$\boxed{L26}$

Today 8.5, 8.6

L26

Today 8.5, 8.6

Center of mass

L26

Today 8.5, 8.6

$\underbrace{8.5}_{\text{Center of mass}}$, $\underbrace{8.6}_{\text{Rocket propulsion}}$

Center of mass

Rocket propulsion

$\boxed{L26}$

Today 8.5, 8.6

Wednesday 9.1, 9.2

$\boxed{L26}$

Today 8.5, 8.6

Wednesday $\underbrace{9.1}$, 9.2

Angular velocity and acceleration

$\boxed{L26}$

Today 8.5, 8.6

Wednesday 9.1, 9.2

9.1 → Angular velocity and acceleration

9.2 → Rotation with constant angular acceleration

$\boxed{L26}$

Today 8.5, 8.6

Wednesday 9.1, 9.2

SI session :

Today 5pm to 6pm

L26

Today 8.5, 8.6

Wednesday 9.1, 9.2

SI session :

Today 5pm to 6pm

* §8.3: Momentum conservation & collisions

$\boxed{L26}$

Today 8.5, 8.6

Wednesday 9.1, 9.2

SI session :

Today 5pm to 6pm

- * §8.3: Momentum conservation & collisions
- * §8.4: Elastic collisions

Center of mass

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

Center of mass

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} \quad \& \quad y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$$

Center of mass

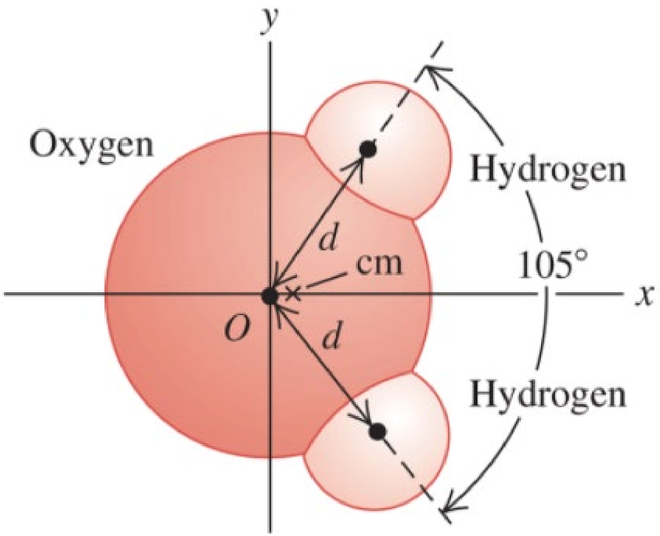
$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$ & $y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$ & $z_{cm} = \frac{\sum m_i z_i}{\sum m_i}$

Center of mass

$X_{cm} = \frac{\sum m_i x_i}{\sum m_i}$ \& $y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$ \& $z_{cm} = \frac{\sum m_i z_i}{\sum m_i}$

OR $\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$

Figure 8.28 shows a simple model of a water molecule. The oxygen–hydrogen separation is $d = 9.57 \times 10^{-11}$ m. Each hydrogen atom has mass 1.0 u, and the oxygen atom has mass 16.0 u. Find the position of the center of mass.



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$$\vec{r}_1 = d[\hat{i}\cos\theta + \hat{j}\cos\theta]$$

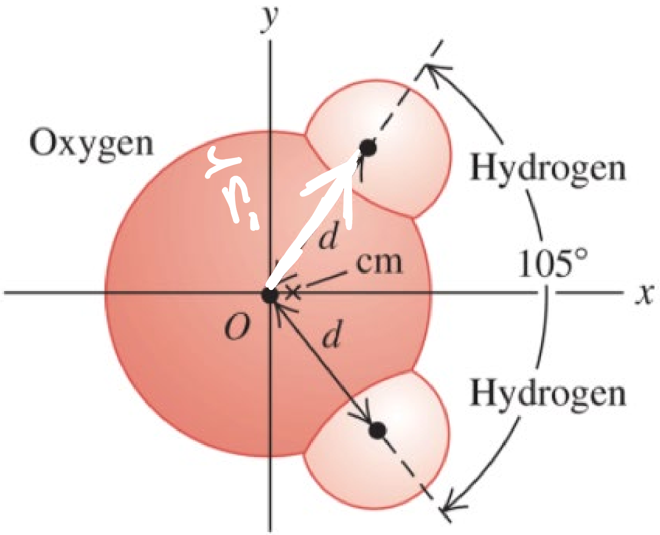

Figure 8.28 shows a simple model of a water molecule. The oxygen–hydrogen separation is $d = 9.57 \times 10^{-11}$ m. Each hydrogen atom has mass 1.0 u, and the oxygen atom has mass 16.0 u. Find the position of the center of mass. $\theta = 105°/2 = 52.5°$

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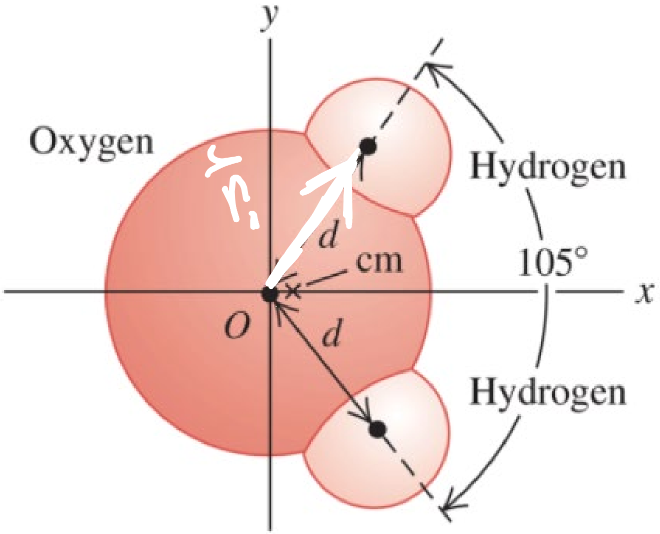

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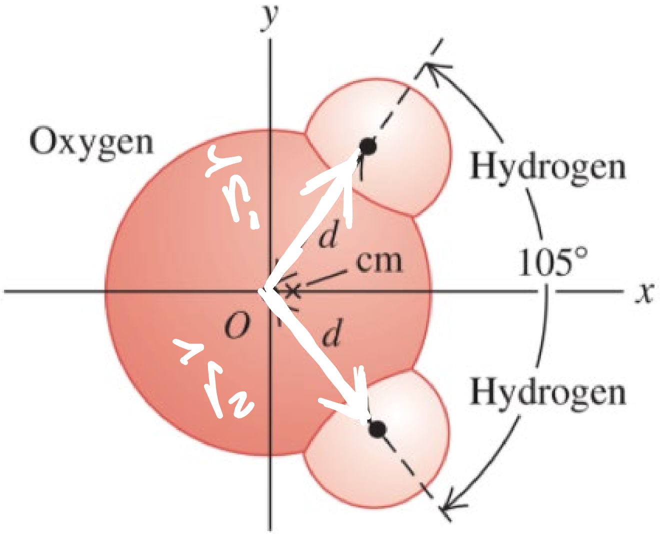

