Today 9.1, 9.2

$\boxed{L27}$

Today 9.1, 9.2

L27

Angular velocity and acceleration

L27

Today 9.1, 9.2

Angular velocity and acceleration

Rotation with constant angular acceleration

L27

Today 9.1, 9.2

Friday 9.3, 9.4

Today 9.1, 9.2

$\boxed{L27}$

Friday 9.3, 9.4

Relating linear and angular Kinematics

Today 9.1, 9.2

Friday 9.3, 9.4

Relating linear and angular kinematics

Energy & rotational motion

$\boxed{L27}$

Today 9.1, 9.2

$\boxed{L27}$

Friday 9.3, 9.4

SI sessions:

Today 9.1, 9.2

L27

Friday 9.3, 9.4

SI sessions:

Today 5 to 6pm

L27

Today 9.1, 9.2

Friday 9.3, 9.4

SI sessions:

Today 5 to 6 pm

* §8.5 (center of mass)

ASU

Today 9.1, 9.2

Friday 9.3, 9.4

$\boxed{L27}$

SI sessions:

Today 5 to 6 pm

* §8.5 (center of mass)
* §8.6 (Rocket propulsion)

Today 9.1, 9.2

L27

Friday 9.3, 9.4

SI sessions:

Today 5 to 6 pm

* §8.5 (center of mass)

* §8.6 (Rocket propulsion)

Friday 9 to 10 am

Today 9.1, 9.2

L27

Friday 9.3, 9.4

SI sessions:

Today 5 to 6 pm

* §8.5 (center of mass)

* §8.6 (Rocket propulsion)

Friday 9 to 10 am

* §9.1 (Angular velocity & acceleration)

ASU

Today 9.1, 9.2 L27

Friday 9.3, 9.4

SI sessions:

Today 5 to 6 pm

* §8.5 (center of mass)
* §8.6 (Rocket propulsion)

Friday 9 to 10 am

* §9.1 (Angular velocity & acceleration)
* §9.2 (Rotation with $\alpha = \text{const.}$)

Today 9.1, 9.2

Friday 9.3, 9.4

L27

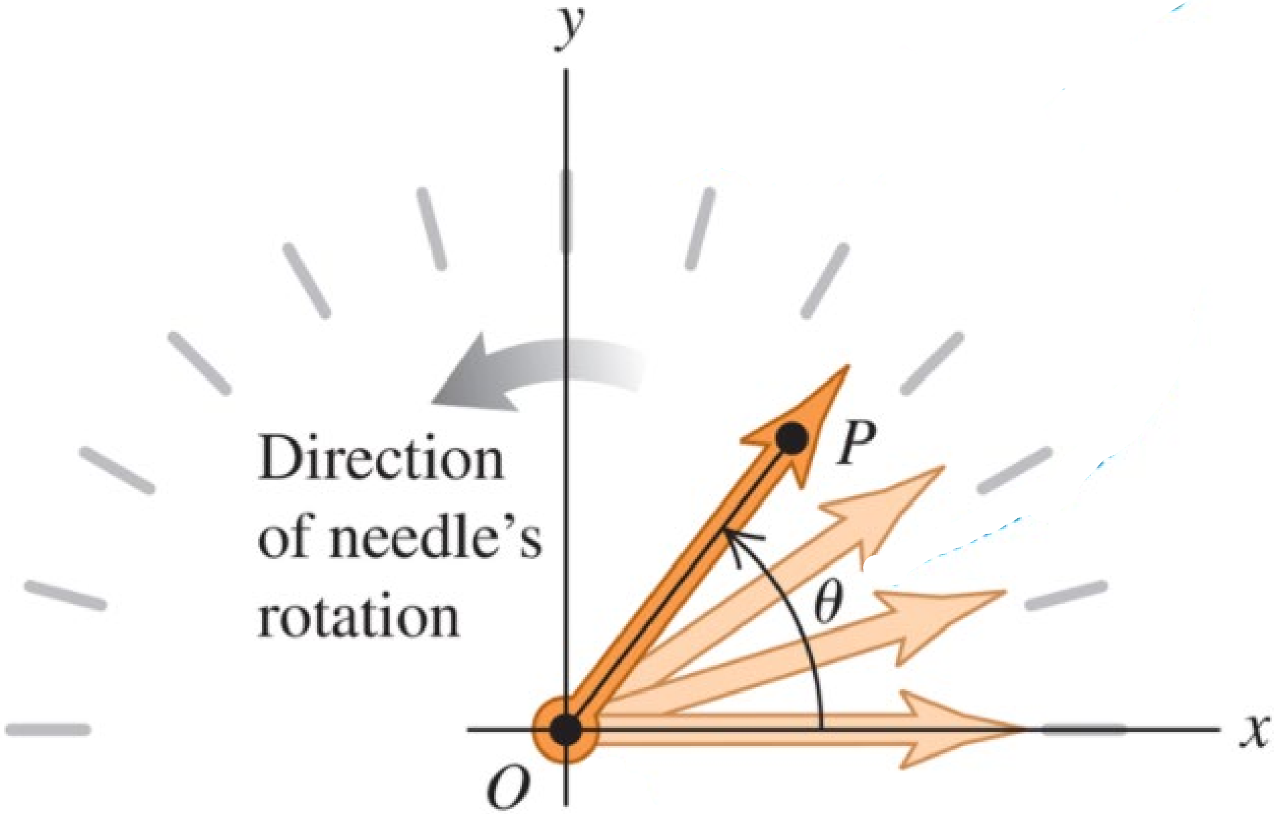
SI sessions:

Today 5 to 6 pm
* §8.5 (center of mass)
* §8.6 (Rocket propulsion)

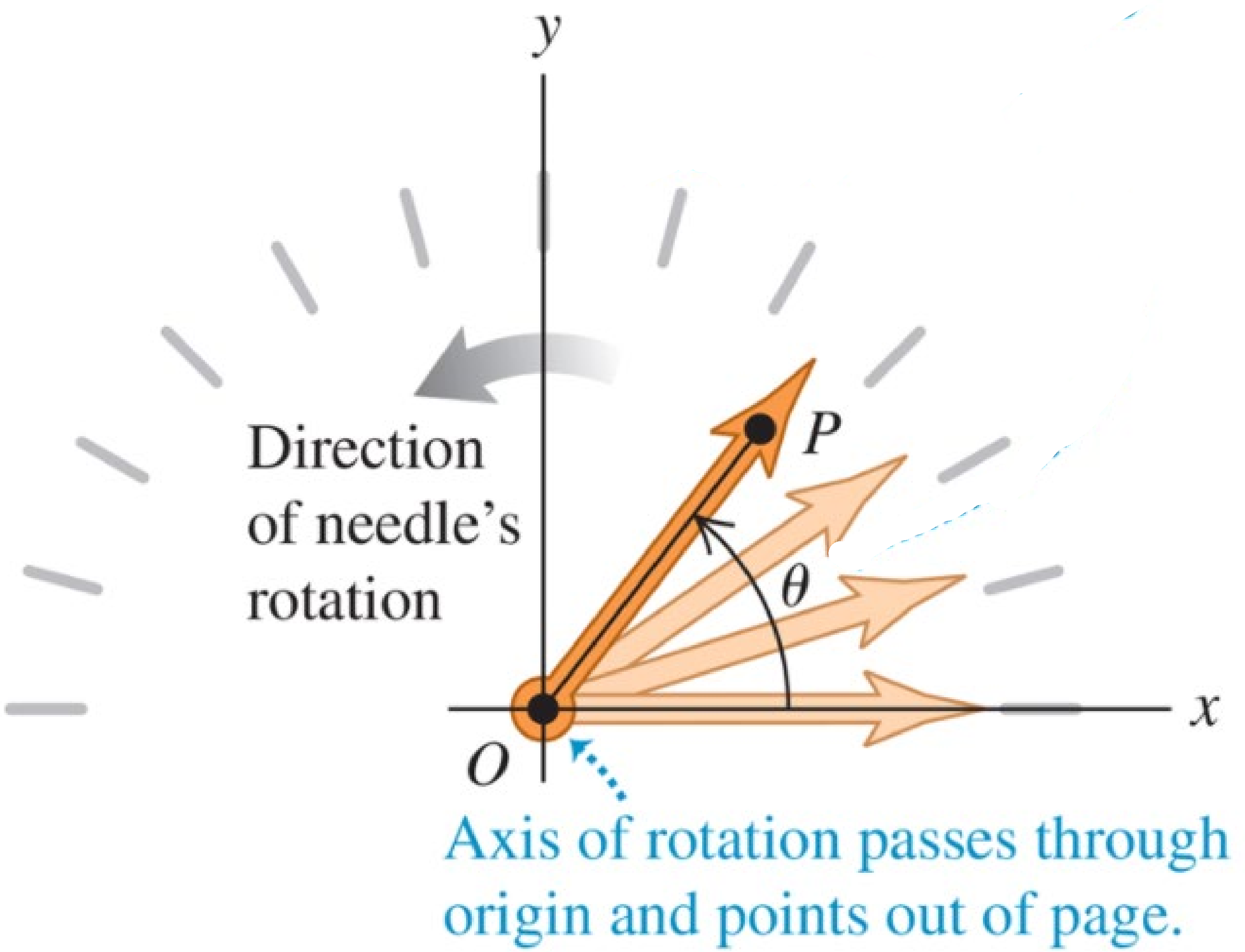
Friday 9 to 10 am
* §9.1 (Angular velocity & acceleration)
* §9.2 (Rotation with α = const.)

