$$



(f) Solid cylinder

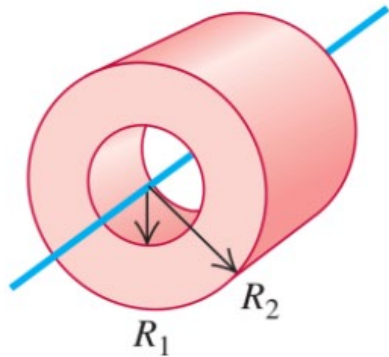
$$I = \frac{1}{2}MR^2$$



(g) Thin-walled hollow

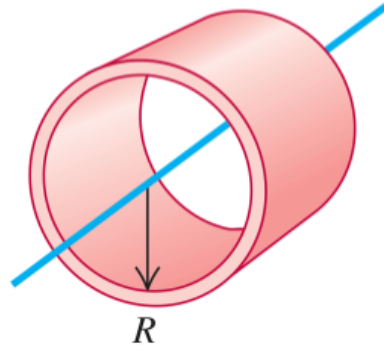
(e) Hollow cylinder

$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$



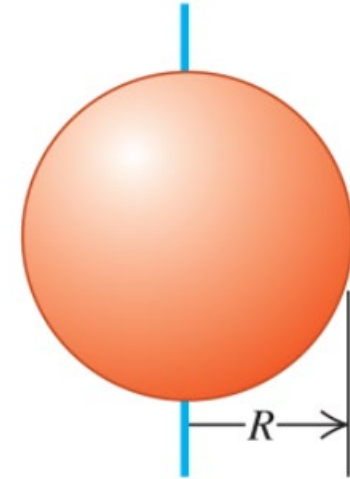
(g) Thin-walled hollow cylinder

$$I = MR^2$$



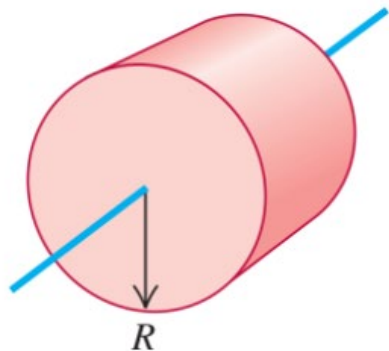
(h) Solid sphere

$$I = \frac{2}{5}MR^2$$



(f) Solid cylinder

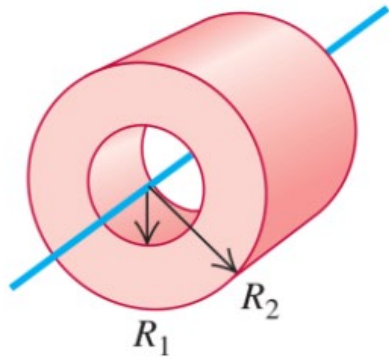
$$I = \frac{1}{2}MR^2$$



(g) Thin-walled hollow

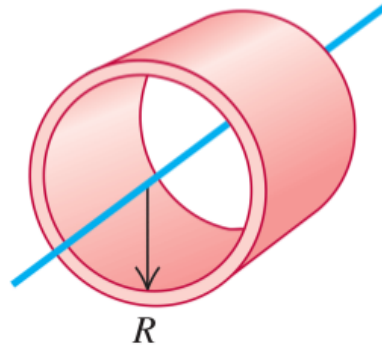
(e) Hollow cylinder

$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$



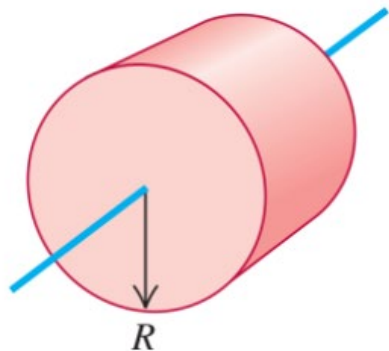
(g) Thin-walled hollow cylinder

$$I = MR^2$$



(f) Solid cylinder

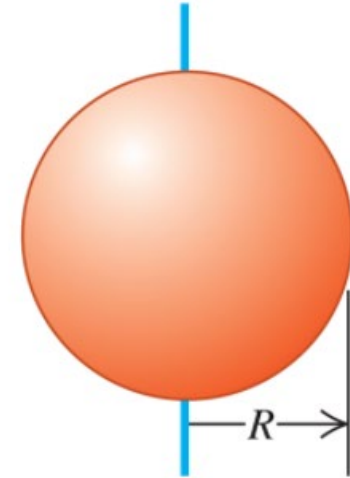
$$I = \frac{1}{2}MR^2$$



(g) Thin-walled hollow

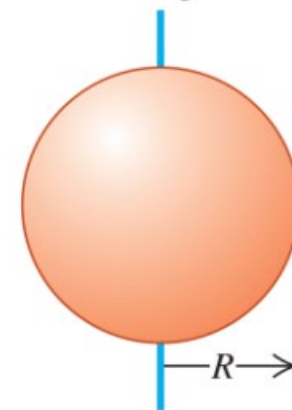