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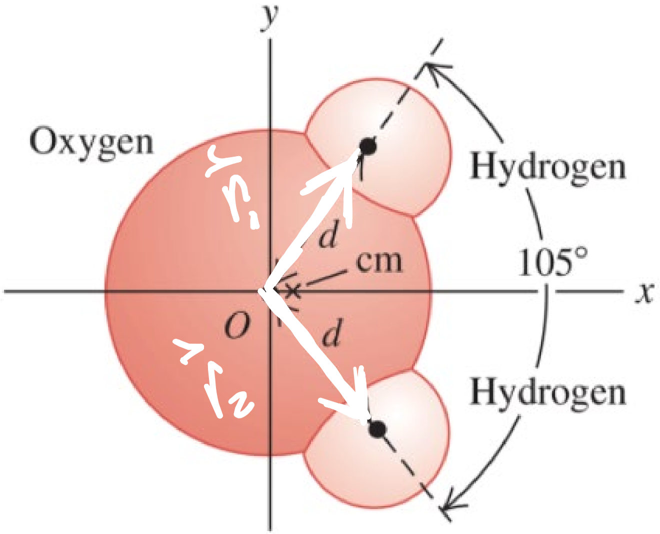
$$\vec{r}_1 = d[\hat{\imath}\cos\theta + \hat{\jmath}\cos\theta]$$

$$\vec{r}_2 = d[\hat{\imath}\cos\theta - \hat{\jmath}\cos\theta] \quad \text{so}$$

$$\vec{r}_{cm} = \frac{M_O\vec{r}_O + M_H\vec{r}_1 + M_H\vec{r}_2}{M_O + M_H + M_H}$$



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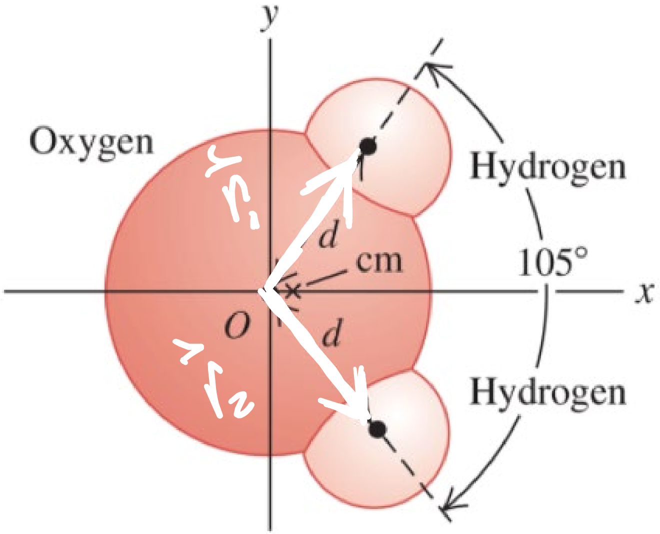

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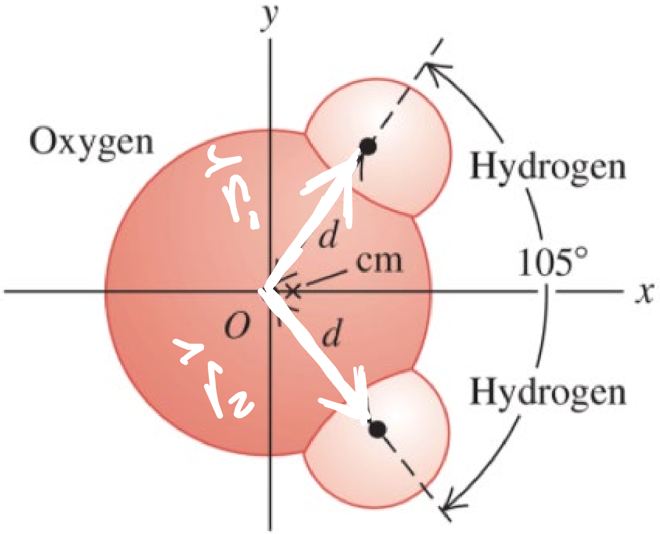
$$\vec{r}_{cm} = \frac{\overset{0}{\cancel{M_O\vec{r}_O}} + M_H\vec{r}_1 + M_H\vec{r}_2}{M_O + M_H + M_H} \Rightarrow$$

$$\vec{r}_{cm} = \left(\frac{M_H d}{M_O + 2M_H}\right)(2\hat{\imath}\cos\theta)$$



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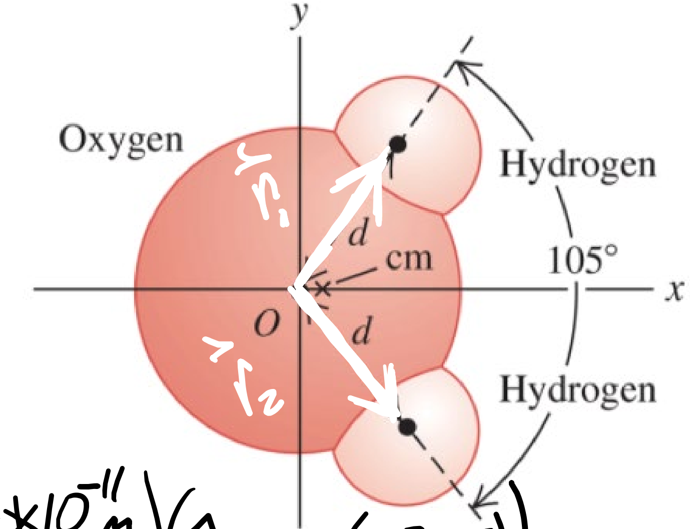

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$$\vec{r}_{cm} = \frac{\cancelto{0}{M_O\vec{r}_O} + M_H\vec{r}_1 + M_H\vec{r}_2}{M_O + M_H + M_H} \Rightarrow$$

$$\vec{r}_{cm} = \left(\frac{M_H d}{M_O + 2M_H}\right)(2\hat{i}\cos\theta) = \left(\frac{2 * 9.57 * 10^{-11}\,m}{18}\right)\left(\hat{i}\cos(52.5°)\right)$$

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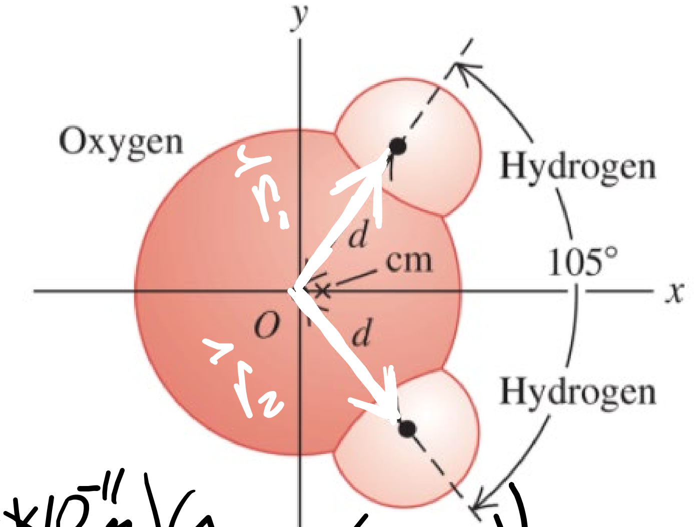