Angular velocity and acceleration

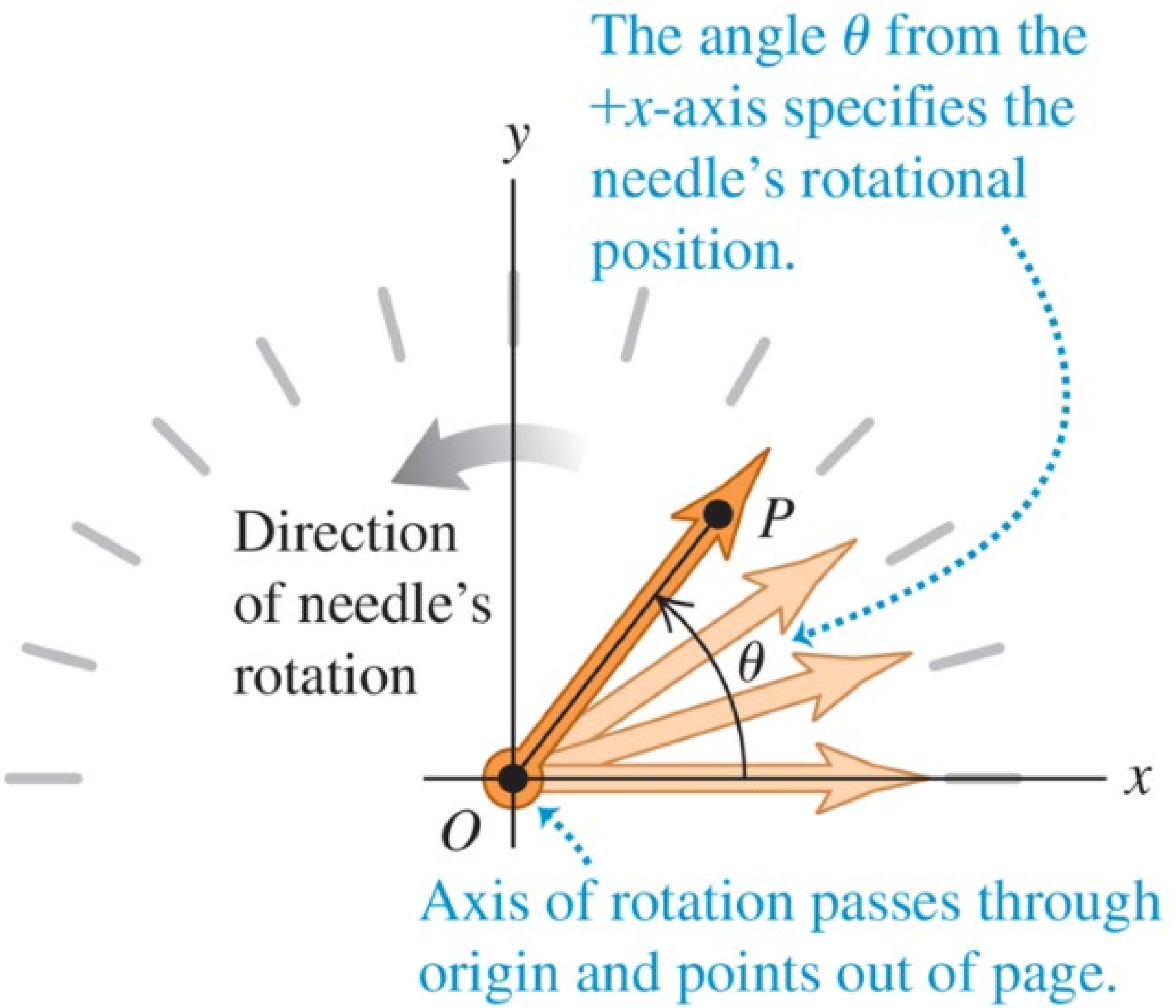
Angular velocity and acceleration



Angular velocity and acceleration



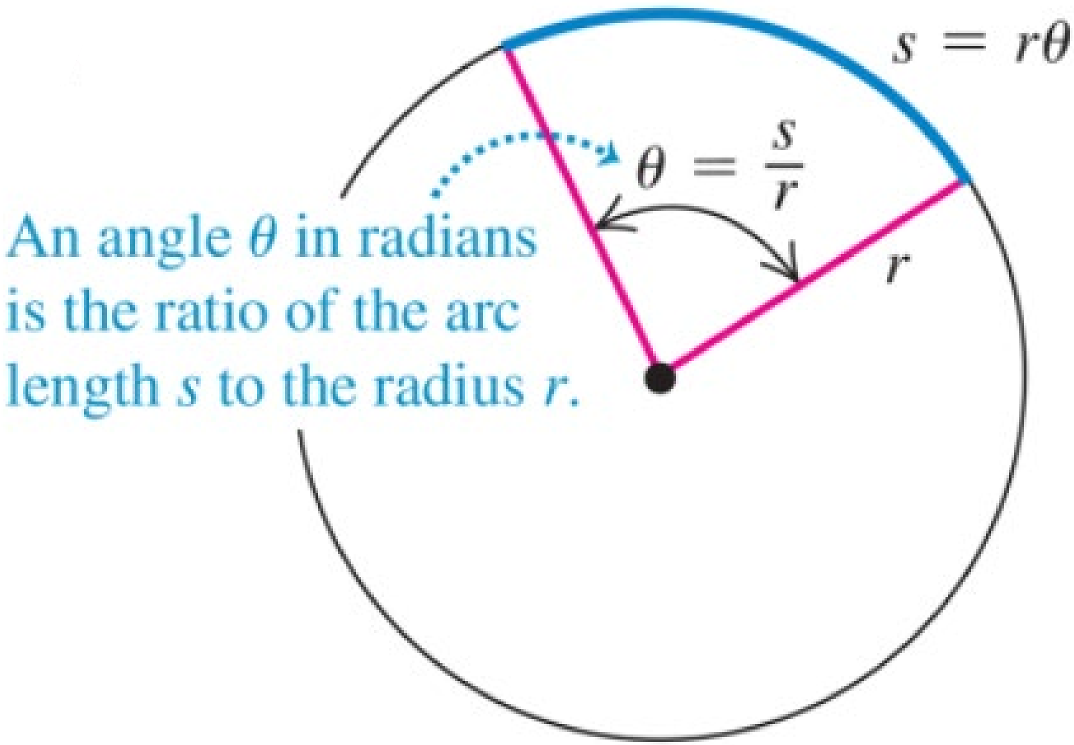
Angular velocity and acceleration



ASU

Angular velocity and acceleration

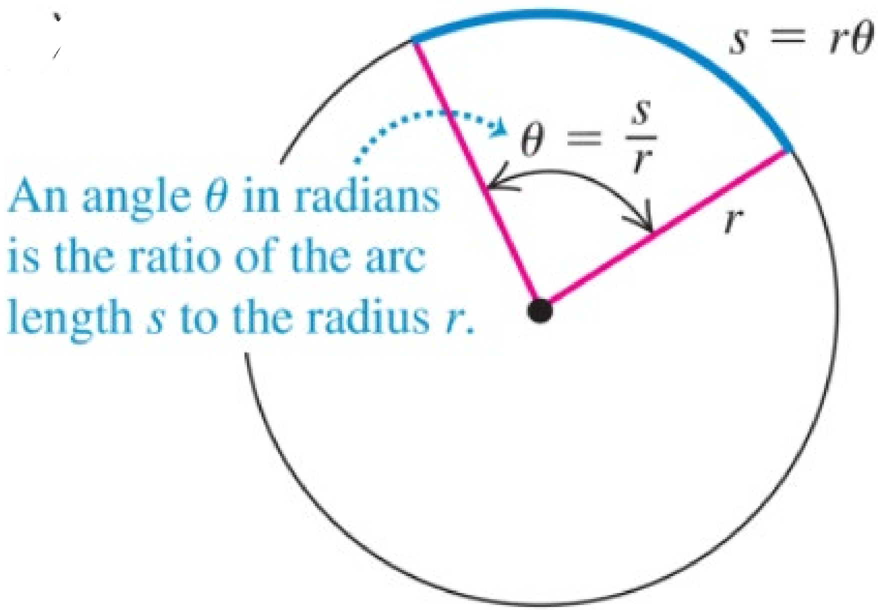
Let $s \equiv$ arc length



Angular velocity and acceleration

Let $s \equiv$ arc length

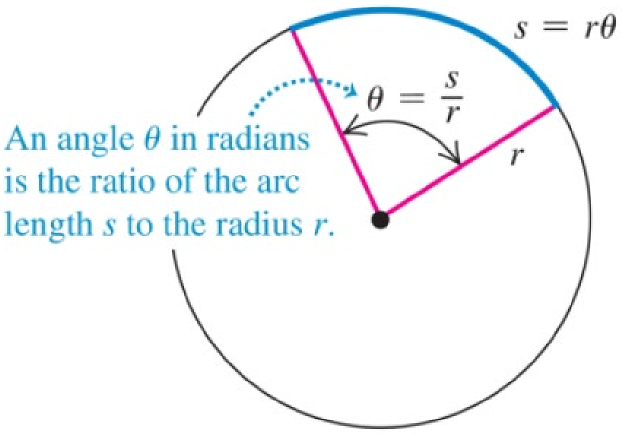
$s = r\theta$



Angular velocity and acceleration

Let $s \equiv$ arc length

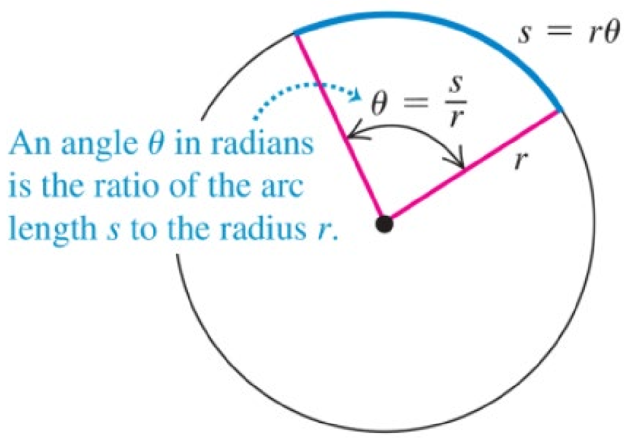
$s = r\theta$, where θ **must** be measured in radians