(h) Solid sphere

$$I = \frac{2}{5}MR^2$$

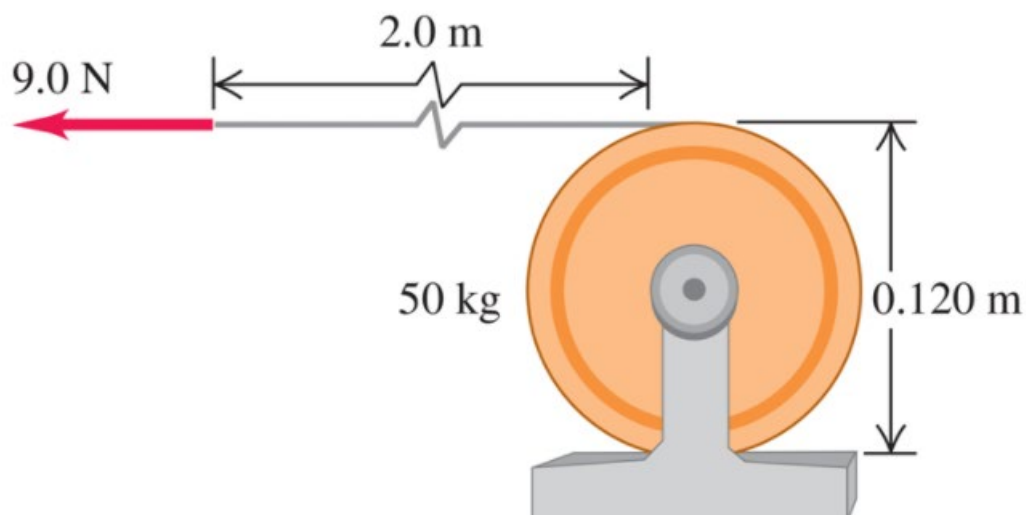


(i) Thin-walled hollow sphere

$$I = \frac{2}{3}MR^2$$

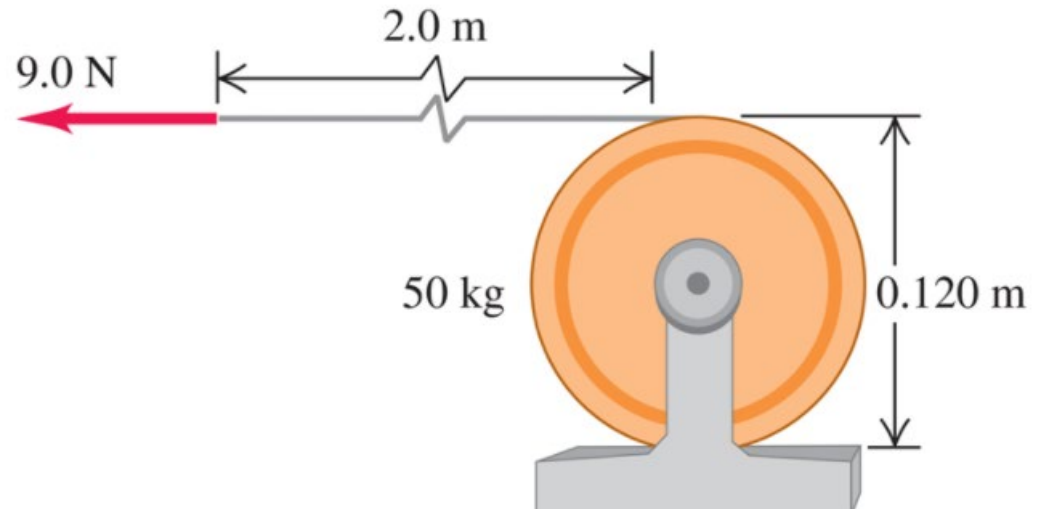


We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16). We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.



We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16). We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

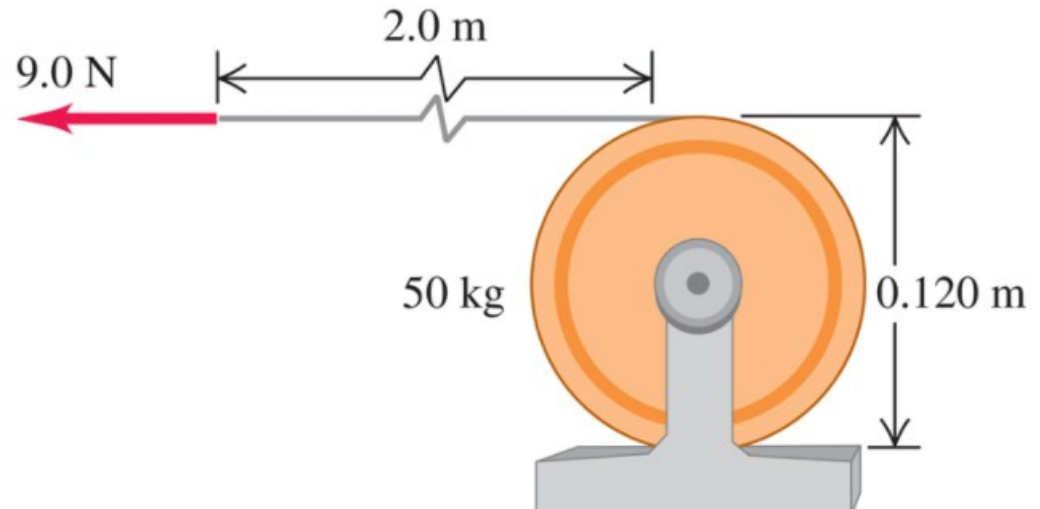
$$I = \frac{1}{2}MR^2$$



We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16□).

We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

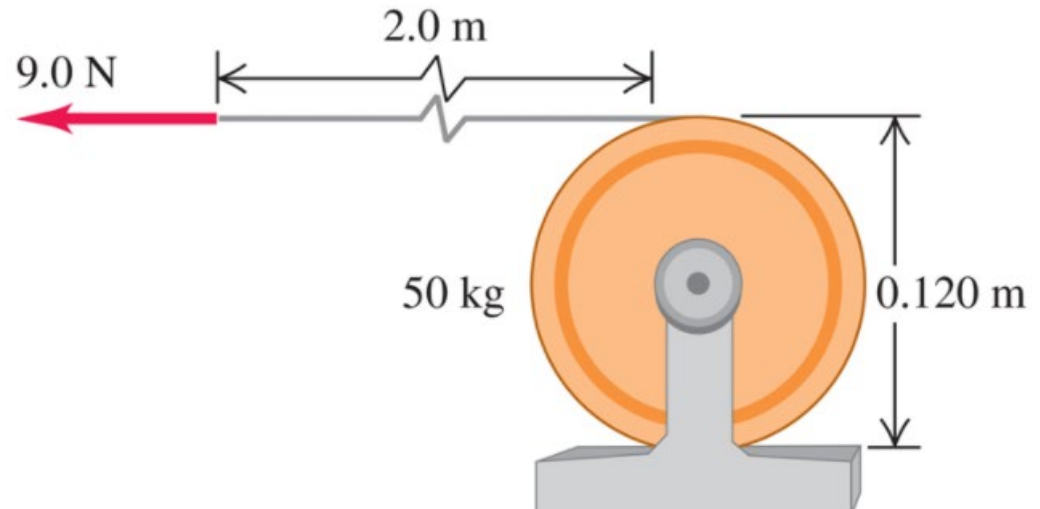


We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16□).

We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \ \& \ d = 2 \text{ m}$$



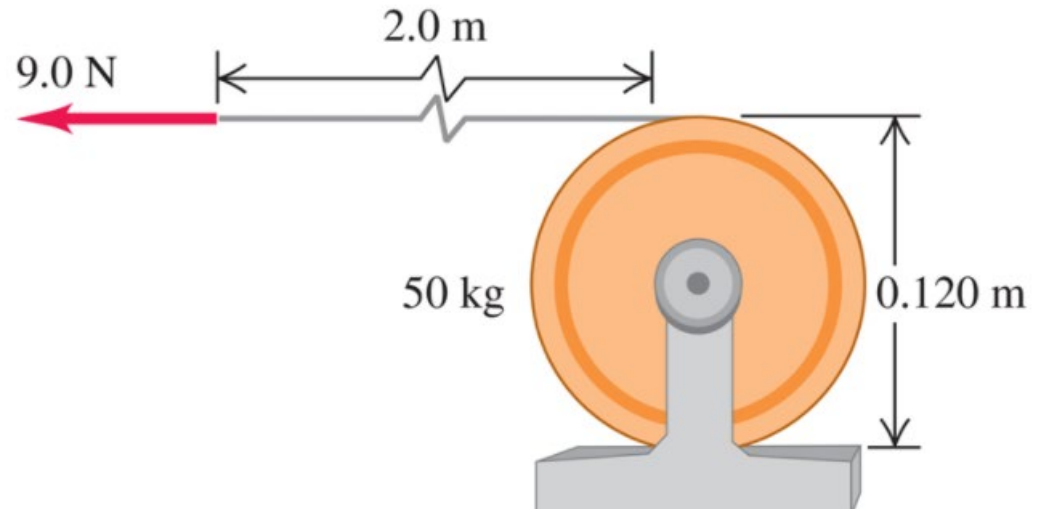
We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16).