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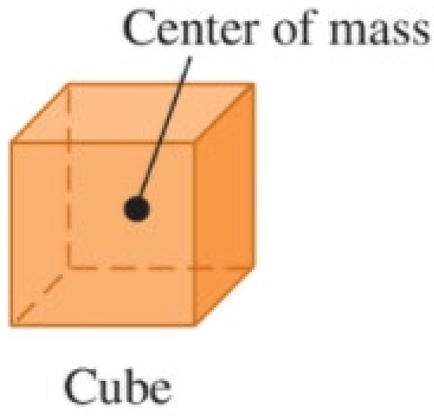
$$\Rightarrow \quad \vec{r}_{cm} = \hat{\imath}\, 6.47 * 10^{-12}\, m$$

If a homogeneous object has a geometric center, that is where the center of mass is located.



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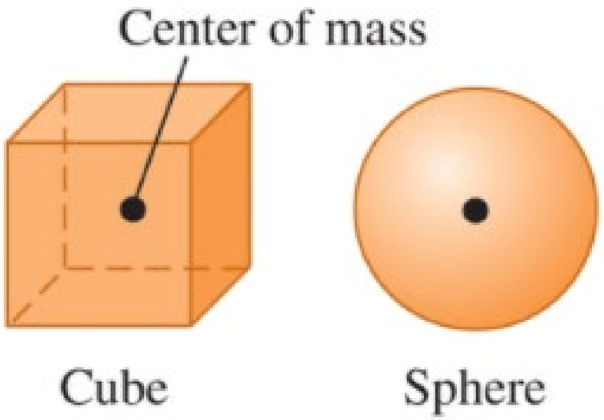

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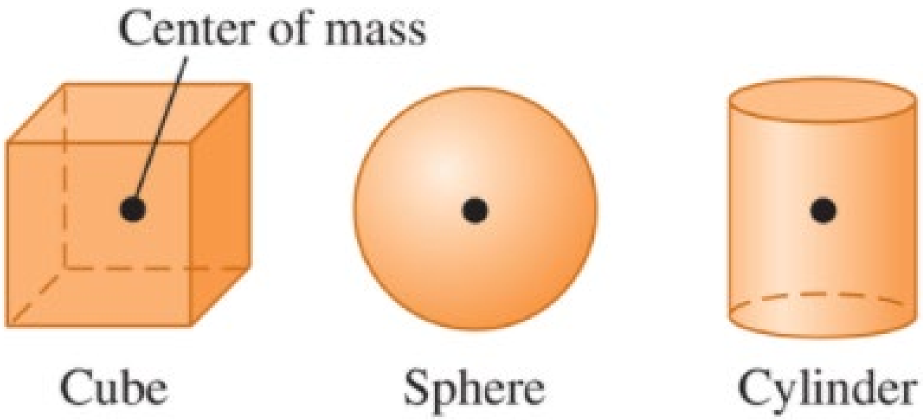

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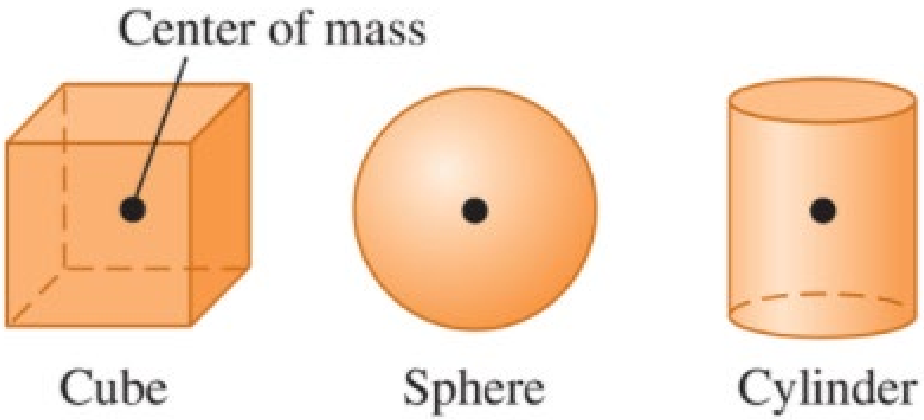
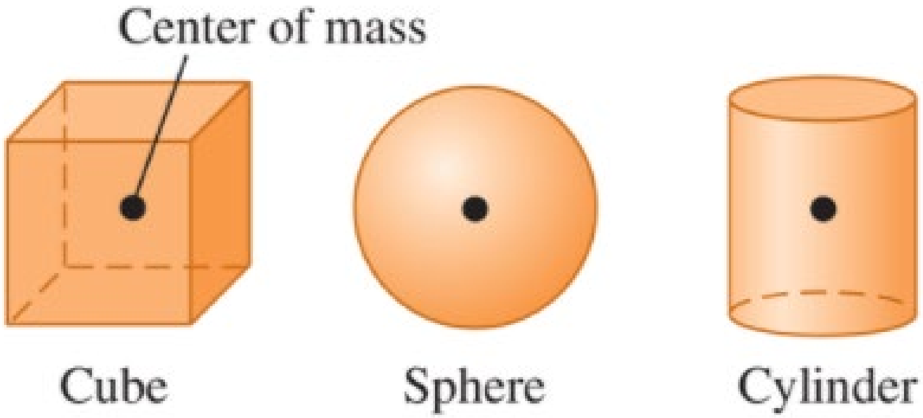


If a homogeneous object has a geometric center, that is where the center of mass is located.

If an object has an axis of symmetry, the center of mass lies along it.



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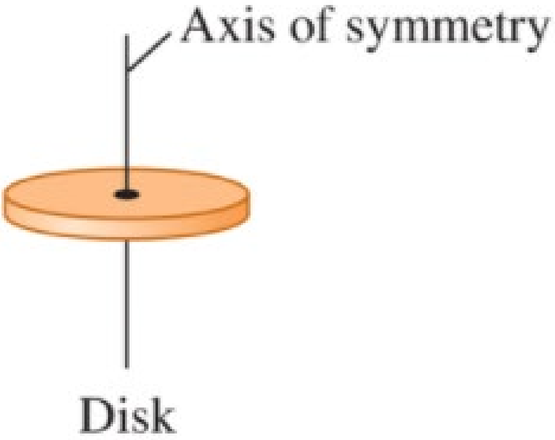