Angular velocity and acceleration

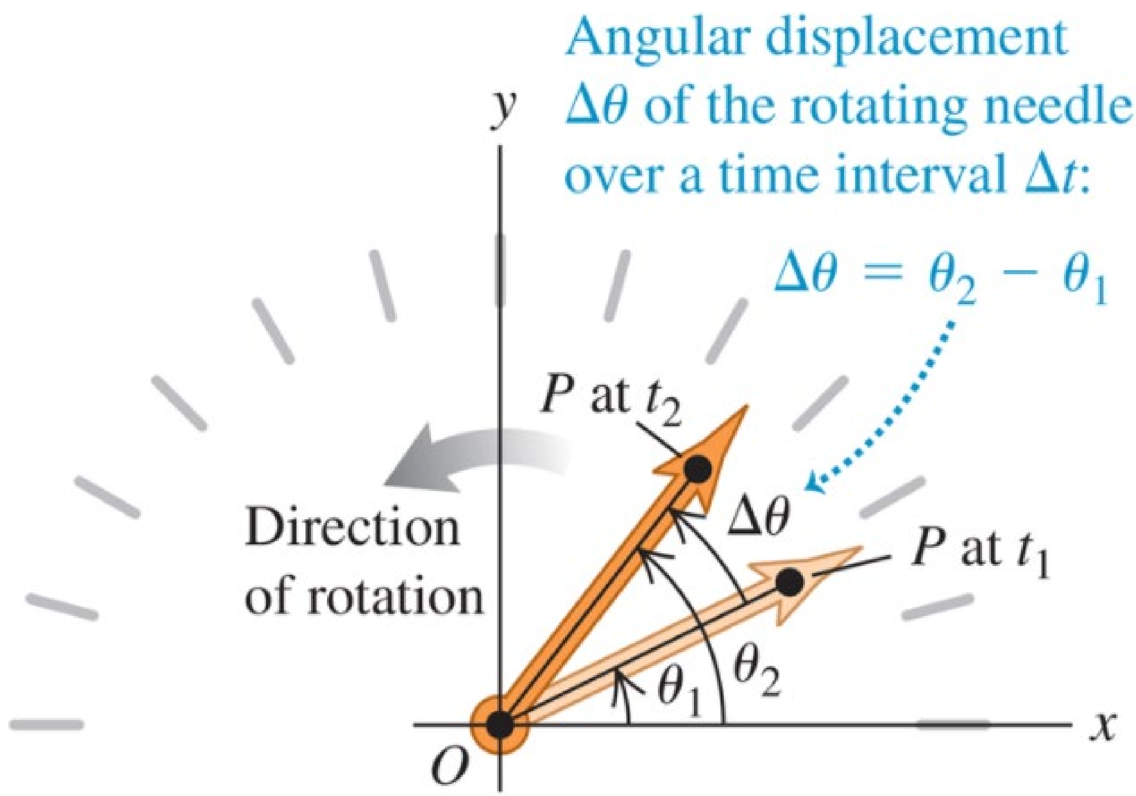
Let $s \equiv$ arc length

$s = r\theta$, where θ **must** be measured in radians



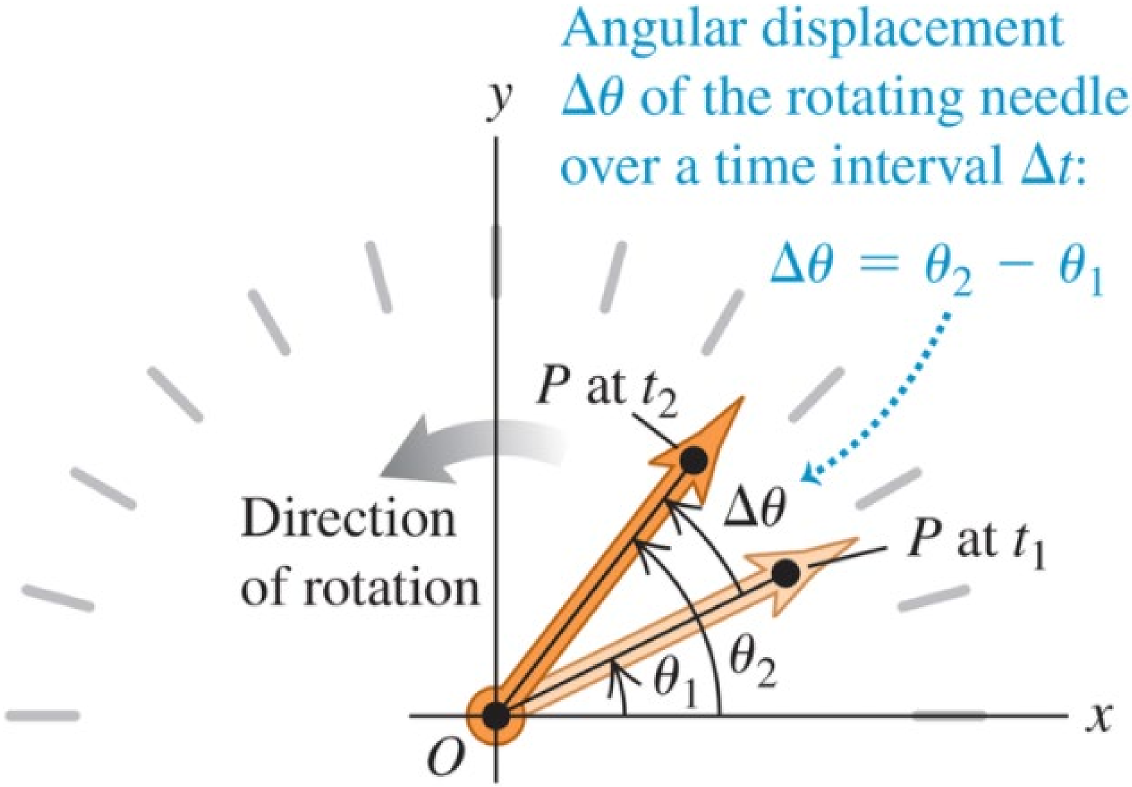
Not degrees

Angular velocity and acceleration



Angular velocity and acceleration

$$\omega_{AVE} = \frac{\Delta\theta}{\Delta t}$$

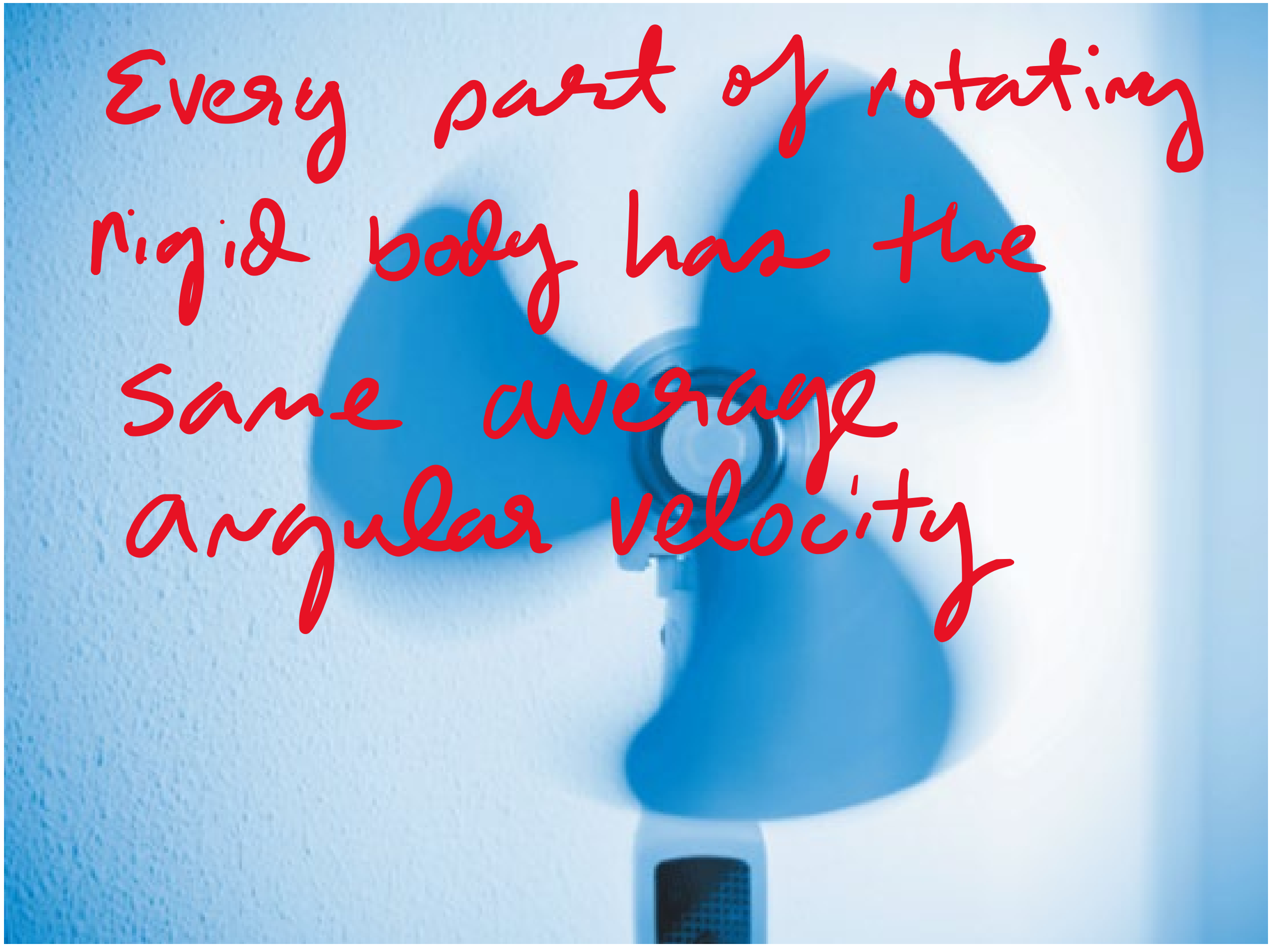


Angular velocity and acceleration

$$\omega_{AVE} = \frac{\Delta\theta}{\Delta t}$$

Angular velocity and acceleration

$$\omega_{ave} = \frac{\Delta\theta}{\Delta t}$$



Every part of rotating rigid body has the same average angular velocity

Angular velocity and acceleration

$$\omega_{AVE} = \frac{\Delta\theta}{\Delta t}, \quad \omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

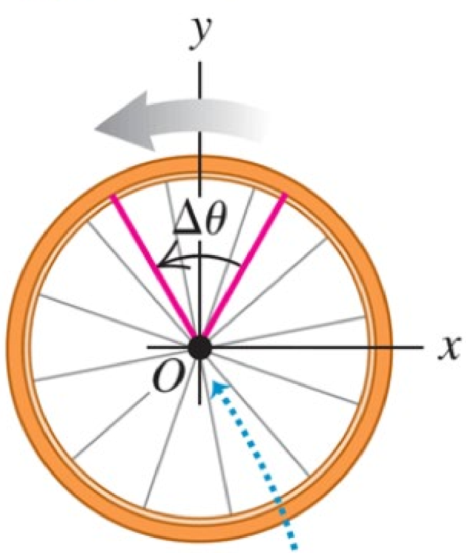
Angular velocity and acceleration

We choose the angle θ to increase in the counterclockwise rotation.

Angular velocity and acceleration

We choose the angle θ to increase in the counterclockwise rotation.

Counterclockwise rotation:

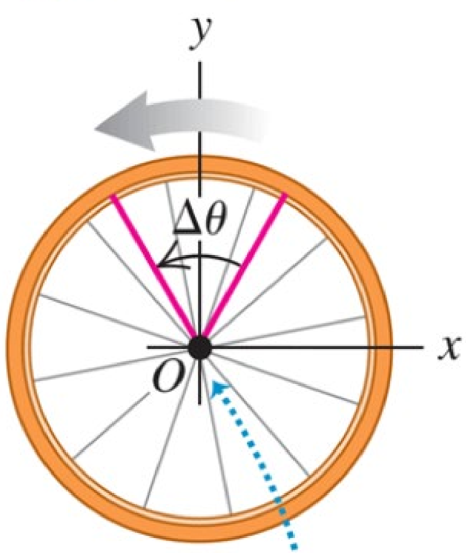


Axis of rotation (z-axis) passes through origin and points out of page.

Angular velocity and acceleration

We choose the angle θ to increase in the counterclockwise rotation.

Counterclockwise rotation:
θ increases, so angular velocity is positive.
$\Delta\theta > 0$, so
$\omega_{\text{av-}z} = \Delta\theta/\Delta t > 0$



Axis of rotation (z-axis) passes through origin and points out of page.

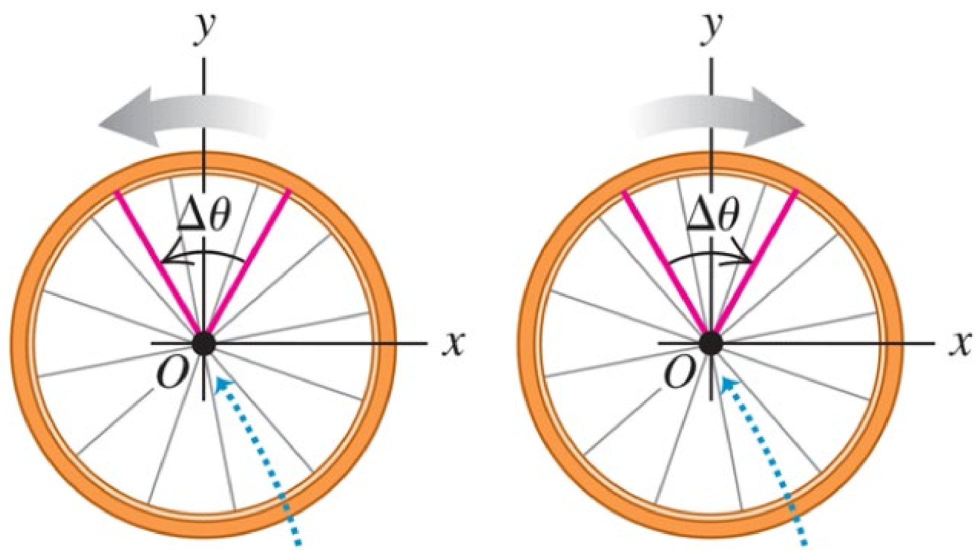
Angular velocity and acceleration

We choose the angle θ to increase in the counterclockwise rotation.

Counterclockwise rotation:

θ increases, so angular velocity is positive.
$\Delta\theta > 0$, so
$\omega_{\text{av-}z} = \Delta\theta/\Delta t > 0$

Clockwise rotation:



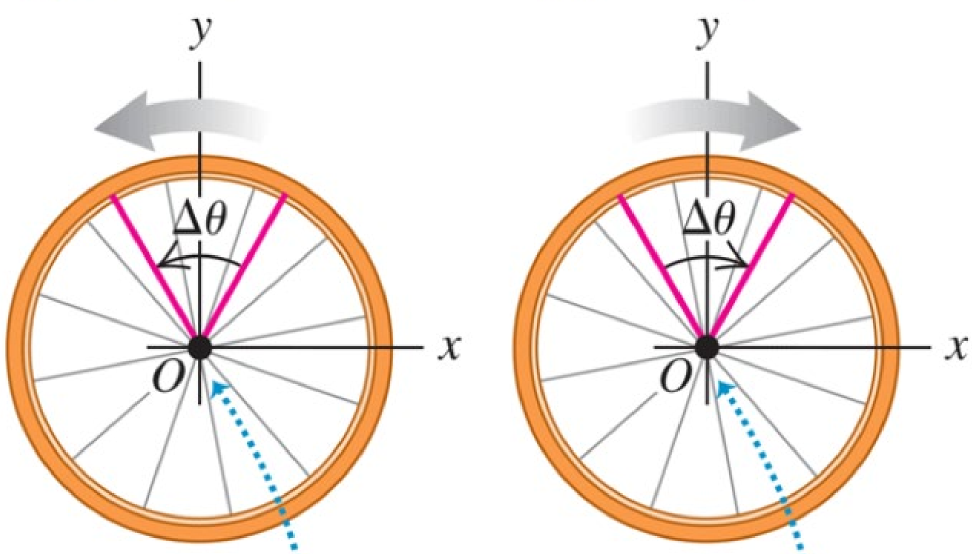
Axis of rotation (z-axis) passes through origin and points out of page.