We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \ \& \ d = 2 \text{ m}$$

$$d = R\theta$$



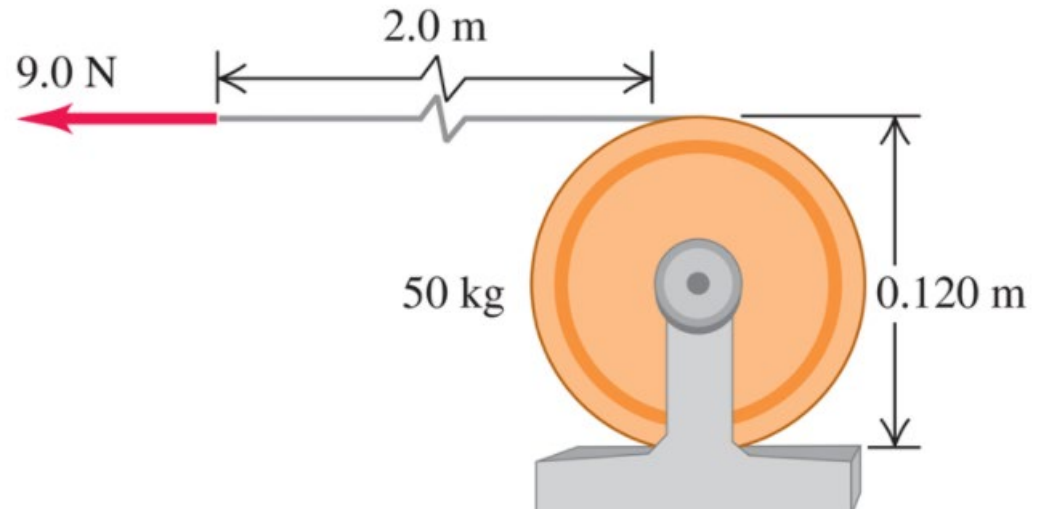
We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16).

We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \quad \& \quad d = 2 \text{ m}$$

$$d = R\theta \quad W = K_2 - K_1$$



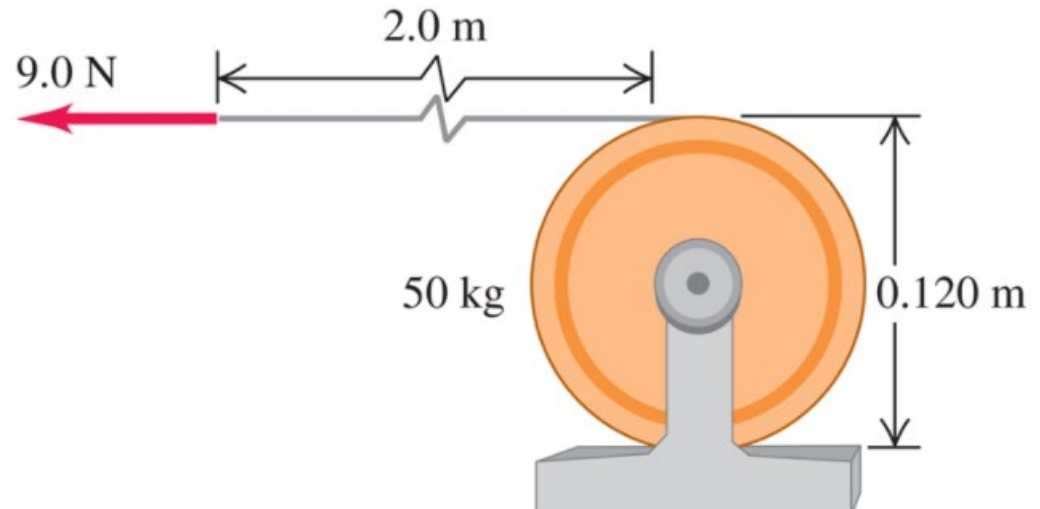
We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16).

We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \ \& \ d = 2 \text{ m}$$

$$d = R\theta \quad W = K_2 - K_1$$



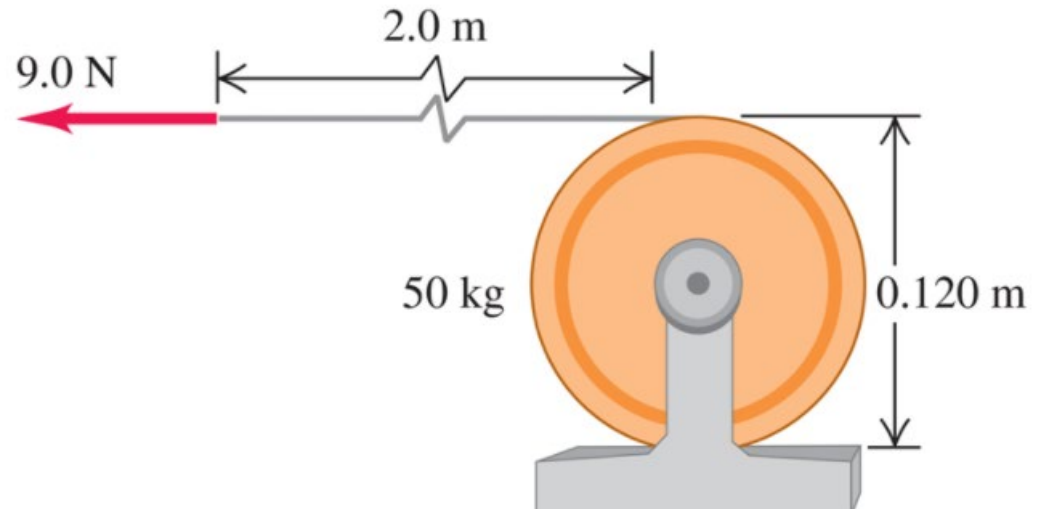
We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16).

We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \ \& \ d = 2 \text{ m}$$

$$Q = R\Theta \quad W = K_2 - \cancel{K_1} \quad \text{but} \quad W = Fd$$



We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16).