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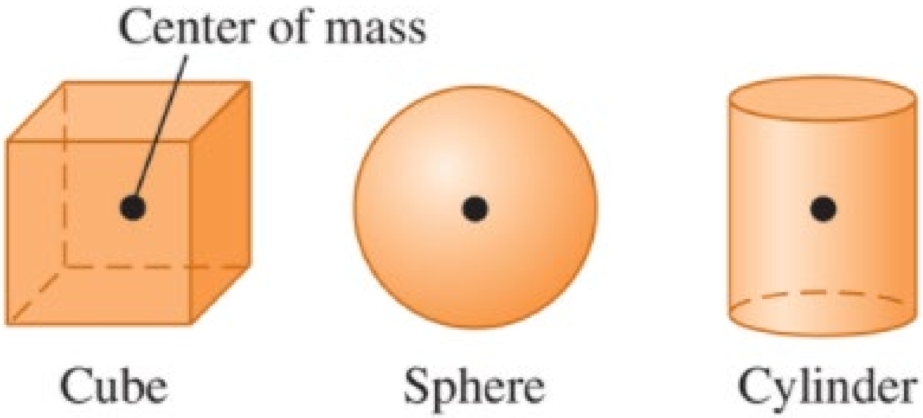

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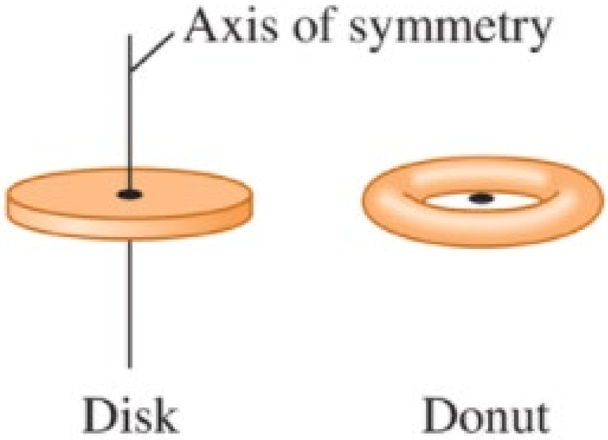

If an object has an axis of symmetry, the center of mass lies along it. As in the case of the donut, the center of mass may not be within the object.

Motion of center of mass

Motion of center of mass

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

Motion of center of mass

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} \Rightarrow \frac{d\vec{r}_{cm}}{dt} = \frac{d}{dt}\left\{\frac{\sum m_i \vec{r}_i}{\sum m_i}\right\}$$

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$$\Rightarrow \vec{V}_{cm} = \frac{\sum m_i \vec{V}_i}{\sum m_i}$$

Motion of center of mass

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$$\Rightarrow \vec{V}_{cm} = \frac{\sum m_i \vec{V}_i}{\sum m_i}$$

Let $M \equiv \sum m_i$

Motion of center of mass

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$$\Rightarrow M\vec{V}_{cm} = \sum m_i \vec{V}_i$$

Motion of center of mass

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$$\Rightarrow M\vec{V}_{cm} = \sum m_i \vec{V}_i = \vec{P}$$

Motion of center of mass

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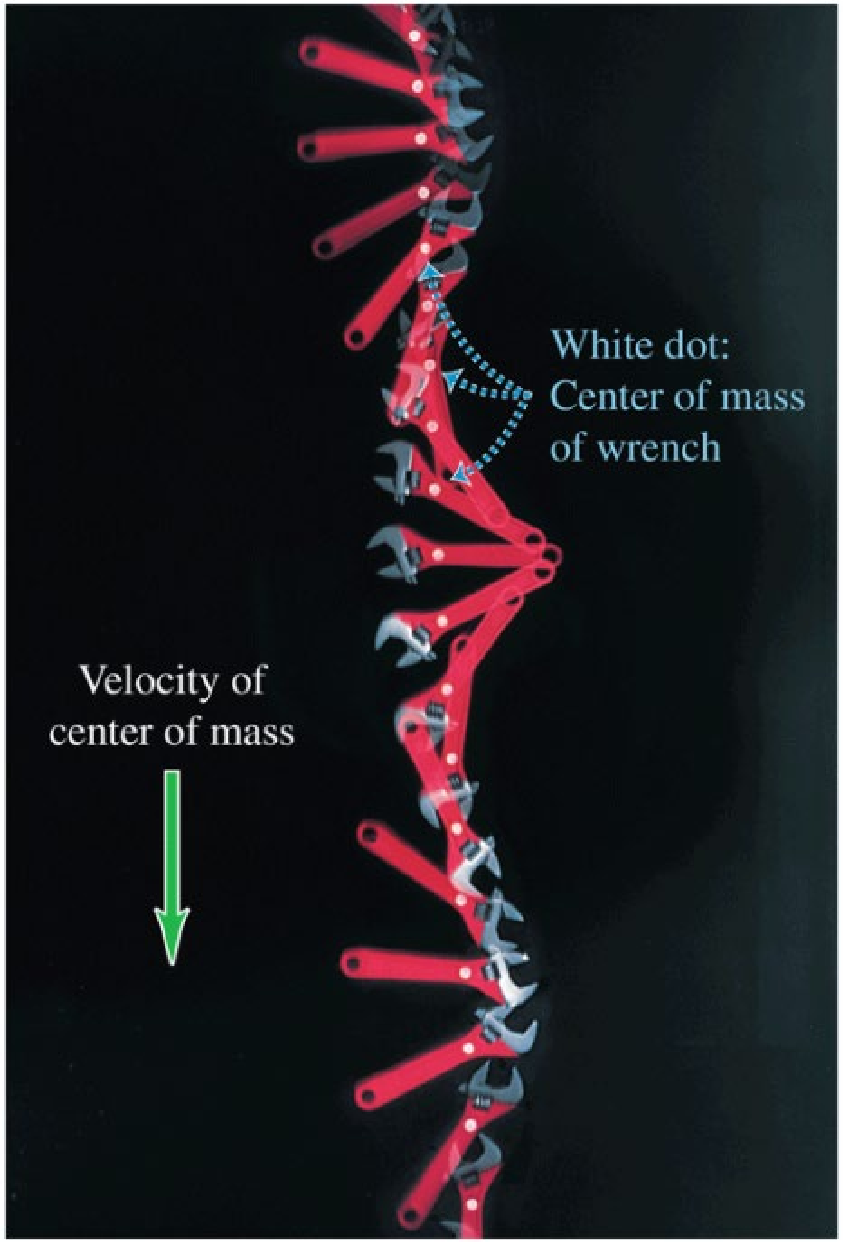
$$\Rightarrow \vec{V}_{cm} = \frac{\sum m_i \vec{V}_i}{\sum m_i}$$

Let $M \equiv \sum m_i$

$$\Rightarrow M\vec{V}_{cm} = \sum m_i \vec{V}_i = \underbrace{\vec{P}}_{\text{Total momentum of system}}$$

Motion of center of mass



External forces & cm motion

Since $\vec{a} = d\vec{v}/dt$

External forces & cm motion

Since $\vec{a} = d\vec{v}/dt$ then

$$M\vec{a}_{cm} = \sum m_i \vec{a}_i$$

External forces & cm motion

Since $\vec{a} = d\vec{v}/dt$ then

$M\vec{a}_{cm} = \sum m_i \vec{a}_i$ & since $\vec{F} = m\vec{a}$

External forces & cm motion

Since $\vec{a} = d\vec{v}/dt$ then

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then $\sum \vec{F} = \sum m_i \vec{a}$