Angular velocity and acceleration

We choose the angle θ to increase in the counterclockwise rotation.

Counterclockwise rotation:
θ increases, so angular velocity is positive.
$\Delta\theta > 0$, so
$\omega_{av\text{-}z} = \Delta\theta/\Delta t > 0$

Clockwise rotation:
θ decreases, so angular velocity is negative.
$\Delta\theta < 0$, so
$\omega_{av\text{-}z} = \Delta\theta/\Delta t < 0$



Axis of rotation (z-axis) passes through origin and points out of page.

ASU

Angular velocity and acceleration

$1 \text{ rev} = 2\pi \text{ rad}$

Angular velocity and acceleration

$$1 \text{ rev} = 2\pi \text{ rad} = 360^\circ$$

Angular velocity and acceleration

$1\,\text{rev} = 2\pi\,\text{rad} = 360°$ So

$\frac{\text{rev}}{2\pi} = 1$

Angular velocity and acceleration

$1\,\text{rev} = 2\pi\,\text{rad} = 360°$ So

$\frac{\text{rev}}{2\pi} = 1$ & $\frac{360°}{2\pi\,\text{rad}} = \frac{180°}{\pi\,\text{rad}} = 1$

Angular velocity and acceleration

$$1\,\text{rev} = 2\pi\,\text{rad} = 360^\circ \quad \text{so}$$

$$\frac{\text{rev}}{2\pi} = 1 \quad \& \quad \frac{360^\circ}{2\pi\,\text{rad}} = \frac{180^\circ}{\pi\,\text{rad}} = 1$$

$$\& \quad 1\,\text{min} = 60\,\text{s}$$

Angular velocity and acceleration

$1\,\text{rev} = 2\pi\,\text{rad} = 360°$ so

$\frac{\text{rev}}{2\pi} = 1$ & $\frac{360°}{2\pi\,\text{rad}} = \frac{180°}{\pi\,\text{rad}} = 1$

& $1\,\text{min} = 60\,\text{s}$ so $\frac{60\,\text{s}}{\text{min}} = 1$

Angular velocity and acceleration

$1\text{ rev} = 2\pi\text{ rad} = 360^\circ$ so

$\frac{\text{rev}}{2\pi} = 1$ & $\frac{360^\circ}{2\pi\text{ rad}} = \frac{180^\circ}{\pi\text{ rad}} = 1$

& $1\text{ min} = 60\text{ s}$ so $\frac{60\text{ s}}{\text{min}} = 1$

Now $\frac{\text{rad}}{\text{s}} = \frac{\text{rad}}{\text{s}}\left(\frac{\text{rev}}{2\pi\text{ rad}}\right)$

Angular velocity and acceleration

$1\,\text{rev} = 2\pi\,\text{rad} = 360°$ so

$\frac{\text{rev}}{2\pi} = 1$ & $\frac{360°}{2\pi\,\text{rad}} = \frac{180°}{\pi\,\text{rad}} = 1$

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Now $\frac{\text{rad}}{\text{s}} = \frac{\text{rad}}{\text{s}}\left(\frac{\text{rev}}{2\pi\,\text{rad}}\right)\left(\frac{60\,\text{s}}{\text{min}}\right)$

Angular velocity and acceleration

$1\,\text{rev} = 2\pi\,\text{rad} = 360^\circ$ so

$\frac{\text{rev}}{2\pi} = 1$ & $\frac{360^\circ}{2\pi\,\text{rad}} = \frac{180^\circ}{\pi\,\text{rad}} = 1$

& $1\,\text{min} = 60\,\text{s}$ so $\frac{60\,\text{s}}{\text{min}} = 1$

Now $\frac{\text{rad}}{\text{s}} = \frac{\text{rad}}{\text{s}}\left(\frac{\text{rev}}{2\pi\,\text{rad}}\right)\left(\frac{60\,\text{s}}{\text{min}}\right)$

$\Rightarrow$ $\frac{\text{rad}}{\text{s}} = \left(\frac{60}{2\pi}\right)\frac{\text{rev}}{\text{min}}$

Angular velocity and acceleration

$$1\text{ rev} = 2\pi\text{ rad} = 360^\circ \quad \text{so}$$

$$\frac{\text{rev}}{2\pi} = 1 \quad \& \quad \frac{360^\circ}{2\pi\text{ rad}} = \frac{180^\circ}{\pi\text{ rad}} = 1$$

$$\& \quad 1\text{ min} = 60\text{ s} \quad \text{so} \quad \frac{60\text{ s}}{\text{min}} = 1$$

Now $$\frac{\text{rad}}{\text{s}} = \frac{\text{rad}}{\text{s}}\left(\frac{\text{rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{\text{min}}\right)$$

$$\Rightarrow \quad \frac{\text{rad}}{\text{s}} = \left(\frac{60}{2\pi}\right)\frac{\text{rev}}{\text{min}} = \left(\frac{60}{2\pi}\right)\text{rpm}$$

Angular velocity and acceleration

$1\,\text{rev} = 2\pi\,\text{rad} = 360°$ so

$\frac{\text{rev}}{2\pi} = 1$ & $\frac{360°}{2\pi\,\text{rad}} = \frac{180°}{\pi\,\text{rad}} = 1$

& $1\,\text{min} = 60\,\text{s}$ so $\frac{60\,\text{s}}{\text{min}} = 1$

Now $\frac{\text{rad}}{\text{s}} = \frac{\text{rad}}{\text{s}}\left(\frac{\text{rev}}{2\pi\,\text{rad}}\right)\left(\frac{60\,\text{s}}{\text{min}}\right)$

$\Rightarrow \frac{\text{rad}}{\text{s}} = \left(\frac{60}{2\pi}\right)\frac{\text{rev}}{\text{min}} = \left(\frac{60}{2\pi}\right)\text{rpm}$

revolutions per minute

The angular position θ of a 0.36-m-diameter flywheel is given by

$$\theta = (2.0\ \mathrm{rad/s^3})t^3$$

(a) Find θ, in radians and in degrees, at $t_1 = 2.0\ \mathrm{s}$ and $t_2 = 5.0\ \mathrm{s}$. (b) Find the distance that a particle on the flywheel rim moves from $t_1 = 2.0\ \mathrm{s}$ to $t_2 = 5.0\ \mathrm{s}$. (c) Find the average angular velocity, in rad / s and in rev / min, over that interval. (d) Find the instantaneous angular velocities at $t_1 = 2.0\ \mathrm{s}$ and $t_2 = 5.0\ \mathrm{s}$.

ASU

The angular position θ of a 0.36-m-diameter flywheel is given by

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$\theta(2s) = (2 * 8)$ rad

The angular position θ of a 0.36-m-diameter flywheel is given by

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$\theta(2s) = (2*8)\,\mathrm{rad} = 16\,\mathrm{rad}$

$\& \quad \theta(5s) = (2*125)\,\mathrm{rad}$

The angular position θ of a 0.36-m-diameter flywheel is given by

$$\theta = (2.0\ \mathrm{rad/s^3})t^3$$

(a) Find θ, in radians and in degrees, at $t_1 = 2.0$ s and $t_2 = 5.0$ s. (b) Find the distance that a particle on the flywheel rim moves from $t_1 = 2.0$ s to $t_2 = 5.0$ s. (c) Find the average angular velocity, in rad / s and in rev / min, over that interval. (d) Find the instantaneous angular velocities at $t_1 = 2.0$ s and $t_2 = 5.0$ s.

$\theta(2s) = (2 * 8)$ rad $= 16$ rad

& $\theta(5s) = (2 * 125)$ rad $= 250$ rad

ASU

The angular position θ of a 0.36-m-diameter flywheel is given by

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(a) Find θ, in radians and in degrees, at $t_1 = 2.0$ s and $t_2 = 5.0$ s. (b) Find the distance that a particle on the flywheel rim moves from $t_1 = 2.0$ s to $t_2 = 5.0$ s. (c) Find the average angular velocity, in rad / s and in rev / min, over that interval. (d) Find the instantaneous angular velocities at $t_1 = 2.0$ s and $t_2 = 5.0$ s.

$\theta(2s) = (2 * 8)\,\text{rad} = 16\,\text{rad}$

& $\theta(5s) = (2 * 125)\,\text{rad} = 250\,\text{rad}$ & $180° = \pi\,\text{rad}$

ASU

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$\theta(2s) = (2 * 8)\ \mathrm{rad} = 16\ \mathrm{rad}$

& $\theta(5s) = (2 * 125)\ \mathrm{rad} = 250\ \mathrm{rad}$ & $180° = \pi\ \mathrm{rad}$

so $\theta(2s) = (16\ \mathrm{rad})\left(\frac{180°}{\pi\ \mathrm{rad}}\right)$

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$\theta(2s) = (2 * 8)\,\mathrm{rad} = 16\,\mathrm{rad}$

& $\theta(5s) = (2 * 125)\,\mathrm{rad} = 250\,\mathrm{rad}$ & $180^\circ = \pi\,\mathrm{rad}$

so $\theta(2s) = (16\,\mathrm{rad})\left(\frac{180^\circ}{\pi\,\mathrm{rad}}\right) = 917^\circ$

ASU

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$\theta(2s) = (2 * 8)\ \mathrm{rad} = 16\ \mathrm{rad}$

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$\theta(2s) = (2 * 8)\text{ rad} = 16\text{ rad}$

& $\theta(5s) = (2 * 125)\text{ rad} = 250\text{ rad}$ & $180° = \pi\text{ rad}$

so $\theta(2s) = (16\text{ rad})\left(\frac{180°}{\pi\text{ rad}}\right) = 917°$

& $\theta(5s) = (250\text{ rad})\left(\frac{180°}{\pi\text{ rad}}\right) = 14300°$

ASU

The angular position θ of a 0.36-m-diameter flywheel is given by

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$\theta(2s) = (2 * 8)\,\mathrm{rad} = 16\,\mathrm{rad}$

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so $\theta(2s) = (16\,\mathrm{rad})\left(\frac{180^\circ}{\pi\,\mathrm{rad}}\right) = 917^\circ$

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ASU

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$\theta(2s) = (2 * 8)\text{ rad} = 16\text{ rad}$

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so $\theta(2s) = (16\text{ rad})\left(\frac{180^\circ}{\pi\text{ rad}}\right) = 917^\circ$

& $\theta(5s) = (250\text{ rad})\left(\frac{180^\circ}{\pi\text{ rad}}\right) = 14300^\circ$

$s = r\Delta\theta$

The angular position θ of a 0.36-m-diameter flywheel is given by

$$\theta = (2.0\ \text{rad/s}^3)t^3$$

(a) Find θ, in radians and in degrees, at $t_1 = 2.0$ s and $t_2 = 5.0$ s. (b) Find the distance that a particle on the flywheel rim moves from $t_1 = 2.0$ s to $t_2 = 5.0$ s. (c) Find the average angular velocity, in rad / s and in rev / min, over that interval. (d) Find the instantaneous angular velocities at $t_1 = 2.0$ s and $t_2 = 5.0$ s.