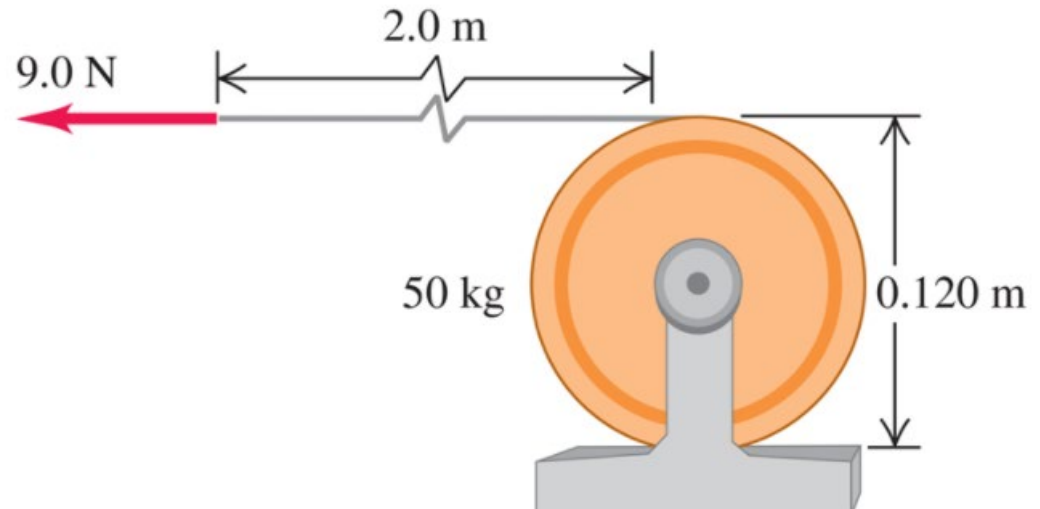
We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \ \& \ d = 2 \text{ m}$$

$$d = R\theta \quad W = k_2 - k_1 \quad \text{but} \quad W = Fd$$

$$\& \ k_2 = \frac{1}{2}I\omega^2$$



We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16).

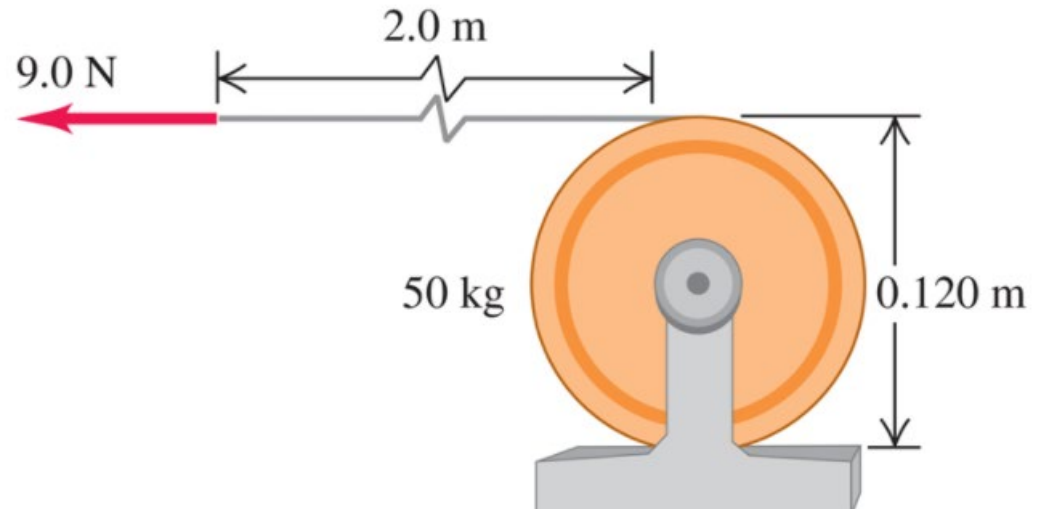
We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \ \& \ d = 2 \text{ m}$$

$$d = R\theta \quad W = k_2 - k_1 \quad \text{but} \quad W = Fd$$

$$\& \ k_2 = \frac{1}{2}I\omega^2 \quad \text{so} \quad Fd = \frac{1}{2}I\omega^2$$



We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16).

We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

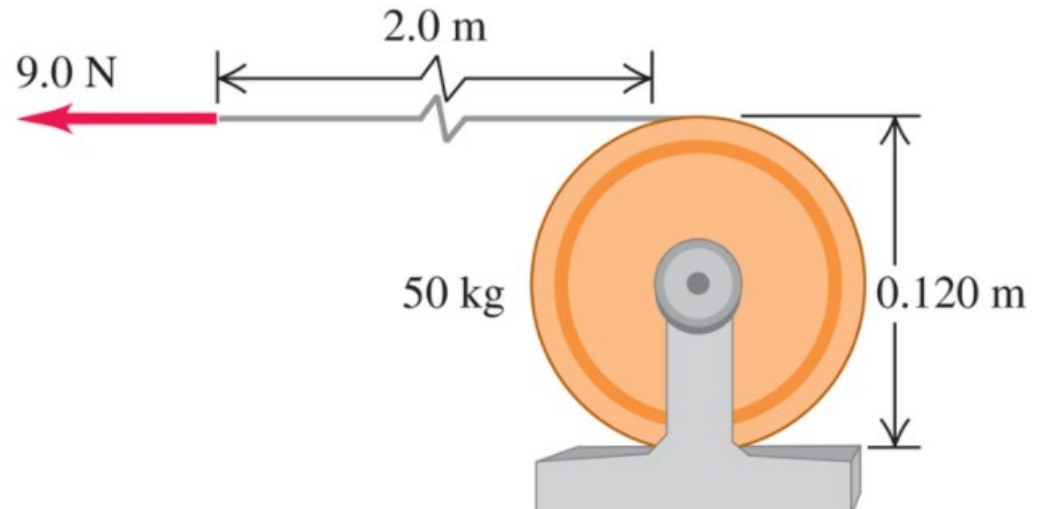
$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \ \& \ d = 2 \text{ m}$$

$$d = R\theta \quad W = k_2 - k_1 \quad \text{but} \quad W = Fd$$

$$\& \ k_2 = \frac{1}{2}I\omega^2 \quad \text{so} \quad Fd = \frac{1}{2}I\omega^2 \Rightarrow$$

$$\omega = \sqrt{\frac{2Fd}{I}}$$



We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16).

We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \ \& \ d = 2 \text{ m}$$

$$d = R\theta \quad W = k_2 - \cancel{k_1} \quad \text{but } W = Fd$$

$$\& \ k_2 = \frac{1}{2}I\omega^2 \quad \text{so} \quad Fd = \frac{1}{2}I\omega^2 \Rightarrow$$

$$\omega = \sqrt{\frac{2Fd}{I}} = \sqrt{\frac{2 \times 9 \times 2}{0.09}} \left(\frac{\text{rad}}{\text{s}}\right)$$

We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16).