External forces & cm motion

Since $\vec{a} = d\vec{v}/dt$ then

$M\vec{a}_{cm} = \sum m_i \vec{a}_i$ & since $\vec{F} = m\vec{a}$

then $\sum \vec{F} = \sum m_i \vec{a} \Rightarrow$ $\sum \vec{F} = M\vec{a}_{cm}$

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Since $\vec{a} = d\vec{v}/dt$ then

$M\vec{a}_{cm} = \sum m_i \vec{a}_i$ & since $\vec{F} = m\vec{a}$

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Let $\sum \vec{F} = \sum \vec{F}_{ext} + \sum \vec{F}_{int}$

External forces & cm motion

Since $\vec{a} = d\vec{v}/dt$ then

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Let $\sum \vec{F} = \sum \vec{F}_{ext} + \sum \vec{F}_{int}$, but Newton's 3rd law says that all internal force pairs are equal & opposite

External forces & cm motion

Since $\vec{a} = d\vec{v}/dt$ then

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External forces & cm motion

Since $\vec{a} = d\vec{v}/dt$ then

$M\vec{a}_{cm} = \Sigma m_i \vec{a}_i$ & since $\vec{F} = m\vec{a}$

then $\Sigma \vec{F} = \Sigma m_i \vec{a} \Rightarrow \Sigma \vec{F} = M\vec{a}_{cm}$

Let $\Sigma \vec{F} = \Sigma \vec{F}_{ext} + \overset{0}{\cancel{\Sigma \vec{F}_{int}}}$, but Newton's 3rd law says that all internal force pairs are equal & opposite $\Rightarrow \Sigma F_{int} = 0$

So $\Sigma \vec{F}_{ext} = M\vec{a}_{cm}$

External forces & cm motion

Since $\vec{a} = d\vec{v}/dt$ then

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So $\sum \vec{F}_{ext} = M\vec{a}_{cm}$ or $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$

External forces & cm motion

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So $\sum \vec{F}_{ext} = M\vec{a}_{cm}$ or $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$

thus $\sum F_{ext} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0$

External forces & cm motion

Since $\vec{a} = d\vec{v}/dt$ then

$M\vec{a}_{cm} = \sum m_i \vec{a}_i$ & since $\vec{F} = m\vec{a}$

then $\sum \vec{F} = \sum m_i \vec{a} \Rightarrow$ $\sum \vec{F} = M\vec{a}_{cm}$

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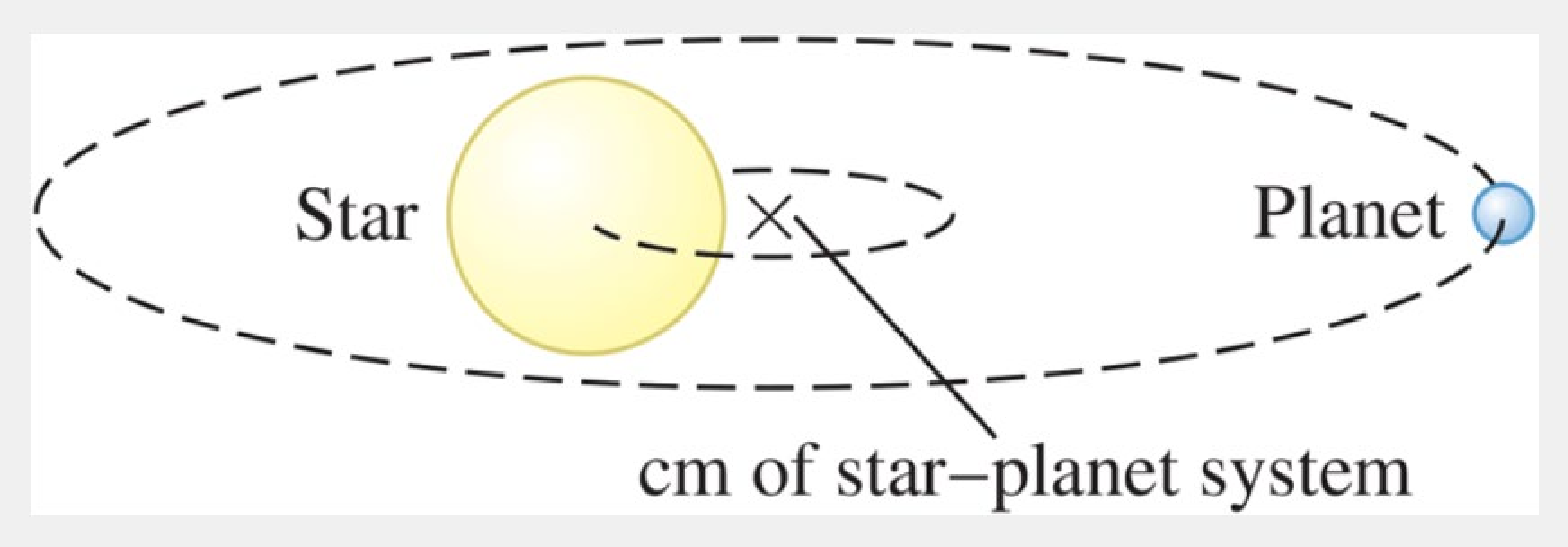
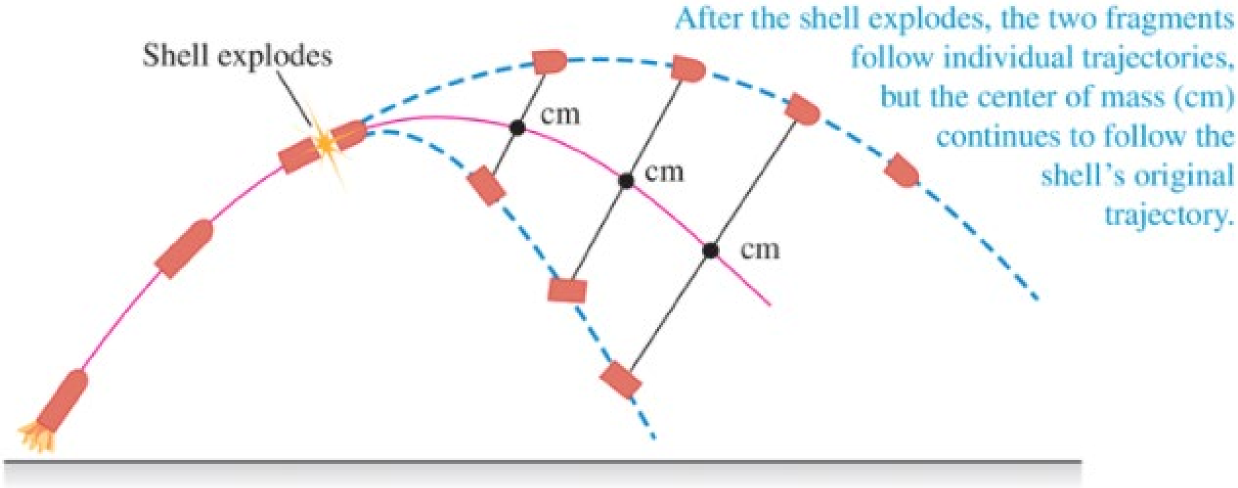
So $\sum \vec{F}_{ext} = M\vec{a}_{cm}$ or $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$

thus $\sum F_{ext} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0$ & $\vec{r}_{cm} = \text{const.}$


Star
Planet
cm of star–planet system



A shell explodes into two fragments in flight. If air resistance is ignored, the center of mass continues on the same trajectory as the shell's path before the explosion.