$\theta(2s) = (2 * 8)\text{ rad} = 16\text{ rad}$

& $\theta(5s) = (2 * 125)\text{ rad} = 250\text{ rad}$ & $180° = \pi\text{ rad}$

so $\theta(2s) = (16\text{ rad})\left(\frac{180°}{\pi\text{ rad}}\right) = 917°$

& $\theta(5s) = (250\text{ rad})\left(\frac{180°}{\pi\text{ rad}}\right) = 14300°$

$s = r\Delta\theta = \left(\frac{0.36}{2}\right)(250-16)\text{ m}$

ASU

The angular position θ of a 0.36-m-diameter flywheel is given by

$$\theta = (2.0 \text{ rad/s}^3)t^3$$

(a) Find θ, in radians and in degrees, at $t_1 = 2.0$ s and $t_2 = 5.0$ s. (b) Find the distance that a particle on the flywheel rim moves from $t_1 = 2.0$ s to $t_2 = 5.0$ s. (c) Find the average angular velocity, in rad / s and in rev / min, over that interval. (d) Find the instantaneous angular velocities at $t_1 = 2.0$ s and $t_2 = 5.0$ s.

$\theta(2s) = (2 * 8)\text{ rad} = 16\text{ rad}$

& $\theta(5s) = (2 * 125)\text{ rad} = 250\text{ rad}$ & $180° = \pi\text{ rad}$

so $\theta(2s) = (16\text{ rad})\left(\frac{180°}{\pi\text{ rad}}\right) = 917°$

& $\theta(5s) = (250\text{ rad})\left(\frac{180°}{\pi\text{ rad}}\right) = 14300°$

$s = r\Delta\theta = \left(\frac{0.36}{2}\right)(250 - 16)\text{ m} = 42.1\text{ m}$

ASU

The angular position θ of a 0.36-m-diameter flywheel is given by

$$\theta = (2.0\ \mathrm{rad/s^3})t^3$$

(a) Find θ, in radians and in degrees, at $t_1 = 2.0$ s and $t_2 = 5.0$ s. (b) Find the distance that a particle on the flywheel rim moves from $t_1 = 2.0$ s to $t_2 = 5.0$ s. (c) Find the average angular velocity, in rad / s and in rev / min, over that interval. (d) Find the instantaneous angular velocities at $t_1 = 2.0$ s and $t_2 = 5.0$ s.

$\theta(2s) = (2 * 8)\ \mathrm{rad} = 16\ \mathrm{rad}$

& $\theta(5s) = (2 * 125)\ \mathrm{rad} = 250\ \mathrm{rad}$

$\omega_{AVE} = \frac{\Delta\theta}{\Delta t}$

The angular position θ of a 0.36-m-diameter flywheel is given by

$$\theta = (2.0\ \mathrm{rad/s^3})t^3$$

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$\theta(2s) = (2 * 8)\,\mathrm{rad} = 16\,\mathrm{rad}$

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$$\omega = \frac{d\theta}{dt}$$

ASU

The angular position θ of a 0.36-m-diameter flywheel is given by

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$$\omega = \frac{d\theta}{dt} = \left(6\frac{\text{rad}}{s^3}\right)t^2$$

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$$\omega = \frac{d\theta}{dt} = \left(6\frac{\text{rad}}{s^3}\right)t^2 \quad \text{so} \quad \omega(2s) = 24\ \text{rad/s}$$

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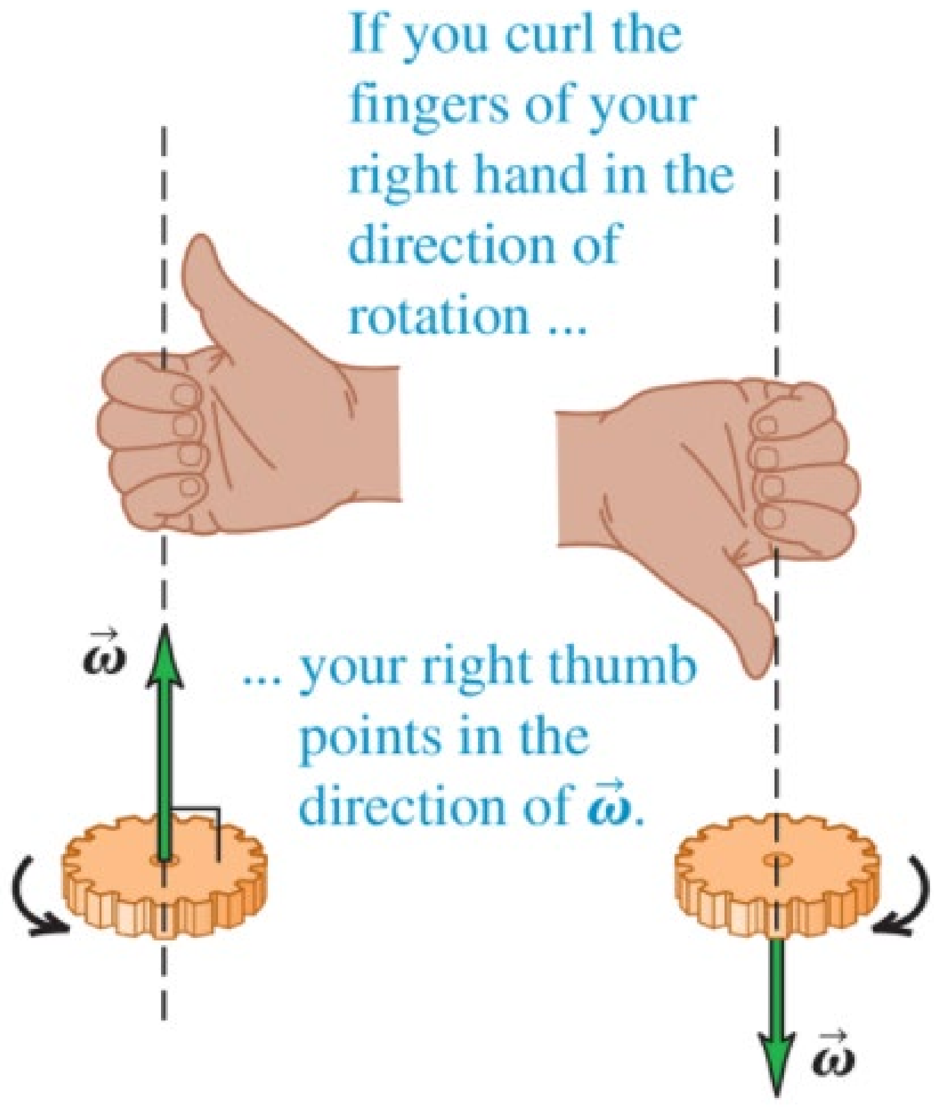
$$\omega = \frac{d\theta}{dt} = \left(6\frac{\mathrm{rad}}{s^3}\right)t^2$$ so $\omega(2s) = 24\ \mathrm{rad/s}$ &