We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \text{ \& } d = 2 \text{ m}$$

$$d = R\theta \quad W = k_2 - \cancel{k_1} \quad \text{but } W = Fd$$

$$\text{\& } k_2 = \frac{1}{2}I\omega^2 \quad \text{so } Fd = \frac{1}{2}I\omega^2 \Rightarrow$$

$$\omega = \sqrt{\frac{2Fd}{I}} = \sqrt{\frac{2 \times 9 \times 2}{0.09}} \left(\frac{\text{rad}}{\text{s}}\right) = 20 \frac{\text{rad}}{\text{s}}$$

We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16).

We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \text{ \& } d = 2 \text{ m}$$

$$d = R\theta \quad W = k_2 - \cancel{k_1} \quad \text{but } W = Fd$$

$$\text{\& } k_2 = \frac{1}{2}I\omega^2 \quad \text{so } Fd = \frac{1}{2}I\omega^2 \Rightarrow$$

$$\omega = \sqrt{\frac{2Fd}{I}} = \sqrt{\frac{2 \times 9 \times 2}{0.09}} \left(\frac{\text{rad}}{\text{s}}\right) = 20 \frac{\text{rad}}{\text{s}}$$

$$v = R\omega$$

We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16).

We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \ \& \ d = 2 \text{ m}$$

$$Q = R\Theta \quad W = k_2 - k_1 \quad \text{but } W = Fd$$

$$\& \ k_2 = \frac{1}{2}I\omega^2 \quad \text{so} \quad Fd = \frac{1}{2}I\omega^2 \Rightarrow$$

$$\omega = \sqrt{\frac{2Fd}{I}} = \sqrt{\frac{2 \times 9 \times 2}{0.09}} \left(\frac{\text{rad}}{\text{s}}\right) = 20 \frac{\text{rad}}{\text{s}}$$

$$v = R\omega = (0.06)(20) \frac{\text{m}}{\text{s}}$$

We wrap a light, nonstretching cable around a solid cylinder, of mass 50 kg and diameter 0.120 m, that rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16).

We pull the free end of the cable with a constant 9.0 N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

$$I = \frac{1}{2}MR^2 = \left(\frac{50}{2}\right)\left(\frac{0.12}{2}\right)^2 \text{ kg}\cdot\text{m}^2$$

$$\Rightarrow I = 0.09 \text{ kg}\cdot\text{m}^2, F = 9 \text{ N} \ \& \ d = 2 \text{ m}$$

$$\Delta = R\Theta \quad W = k_2 - k_1 \quad \text{but} \quad W = Fd$$

$$\& \ k_2 = \frac{1}{2}I\omega^2 \quad \text{so} \quad Fd = \frac{1}{2}I\omega^2 \Rightarrow$$

$$\omega = \sqrt{\frac{2Fd}{I}} = \sqrt{\frac{2 \times 9 \times 2}{0.09}} \left(\frac{\text{rad}}{\text{s}}\right) = 20 \frac{\text{rad}}{\text{s}}$$

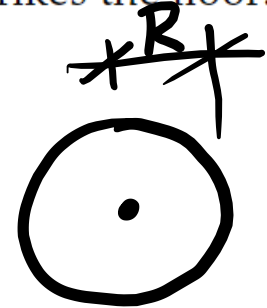
$$v = R\omega = (0.06)(20) \frac{\text{m}}{\text{s}} = 1.2 \frac{\text{m}}{\text{s}}$$

We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.



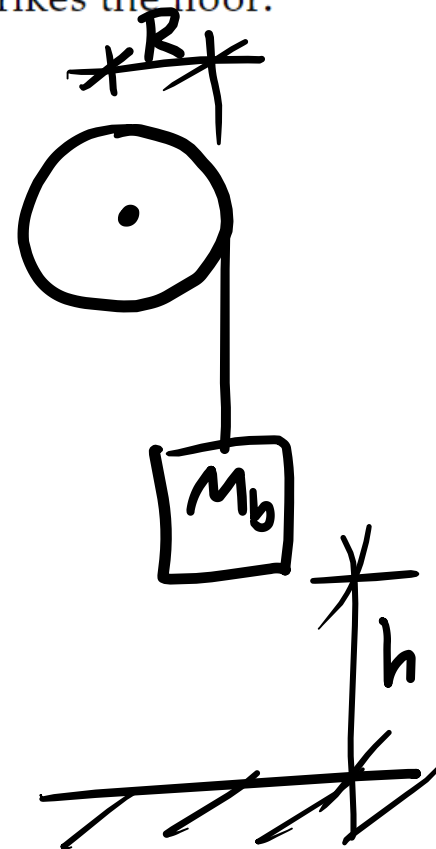
We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

$$I = MR^2/2$$



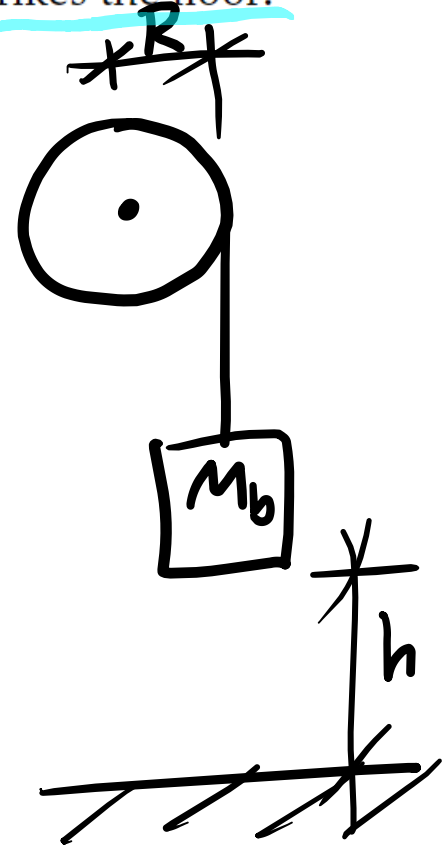
We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

$$I = MR^2/2$$



We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

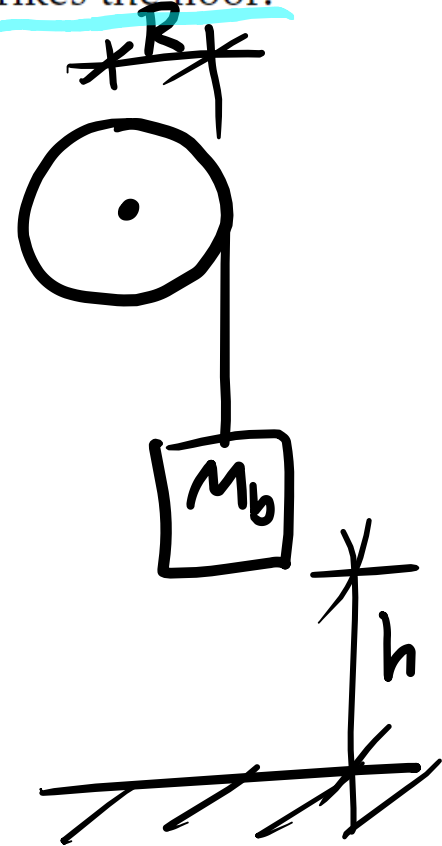
$I = MR^2/2$ ,  $v = R\omega$   
Find  $v$  &  $\omega$  when block hits floor