James (mass 90.0 kg) and Ramon (mass 60.0 kg) are 20.0 m apart on a frozen pond. Midway between them is a mug of their favorite beverage. They pull on the ends of a light rope stretched between them. When James has moved 6.0 m toward the mug, how far and in what direction has Ramon moved?

James (mass 90.0 kg) and Ramon (mass 60.0 kg) are 20.0 m apart on a frozen pond. Midway between them is a mug of their favorite beverage. They pull on the ends of a light rope stretched between them. When James has moved 6.0 m toward the mug, how far and in what direction has Ramon moved?

$M_J = 90 \text{ kg}$

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$M_J = 90\,kg,\ M_R = 60\,kg$

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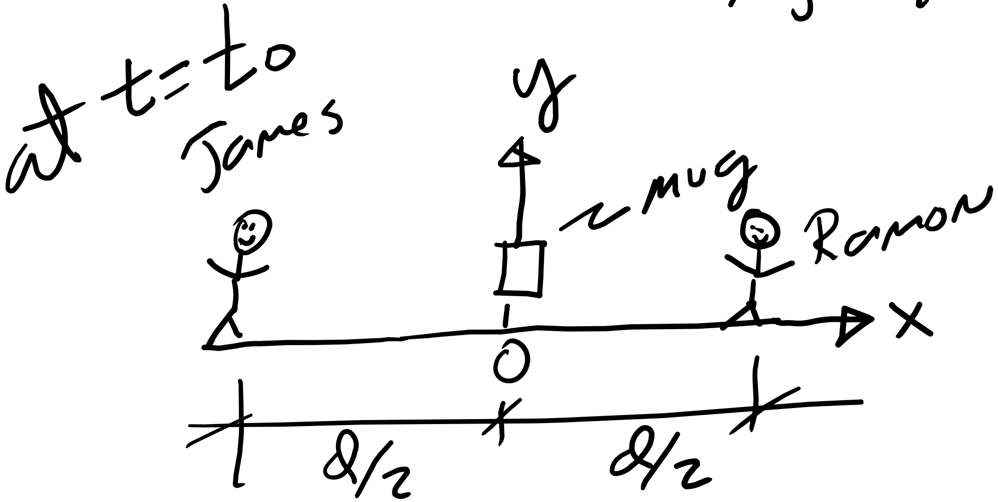

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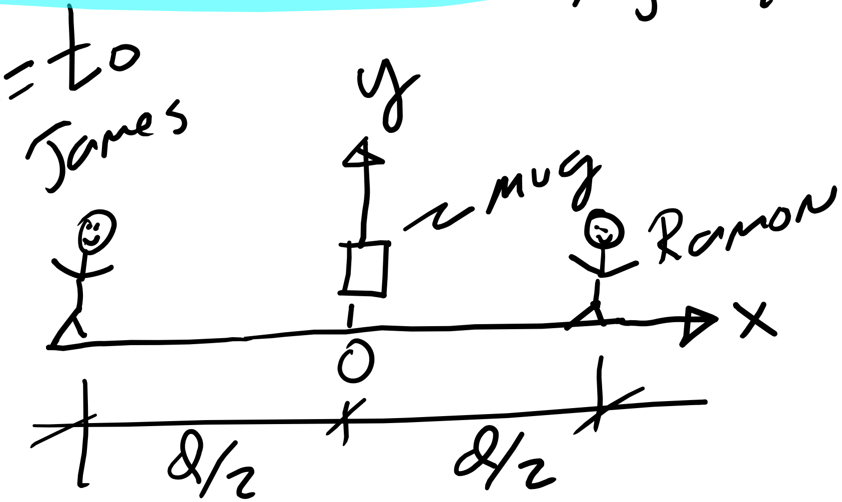
$M_J = 90\,kg,\ M_R = 60\,kg,\ d = 20\,m$

at $t = t_0$



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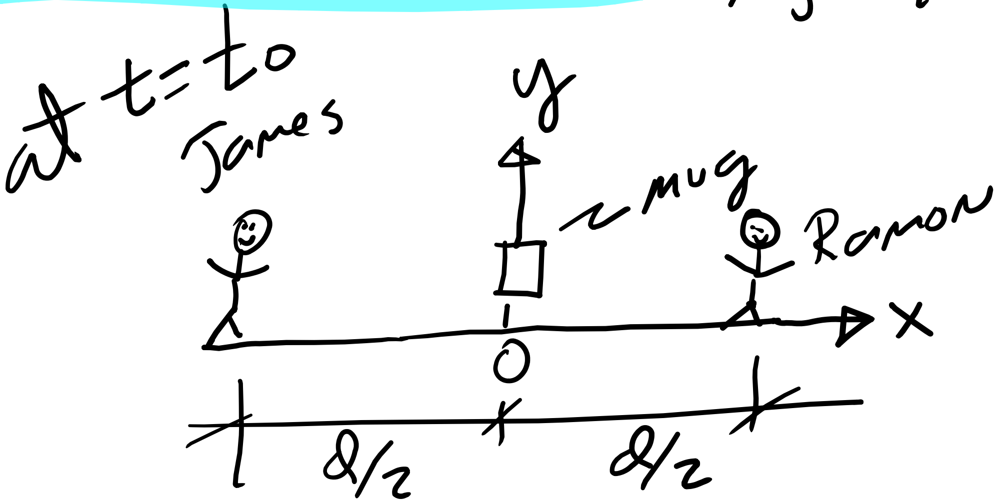

Note: No external forces $\Rightarrow \vec{r}_{cm} = \text{const.}$

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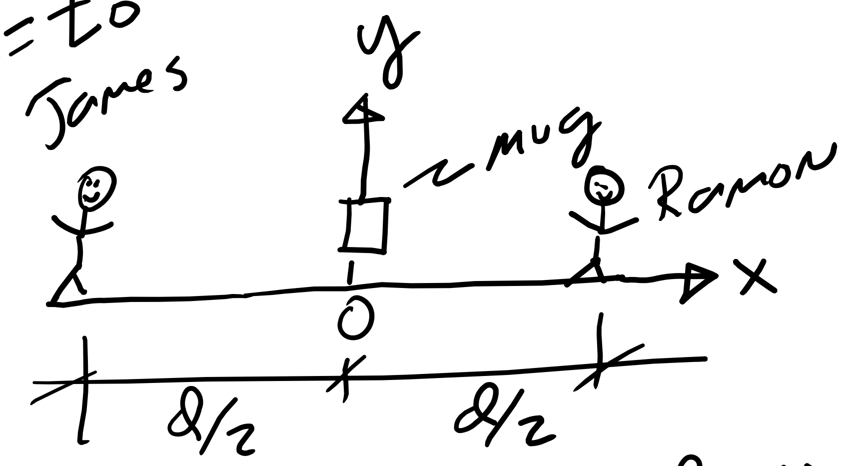