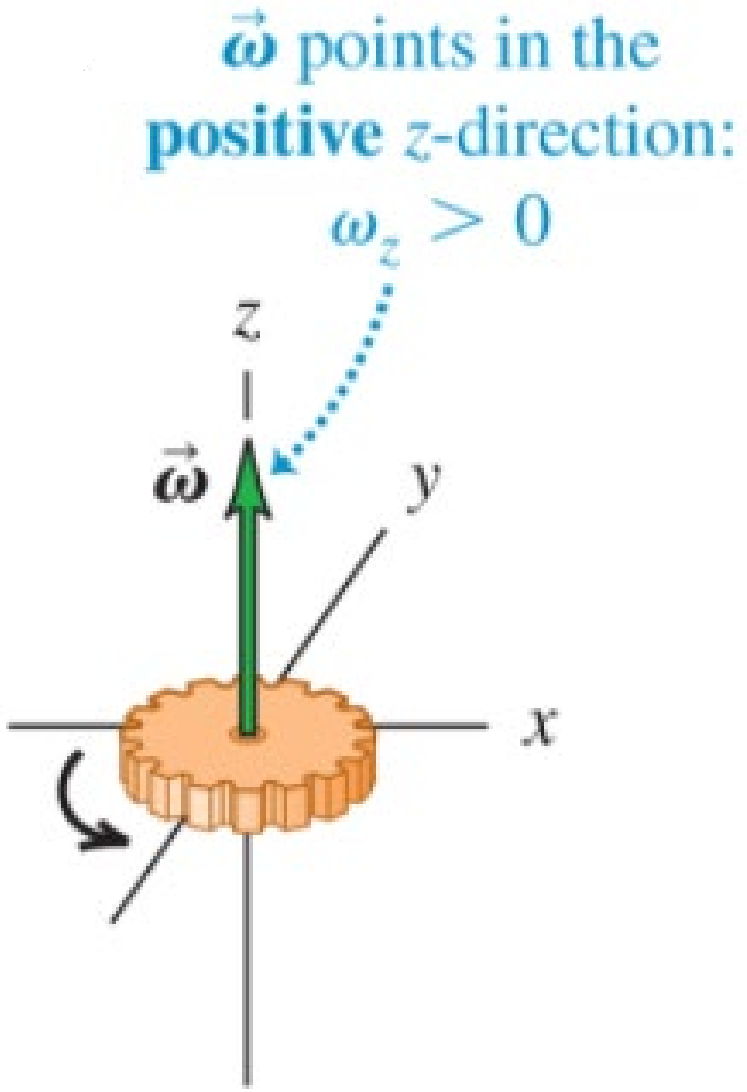
$\omega(5s) = 150\ \mathrm{rad/s}$

Angular velocity as a vector

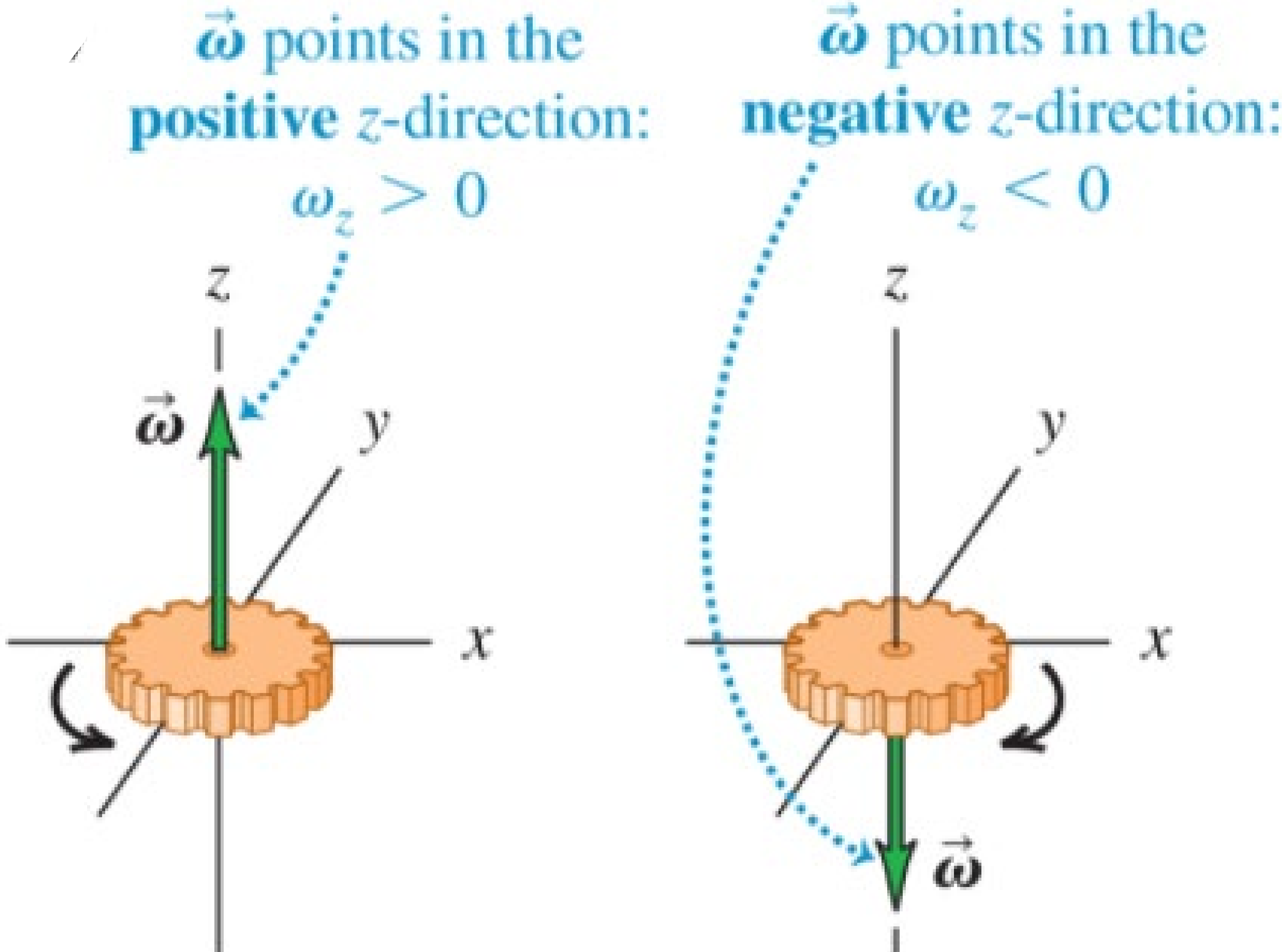
Angular velocity as a vector



Angular velocity as a vector



Angular velocity as a vector



Angular acceleration

$$\alpha_{AVE} = \frac{\Delta \omega}{\Delta t}$$

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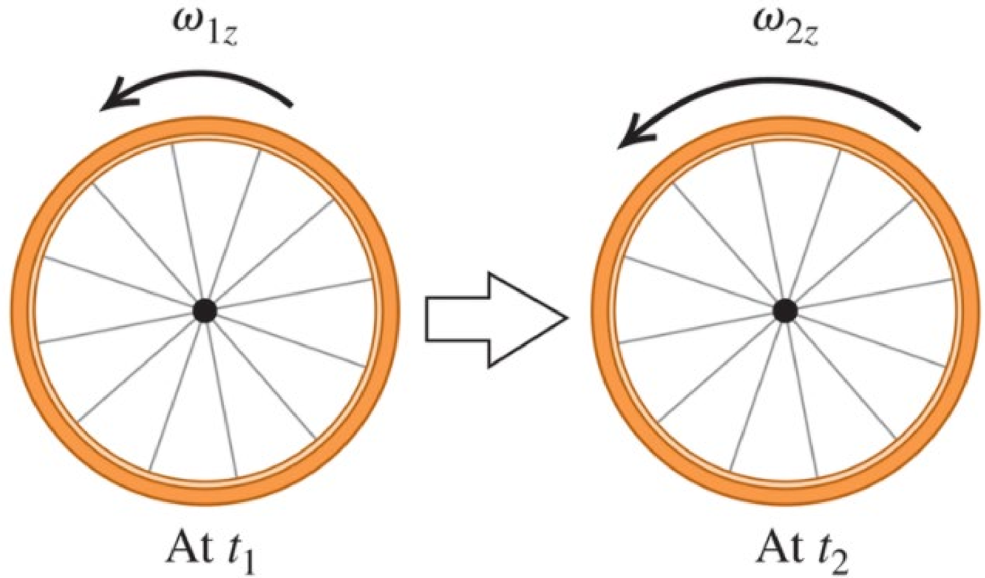
The average angular acceleration is the change in angular velocity divided by the time interval:

$$\alpha_{\text{av-}z} = \frac{\omega_{2z} - \omega_{1z}}{t_2 - t_1} = \frac{\Delta\omega_z}{\Delta t}$$



Angular acceleration

$$\alpha_{AVE} = \frac{\Delta \omega}{\Delta t} \quad \& \quad \alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$$

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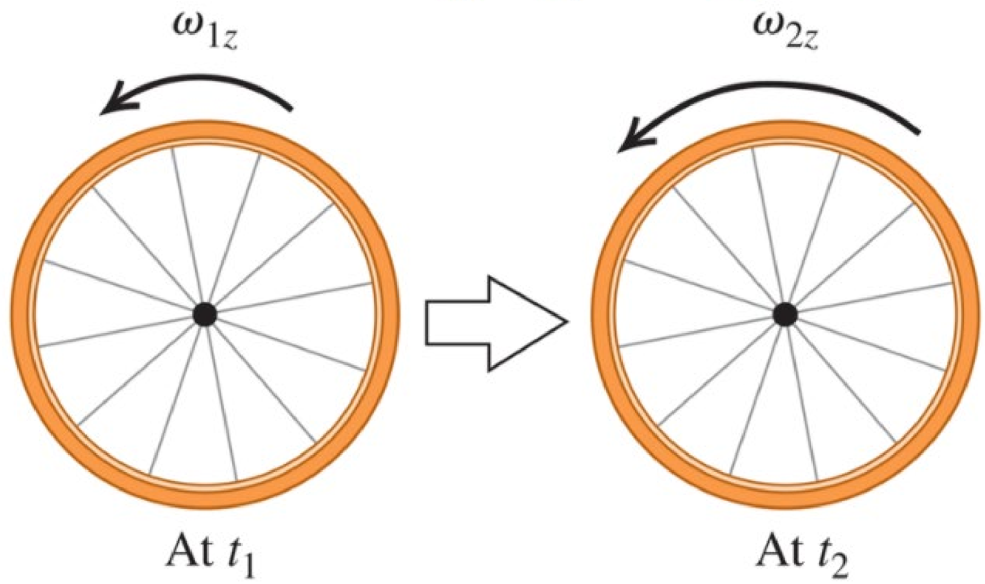

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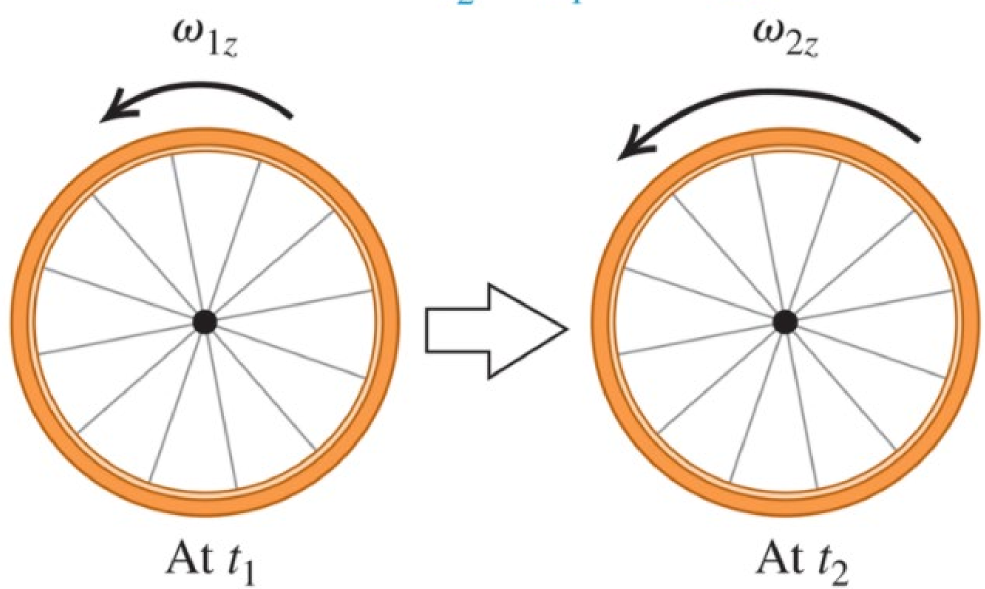

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ASU

Angular acceleration

$$\alpha_{AVE} = \frac{\Delta\omega}{\Delta t} \ \& \ \alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{\partial\omega}{\partial t}$$

$$\& \text{ since } \omega = \frac{\partial\theta}{\partial t}$$

then $$\alpha = \frac{\partial}{\partial t}\left[\frac{\partial\theta}{\partial t}\right]$$

ASU

Angular acceleration

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then $$\alpha = \frac{d}{dt}\left[\frac{d\theta}{dt}\right] = \frac{d^2\theta}{dt^2}$$

ASU

The angular position θ of a 0.36-m-diameter flywheel is given by

$$\theta = (2.0 \text{ rad/s}^3)t^3$$

For the flywheel of Example 9.1, (a) find the average angular acceleration between $t_1 = 2.0$ s and $t_2 = 5.0$ s. (b) Find the instantaneous angular accelerations at $t_1 = 2.0$ s and $t_2 = 5.0$ s.

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Previously we found $\omega(5s) = 150 \frac{rad}{s}$ & $\omega(2s) = 24 \frac{rad}{s}$

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so $\quad \alpha_{AVE} = \frac{\Delta\omega}{\Delta t} = \left(\frac{150-24}{5-3}\right)\frac{rad}{s^2} = 42\ rad/s^2$

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$\alpha = \frac{d^2\theta}{dt^2}$

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So $\alpha_{AVE} = \frac{\Delta\omega}{\Delta t} = \left(\frac{150-24}{5-3}\right)\frac{rad}{s^2} = 42\ rad/s^2$

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{d}{dt}\left[(6\ rad/s^3)t^2\right]$$

The angular position θ of a 0.36-m-diameter flywheel is given by

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$$\alpha = \frac{d^2\theta}{dt^2} = \frac{d}{dt}\left[(6 \, rad/s^3)t^2\right] = (12 \, rad/s^3)t$$

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$$\alpha = \frac{d^2\theta}{dt^2} = \frac{d}{dt}\left[(6\ rad/s^3)t^2\right] = (12\ rad/s^3)t$$

$$\Rightarrow \quad \alpha(2s) = 24\ rad/s^2$$

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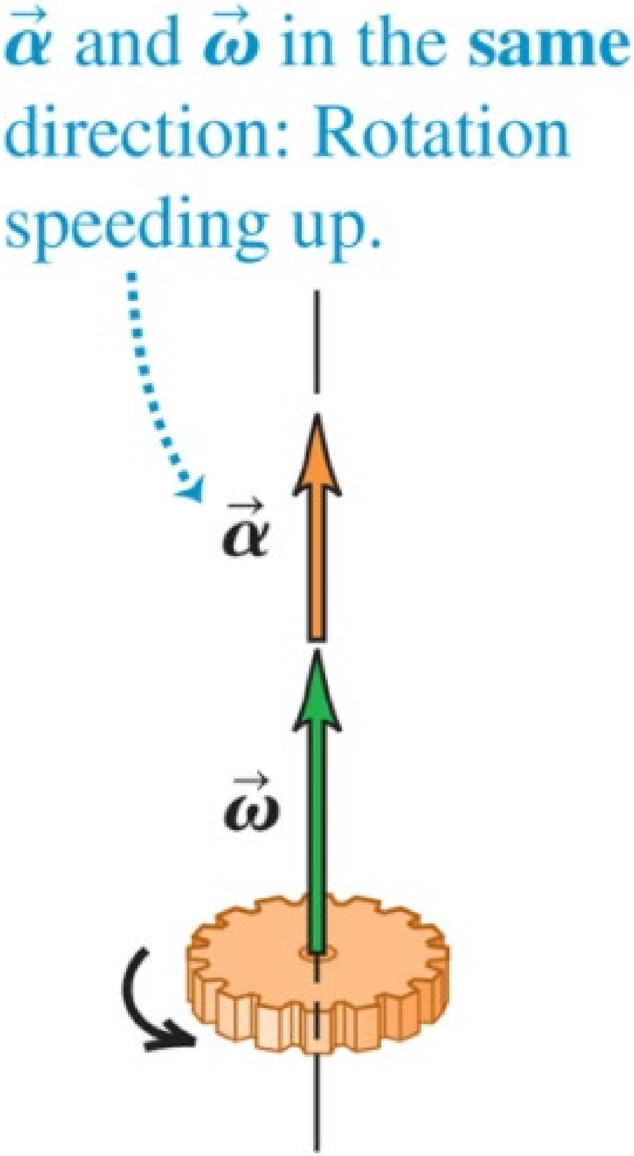
$\alpha = \frac{d^2\theta}{dt^2} = \frac{d}{dt}\left[(6\,rad/s^3)t^2\right] = (12\,rad/s^3)t$