We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

$I = MR^2/2$ ,  $v = R\omega$   
Find  $v$  &  $\omega$  when block hits floor

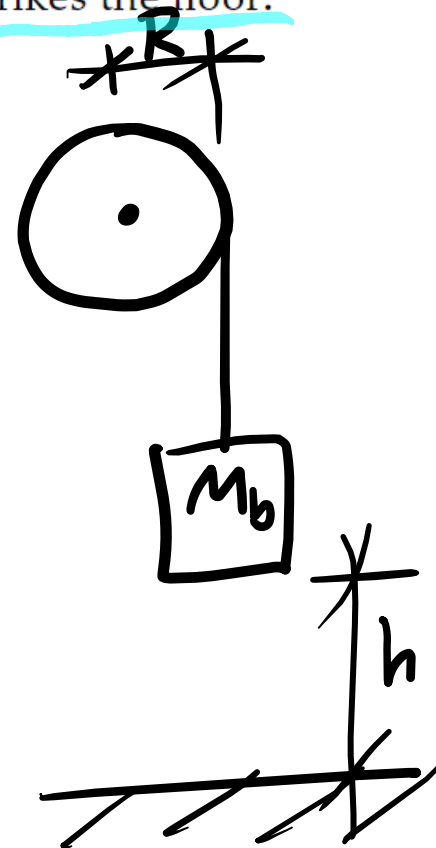
$$U_1 + K_1 = U_2 + K_2$$



We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

$I = MR^2/2$ ,  $v = R\omega$   
Find  $v$  &  $\omega$  when block hits floor

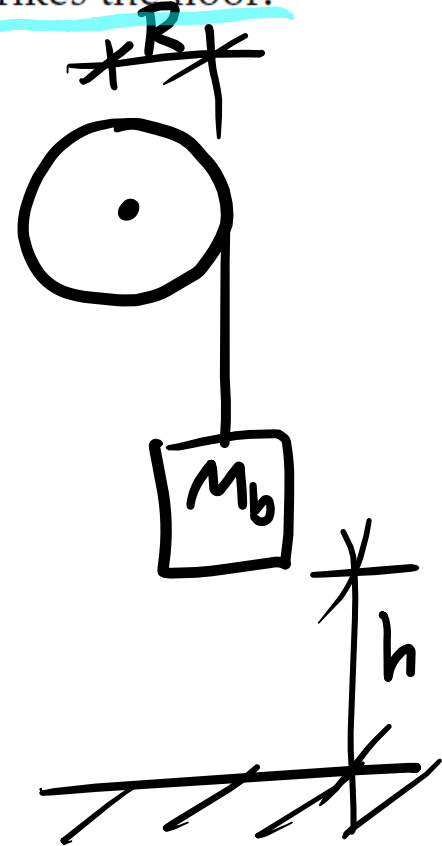
$$U_1 + \cancel{K_1} = U_2 + K_2$$



We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

$I = MR^2/2$ ,  $v = R\omega$   
Find  $v$  &  $\omega$  when block hits floor

$$U_1 + \cancel{K_1} = U_2 + K_2 \Rightarrow U_1 - U_2 = K_2$$



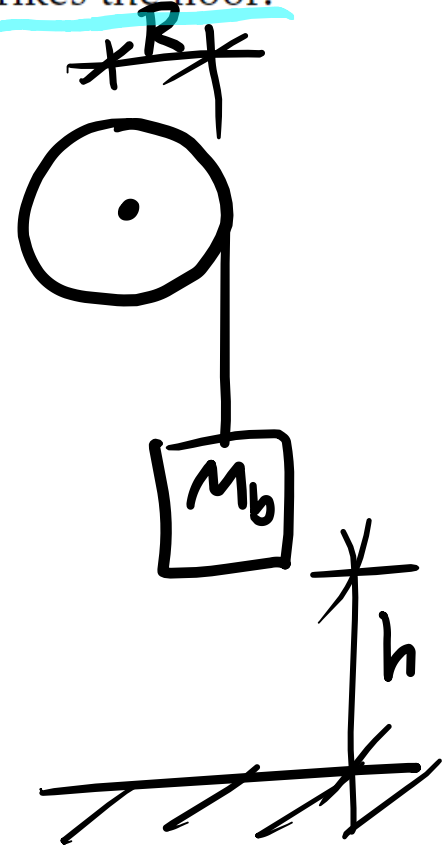
We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

$$I = MR^2/2, \quad v = R\omega$$

Find  $v$  &  $\omega$  when block hits floor

$$U_1 + \cancel{K_1} = U_2 + K_2 \Rightarrow U_1 - U_2 = K_2$$

But  $U_1 - U_2 = mgy_1 - y_2$



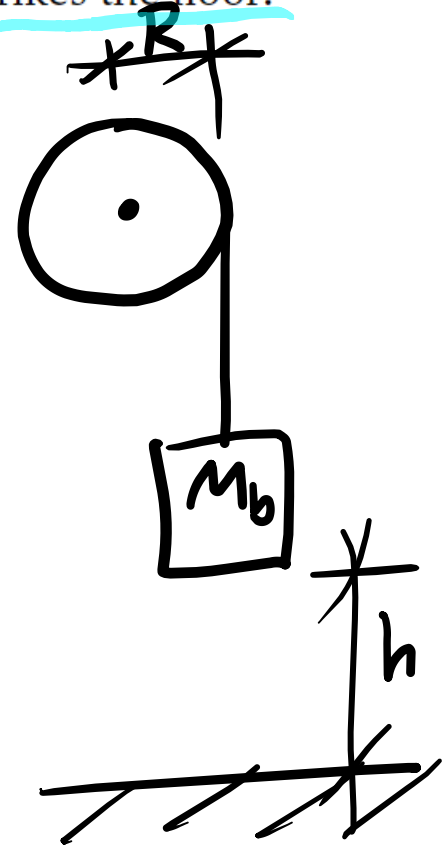
We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