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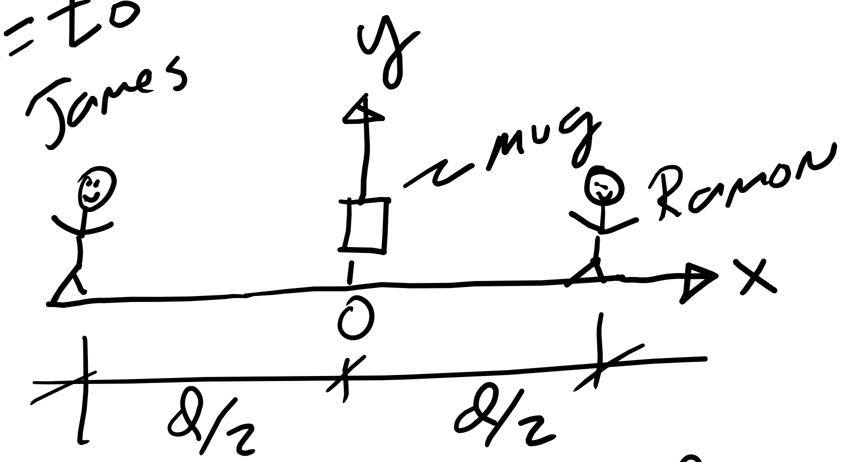

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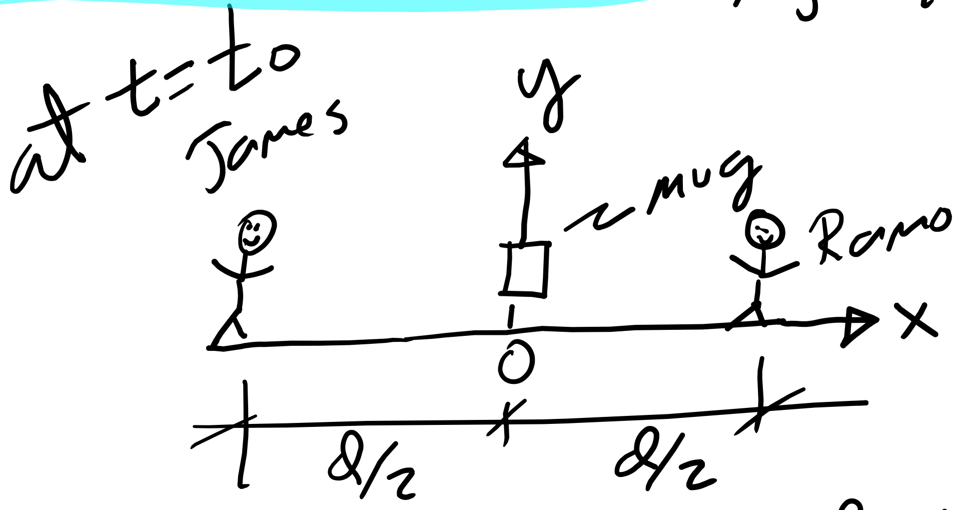

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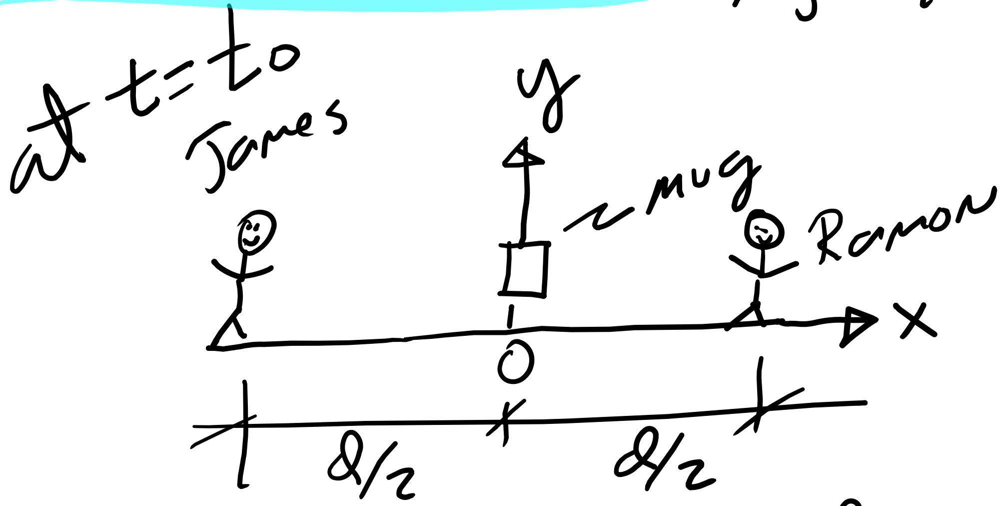

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$\frac{d}{2} - 6m$

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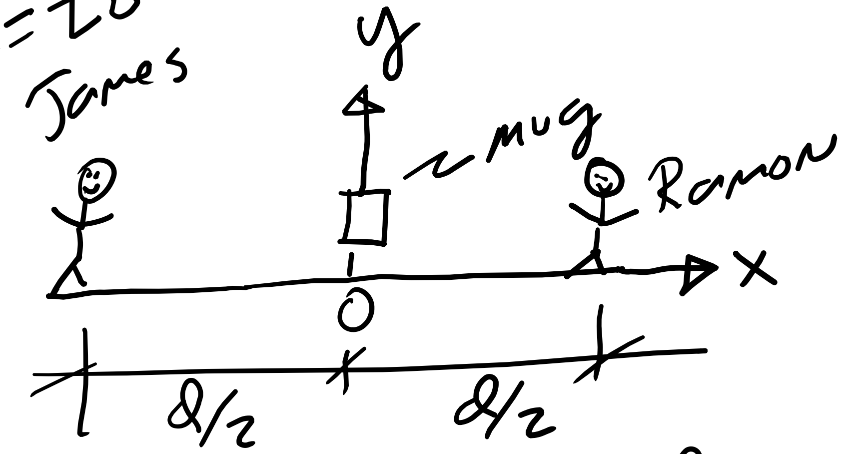