$\Rightarrow \alpha(2s) = 24\,rad/s^2$ & $\alpha(5s) = 60\,rad/s^2$

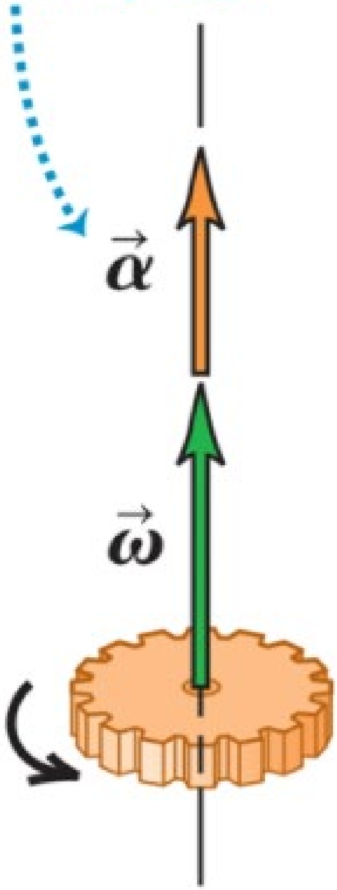
Angular acceleration as a vector

Angular acceleration as a vector

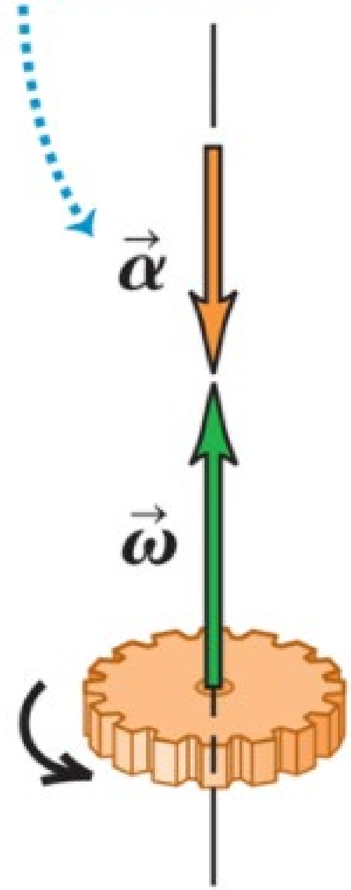


Angular acceleration as a vector

$\vec{\boldsymbol{\alpha}}$ and $\vec{\boldsymbol{\omega}}$ in the **same** direction: Rotation speeding up.



$\vec{\boldsymbol{\alpha}}$ and $\vec{\boldsymbol{\omega}}$ in the **opposite** directions: Rotation slowing down.



Section 9.2

Section 9.2

Rotation with constant angular acceleration

Constant angular acceleration

$$\alpha = \frac{d\omega}{dt} \Rightarrow \int \alpha\, dt = \int d\omega$$

Constant angular acceleration

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so $\quad \alpha = \text{const.} \Rightarrow \alpha \int_0^t dt = \int_{\omega_0}^{\omega} d\omega$

Constant angular acceleration

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so $\quad \alpha = \text{const.} \Rightarrow \alpha \int_0^t dt = \int_{\omega_0}^{\omega} d\omega$

$$\Rightarrow \quad \alpha t = \omega - \omega_0$$

Constant angular acceleration

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$$\Rightarrow \quad \alpha t = \omega - \omega_0 \Rightarrow$$

$$\boxed{\omega = \alpha t + \omega_0}$$

Constant angular acceleration

$$\alpha = \frac{d\omega}{dt} \Rightarrow \int \alpha\, dt = \int d\omega$$

so $\quad \alpha = \text{const.} \Rightarrow \alpha \int_0^t dt = \int_{\omega_0}^{\omega} d\omega$

$$\Rightarrow \quad \alpha t = \omega - \omega_0 \Rightarrow$$

$$\boxed{\omega = \alpha t + \omega_0}$$

compare to $\quad V = at + V_0$

Constant angular acceleration

$\omega = \alpha t + \omega_0$ & $\omega = \frac{d\theta}{dt}$

Constant angular acceleration

$\omega = \alpha t + \omega_0$ & $\omega = \frac{d\theta}{dt} \Rightarrow$

$\frac{d\theta}{dt} = \alpha t + \omega_0$

Constant angular acceleration

$\omega = \alpha t + \omega_0$ & $\omega = \frac{d\theta}{dt} \Rightarrow$

$\frac{d\theta}{dt} = \alpha t + \omega_0 \Rightarrow \int d\theta = \alpha \int t\,dt + \omega_0 \int dt$

Constant angular acceleration

$\omega = \alpha t + \omega_0$ & $\omega = \frac{d\theta}{dt} \Rightarrow$

$\frac{d\theta}{dt} = \alpha t + \omega_0 \Rightarrow \int_{\theta_0}^{\theta} d\theta = \alpha \int_0^t t\,dt + \omega_0 \int_0^t dt$

Constant angular acceleration

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$\frac{d\theta}{dt} = \alpha t + \omega_0 \Rightarrow \int_{\theta_0}^{\theta} d\theta = \alpha \int_0^t t\,dt + \omega_0 \int_0^t dt$

$\Rightarrow \quad \theta - \theta_0 = \frac{1}{2}\alpha t^2 + \omega_0 t$

Constant angular acceleration

$\omega = \alpha t + \omega_0$ & $\omega = \frac{d\theta}{dt} \Rightarrow$

$\frac{d\theta}{dt} = \alpha t + \omega_0 \Rightarrow \int_{\theta_0}^{\theta} d\theta = \alpha \int_0^t t\,dt + \omega_0 \int_0^t dt$

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$$\boxed{\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0}$$

Constant angular acceleration

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$$\boxed{\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0}$$

compare to

$x = \frac{1}{2} a t^2 + v_0 t + x_0$

Constant angular acceleration

$$\alpha = \frac{d\omega}{dt}$$

Constant angular acceleration

$\alpha = \frac{d\omega}{dt}$ but $\frac{d\omega}{dt} = \left(\frac{d\omega}{d\theta}\right)\left(\frac{d\theta}{dt}\right)$

Constant angular acceleration

$$\alpha = \frac{d\omega}{dt} \qquad \text{but} \qquad \frac{d\omega}{dt} = \underbrace{\left(\frac{d\omega}{d\theta}\right)\left(\frac{d\theta}{dt}\right)}_{\text{chain rule}}$$

Constant angular acceleration

$\alpha = \frac{d\omega}{dt}$ but $\frac{d\omega}{dt} = \left(\frac{d\omega}{d\theta}\right)\left(\frac{d\theta}{dt}\right)$

$\& \frac{d\theta}{dt} = \omega$

Constant angular acceleration

$$\alpha = \frac{d\omega}{dt} \quad \text{but} \quad \frac{d\omega}{dt} = \left(\frac{d\omega}{d\theta}\right)\left(\frac{d\theta}{dt}\right)$$

$$\& \ \frac{d\theta}{dt} = \omega \quad \text{so} \quad \alpha = \omega \frac{d\omega}{d\theta}$$

![ASU logo]

Constant angular acceleration

$$\alpha = \frac{d\omega}{dt} \qquad \text{but} \quad \frac{d\omega}{dt} = \left(\frac{d\omega}{d\theta}\right)\left(\frac{d\theta}{dt}\right)$$

$$\text{\& } \frac{d\theta}{dt} = \omega \quad \text{so} \qquad \alpha = \omega \frac{d\omega}{d\theta} \Rightarrow$$

$$\int_{\theta_0}^{\theta} \alpha \, d\theta = \int_{\omega_0}^{\omega} \omega \, d\omega$$

Constant angular acceleration

$$\alpha = \frac{d\omega}{dt} \qquad \text{but} \quad \frac{d\omega}{dt} = \left(\frac{d\omega}{d\theta}\right)\left(\frac{d\theta}{dt}\right)$$

$$\text{\&} \quad \frac{d\theta}{dt} = \omega \quad \text{so} \quad \alpha = \omega \frac{d\omega}{d\theta} \Rightarrow$$

$$\int_{\theta_0}^{\theta} \alpha \, d\theta = \int_{\omega_0}^{\omega} \omega \, d\omega$$

Constant angular acceleration

$\alpha = \frac{d\omega}{dt}$ but $\frac{d\omega}{dt} = \left(\frac{d\omega}{d\theta}\right)\left(\frac{d\theta}{dt}\right)$

& $\frac{d\theta}{dt} = \omega$ so $\alpha = \omega \frac{d\omega}{d\theta} \Rightarrow$

$$\alpha \int_{\theta_0}^{\theta} d\theta = \int_{\omega_0}^{\omega} \omega \, d\omega$$

Constant angular acceleration

$$\alpha = \frac{d\omega}{dt} \quad \text{but} \quad \frac{d\omega}{dt} = \left(\frac{d\omega}{d\theta}\right)\left(\frac{d\theta}{dt}\right)$$

$$\text{\& } \frac{d\theta}{dt} = \omega \quad \text{so} \quad \alpha = \omega\frac{d\omega}{d\theta} \Rightarrow$$

$$\alpha\int_{\theta_0}^{\theta} d\theta = \int_{\omega_0}^{\omega} \omega\, d\omega \Rightarrow$$

$$\alpha\Delta\theta = \frac{1}{2}\left(\omega^2 - \omega_0^2\right)$$

Constant angular acceleration

$$\alpha = \frac{d\omega}{dt} \quad \text{but} \quad \frac{d\omega}{dt} = \left(\frac{d\omega}{d\theta}\right)\left(\frac{d\theta}{dt}\right)$$

$$\& \ \frac{d\theta}{dt} = \omega \quad \text{so} \quad \alpha = \omega \frac{d\omega}{d\theta} \Rightarrow$$

$$\alpha \int_{\theta_0}^{\theta} d\theta = \int_{\omega_0}^{\omega} \omega \, d\omega \Rightarrow$$

$$\alpha \Delta\theta = \tfrac{1}{2}\left(\omega^2 - \omega_0^2\right) \Rightarrow$$

$$\boxed{\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)}$$

Constant angular acceleration

$\alpha = \frac{d\omega}{dt}$ but $\frac{d\omega}{dt} = \left(\frac{d\omega}{d\theta}\right)\left(\frac{d\theta}{dt}\right)$