$$I = MR^2/2, \quad v = R\omega$$

Find  $v$  &  $\omega$  when block hits floor

$$U_1 + \cancel{K_1} = U_2 + K_2 \Rightarrow U_1 - U_2 = K_2$$

But  $U_1 - U_2 = mg(y_1 - y_2) = mgh$



We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

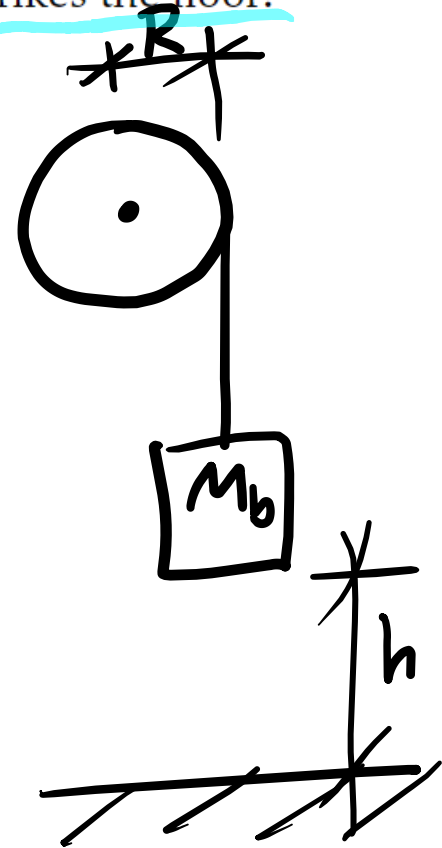
$$I = MR^2/2, \quad v = R\omega$$

Find  $v$  &  $\omega$  when block hits floor

$$U_1 + \cancel{K_1} = U_2 + K_2 \Rightarrow U_1 - U_2 = K_2$$

But  $U_1 - U_2 = mg(y_1 - y_2) = mgh$  &

$$K_2 = \frac{1}{2}m_b v^2 + \frac{1}{2}I\omega^2$$



We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

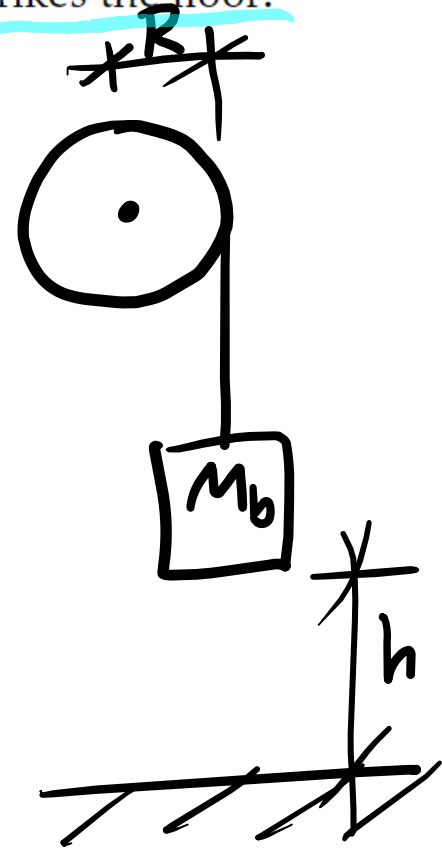
$$I = MR^2/2, \quad v = R\omega$$

Find  $v$  &  $\omega$  when block hits floor

$$U_1 + \cancel{K_1} = U_2 + K_2 \Rightarrow U_1 - U_2 = K_2$$

But  $U_1 - U_2 = mg(y_1 - y_2) = mgh$  &

$$K_2 = \frac{1}{2}m_b v^2 + \frac{1}{2}I\omega^2$$



We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

$$I = MR^2/2, \quad v = R\omega$$

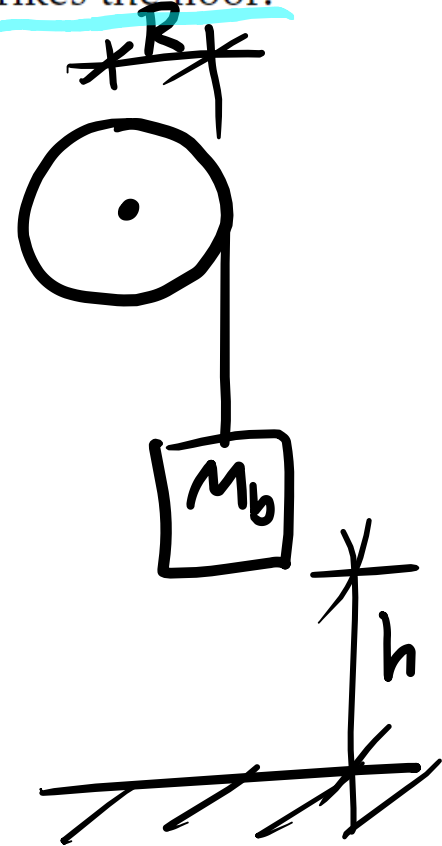
Find  $v$  &  $\omega$  when block hits floor

$$U_1 + \cancel{K_1} = U_2 + K_2 \Rightarrow U_1 - U_2 = K_2$$

But  $U_1 - U_2 = mg(y_1 - y_2) = mgh$  &

$$K_2 = \frac{1}{2}m_b v^2 + \frac{1}{2}I\omega^2$$

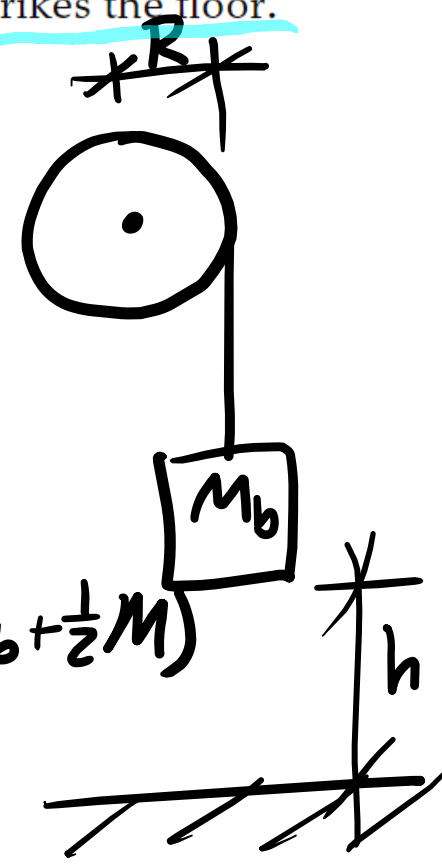
$$= \frac{1}{2}m_b v^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$



We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

$$I = MR^2/2, \quad v = R\omega$$

Find  $v$  &  $\omega$  when block hits floor



$$U_1 + K_1 = U_2 + K_2 \Rightarrow U_1 - U_2 = K_2$$

$$\text{But } U_1 - U_2 = mg(y_1 - y_2) = mgh \quad \&$$

$$K_2 = \frac{1}{2}m_b v^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}m_b v^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}v^2\left(m_b + \frac{1}{2}M\right)$$

We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

$$I = MR^2/2, \quad v = R\omega$$

Find  $v$  &  $\omega$  when block hits floor

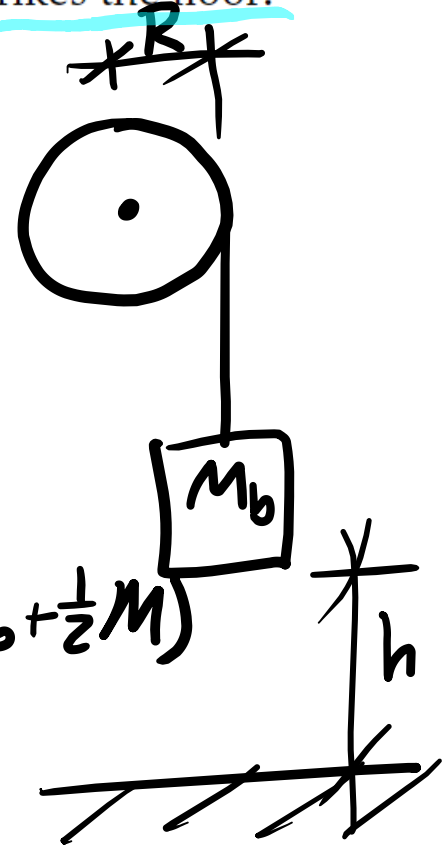
$$U_1 + \cancel{K_1} = U_2 + K_2 \Rightarrow U_1 - U_2 = K_2$$

But  $U_1 - U_2 = mg(y_1 - y_2) = mgh$  &

$$K_2 = \frac{1}{2}m_b v^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}m_b v^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}v^2\left(m_b + \frac{1}{2}M\right)$$

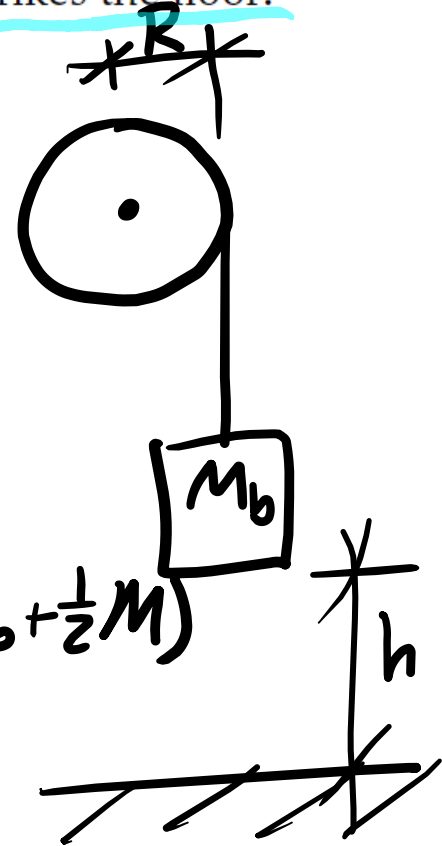
$$\Rightarrow \frac{1}{2}v^2\left(m_b + \frac{1}{2}M\right) = m_bgh$$



We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

$$I = MR^2/2, \quad v = R\omega$$

Find  $v$  &  $\omega$  when block hits floor



$$U_1 + K_1 = U_2 + K_2 \Rightarrow U_1 - U_2 = K_2$$

$$\text{But } U_1 - U_2 = mg(y_1 - y_2) = mgh \quad \&$$

$$K_2 = \frac{1}{2}m_b v^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}m_b v^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}v^2\left(m_b + \frac{1}{2}M\right)$$

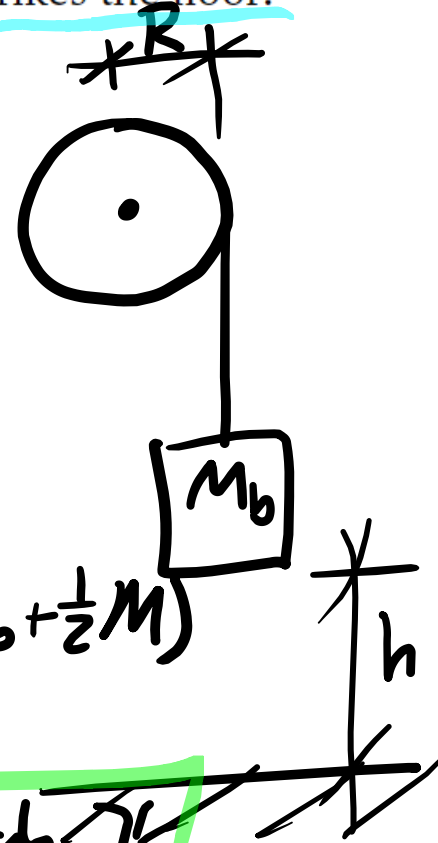
$$\Rightarrow \frac{1}{2}v^2\left(m_b + \frac{1}{2}M\right) = mgh \Rightarrow$$

$$v = \left[ \frac{2m_b g h}{m_b + M/2} \right]^{1/2}$$

We wrap a light, nonstretching cable around a solid cylinder with mass  $M$  and radius  $R$ . The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass  $m$  and release the block from rest at a distance  $h$  above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

$$I = MR^2/2, \quad v = R\omega$$

Find  $v$  &  $\omega$  when block hits floor



$$U_1 + K_1 = U_2 + K_2 \Rightarrow U_1 - U_2 = K_2$$

$$\text{But } U_1 - U_2 = mg(y_1 - y_2) = mgh \quad \&$$

$$K_2 = \frac{1}{2}m_b v^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}m_b v^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}v^2\left(m_b + \frac{1}{2}M\right)$$

$$\Rightarrow \frac{1}{2}v^2\left(m_b + \frac{1}{2}M\right) = mgh \Rightarrow$$

$$v = \left[ \frac{2m_b g h}{m_b + M/2} \right]^{1/2}$$

$$\& \omega = \left( \frac{1}{R} \right) \left[ \frac{2m_b g h}{m_b + M/2} \right]^{1/2}$$

# Gravitational potential energy for rigid body

Gravitational potential energy

for rigid body

$$U = \sum M_i g y_i$$

# Gravitational potential energy for rigid body

$$U = \sum m_i g y_i = g \sum m_i y_i$$

# Gravitational potential energy for rigid body

$$U = \sum m_i g y_i = g \sum m_i y_i = g \left( \frac{\sum m_i y_i}{\sum m_i} \right) \sum m_i$$

# Gravitational potential energy for rigid body

$$U = \sum m_i g y_i = g \sum m_i y_i = g \left( \frac{\sum m_i y_i}{\sum m_i} \right) \sum m_i$$

$$\Rightarrow U = g y_{cm} M$$

# Gravitational potential energy for rigid body

$$U = \sum m_i g y_i = g \sum m_i y_i = g \left( \frac{\sum m_i y_i}{\sum m_i} \right) \sum m_i$$

$$\Rightarrow U = g y_{cm} M, \text{ where } M = \sum m_i$$

# Gravitational potential energy for rigid body

$$U = \sum m_i g y_i = g \sum m_i y_i = g \left( \frac{\sum m_i y_i}{\sum m_i} \right) \sum m_i$$

$$\Rightarrow U = g y_{cm} M, \text{ where } M = \sum m_i$$

or

$$U = M g y_{cm}$$