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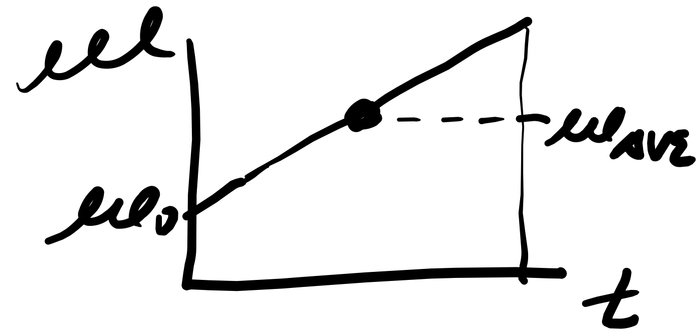
$$\boxed{\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)}$$

compare to

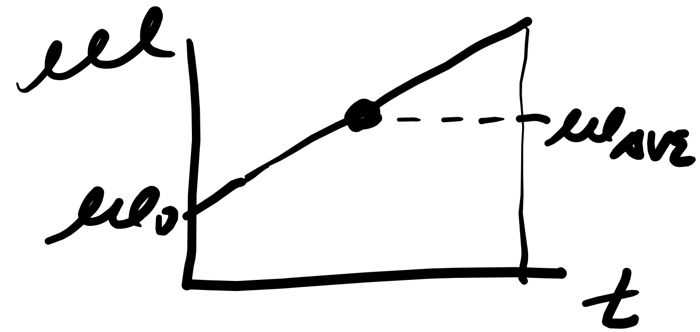
$$v^2 = v_0^2 + 2a(x - x_0)$$

Constant angular acceleration



Constant angular acceleration

$$\omega_{AVE} = \frac{1}{2}(\omega + \omega_0)$$



Constant angular acceleration

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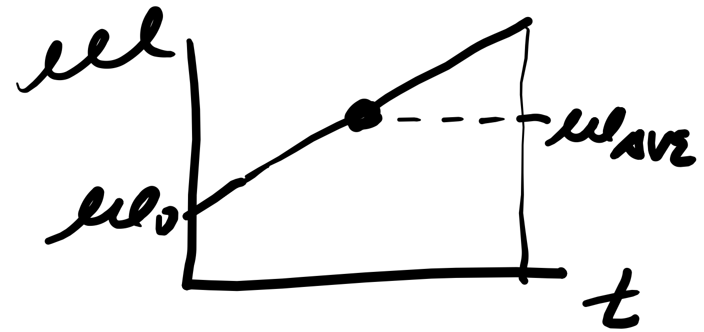
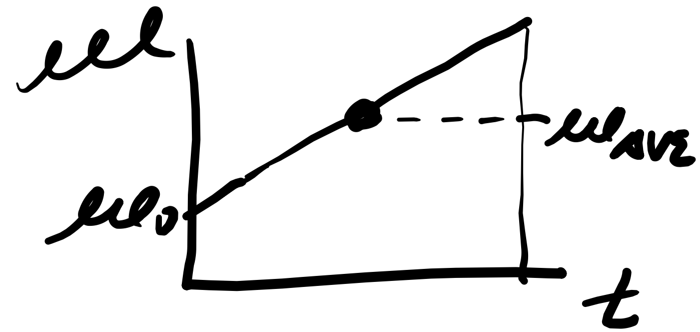


Also

$$\omega_{AVE} = \frac{\Delta\theta}{\Delta t}$$

Constant angular acceleration

$$\omega_{AVE} = \frac{1}{2}(\omega + \omega_0)$$

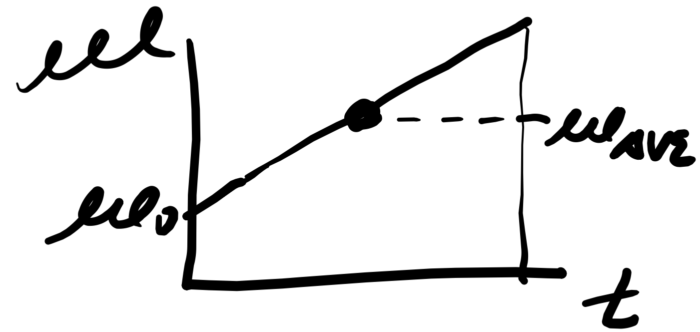


Also

$$\omega_{AVE} = \frac{\Delta\theta}{\Delta t} = \frac{\theta - \theta_0}{t - 0}$$

Constant angular acceleration

$$\omega_{AVE} = \frac{1}{2}(\omega + \omega_0)$$



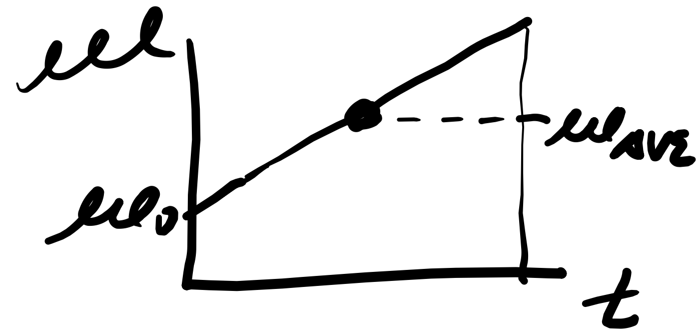
Also

$$\omega_{AVE} = \frac{\Delta\theta}{\Delta t} = \frac{\theta - \theta_0}{t - 0}$$ So

$$\frac{1}{2}(\omega + \omega_0) = (\theta - \theta_0)/t$$

Constant angular acceleration

$$\omega_{AVE} = \frac{1}{2}(\omega + \omega_0)$$



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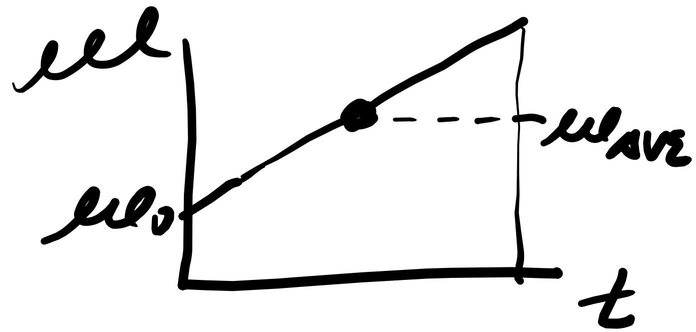
$$\frac{1}{2}(\omega + \omega_0) = (\theta - \theta_0)/t \quad \text{So}$$

$$\boxed{(\theta - \theta_0) = \frac{1}{2}(\omega + \omega_0)t}$$

Constant angular acceleration

$$\omega_{AVE} = \frac{1}{2}(\omega + \omega_0)$$



Also

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$$\frac{1}{2}(\omega + \omega_0) = (\theta - \theta_0)/t \quad \text{So}$$

$$\boxed{(\theta - \theta_0) = \frac{1}{2}(\omega + \omega_0)t}$$

Compare to $(x - x_0) = \frac{1}{2}(v + v_0)t$

Constant angular acceleration

Straight-Line Motion with Constant Linear Acceleration		Fixed-Axis Rotation with Constant Angular Acceleration	
$a_x = \text{constant}$		$\alpha_z = \text{constant}$	
$v_x = v_{0x} + a_x t$	(2.8)	$\omega_z = \omega_{0z} + \alpha_z t$	(9.7)
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	(2.12)	$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$	(9.11)
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	(2.13)	$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$	(9.12)
$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$	(2.14)	$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t$	(9.10)

You have finished watching a movie on Blu-ray and the disc is slowing to a stop. The disc's angular velocity at $t = 0$ is 27.5 rad/s, and its angular acceleration is a constant -10.0 rad/s^2. A line PQ on the disc's surface lies along the $+x$-axis at $t = 0$ (Fig. 9.8). (a) What is the disc's angular velocity at $t = 0.300 \text{ s}$? (b) What angle does the line PQ make with the $+x$-axis at this time?

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$\omega_0 = 27.5 \text{ rad/s}$

You have finished watching a movie on Blu-ray and the disc is slowing to a stop. The disc's angular velocity at $t = 0$ is $27.5\ \mathrm{rad/s}$, and its angular acceleration is a constant $-10.0\ \mathrm{rad/s^2}$. A line PQ on the disc's surface lies along the $+x$-axis at $t = 0$ (Fig. 9.8). (a) What is the disc's angular velocity at $t = 0.300$ s? (b) What angle does the line PQ make with the $+x$-axis at this time?

$\omega_0 = 27.5\ \mathrm{rad/s}$, $\alpha = -10\ \mathrm{rad/s^2}$

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$\omega_0 = 27.5\ \mathrm{rad/s}, \alpha = -10\ \mathrm{rad/s^2}$

$\omega = \alpha t + \omega_0$

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$\omega_0 = 27.5\ \mathrm{rad/s}, \ \alpha = -10\ \mathrm{rad/s^2}$

$\omega = \alpha t + \omega_0 = [-10 * 0.3 + 27.5]\ \mathrm{rad/s}$

You have finished watching a movie on Blu-ray and the disc is slowing to a stop. The disc's angular velocity at $t = 0$ is $27.5\ \mathrm{rad/s}$, and its angular acceleration is a constant $-10.0\ \mathrm{rad/s^2}$. A line PQ on the disc's surface lies along the $+x$-axis at $t = 0$ (Fig. 9.8). (a) What is the disc's angular velocity at $t = 0.300$ s? (b) What angle does the line PQ make with the $+x$-axis at this time?

$\omega_0 = 27.5\ \mathrm{rad/s},\ \alpha = -10\ \mathrm{rad/s^2}$

$$\omega = \alpha t + \omega_0 = [-10 * 0.3 + 27.5]\ \mathrm{rad/s} = 24.5\ \frac{\mathrm{rad}}{\mathrm{s}}$$

You have finished watching a movie on Blu-ray and the disc is slowing to a stop. The disc's angular velocity at $t = 0$ is 27.5 rad/s, and its angular acceleration is a constant -10.0 rad/s^2. A line PQ on the disc's surface lies along the $+x$-axis at $t = 0$ (Fig. 9.8). (a) What is the disc's angular velocity at $t = 0.300$ s? (b) What angle does the line PQ make with the $+x$-axis at this time?

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$$\omega = \alpha t + \omega_0 = [-10 * 0.3 + 27.5]\ \mathrm{rad/s} = 24.5\ \frac{\mathrm{rad}}{\mathrm{s}}$$

$$\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$$

ASU

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$\omega_0 = 27.5\,\mathrm{rad/s}$, $\alpha = -10\,\mathrm{rad/s^2}$

$$\omega = \alpha t + \omega_0 = [-10 * 0.3 + 27.5]\,\mathrm{rad/s} = 24.5\,\frac{\mathrm{rad}}{\mathrm{s}}$$

$$\theta = \tfrac{1}{2}\alpha t^2 + \omega_0 t + \cancel{\theta_0}\ 0$$

You have finished watching a movie on Blu-ray and the disc is slowing to a stop. The disc's angular velocity at $t = 0$ is 27.5 rad/s, and its angular acceleration is a constant $-10.0\ \mathrm{rad/s^2}$. A line PQ on the disc's surface lies along the $+x$-axis at $t = 0$ (Fig. 9.8). (a) What is the disc's angular velocity at $t = 0.300$ s? (b) What angle does the line PQ make with the $+x$-axis at this time?

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$$\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \cancel{\theta_0} \Rightarrow$$

$$\theta(0.3s) = \left[\left(-\frac{10}{2}\right)0.3^2 + 27.5 * 0.3\right]\mathrm{rad}$$

You have finished watching a movie on Blu-ray and the disc is slowing to a stop. The disc's angular velocity at $t = 0$ is 27.5 rad/s, and its angular acceleration is a constant $-10.0\ \mathrm{rad/s^2}$. A line PQ on the disc's surface lies along the $+x$-axis at $t = 0$ (Fig. 9.8). (a) What is the disc's angular velocity at $t = 0.300$ s? (b) What angle does the line PQ make with the $+x$-axis at this time?

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$$\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0 \Rightarrow$$

$\theta_0 = 0$

$$\theta(0.3s) = \left[\left(-\frac{10}{2}\right)0.3^2 + 27.5 * 0.3\right]\mathrm{rad} = 7.8\ \mathrm{rad}$$