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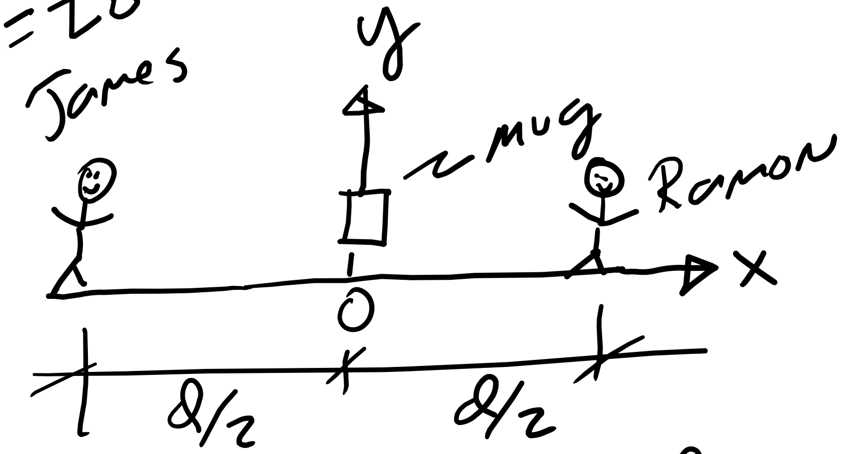

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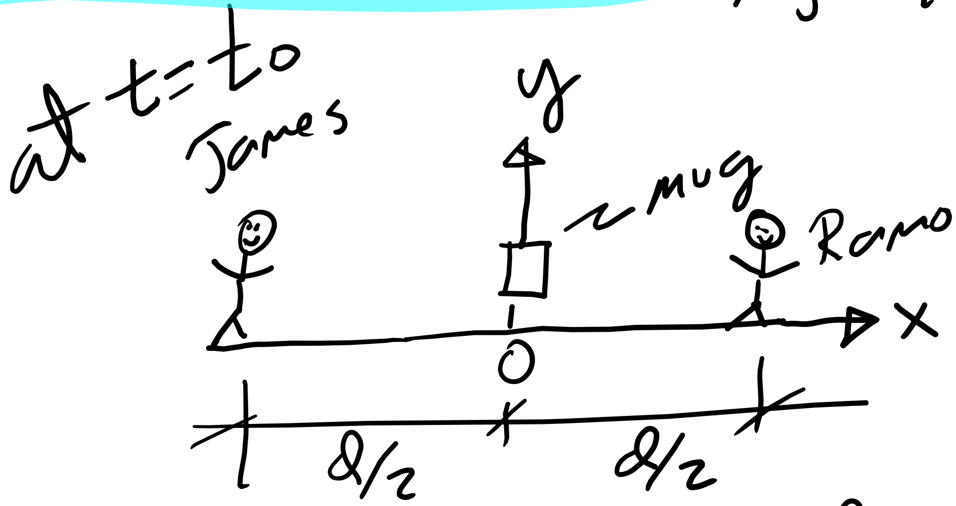

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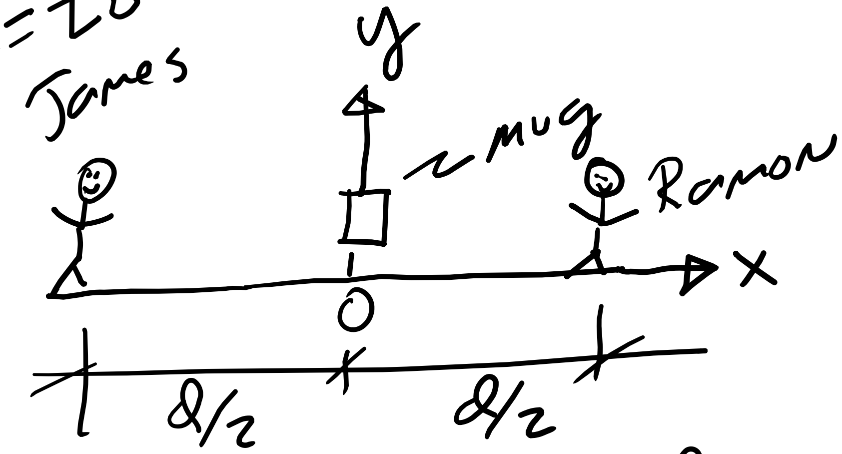

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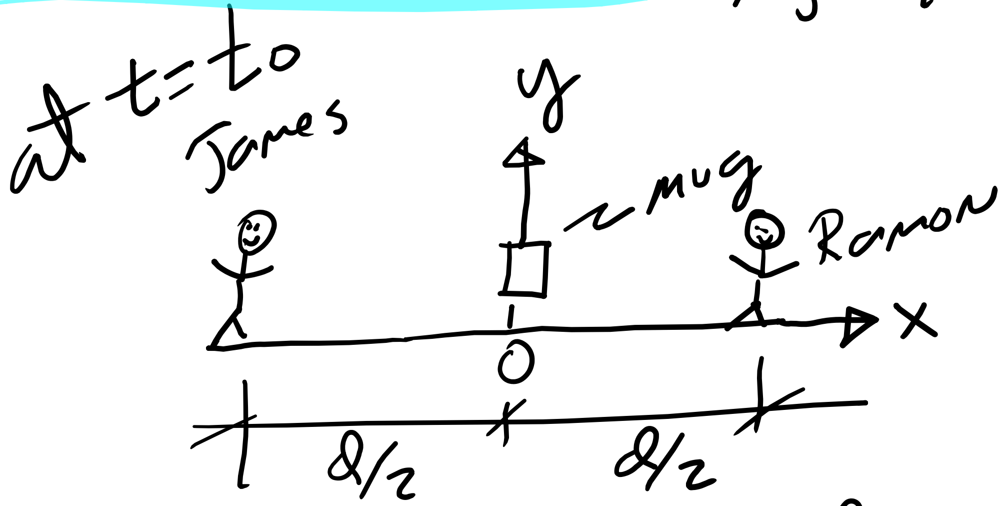

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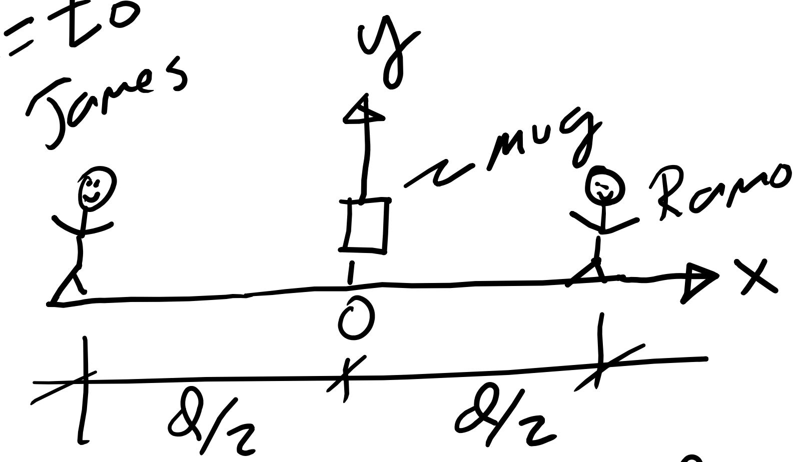

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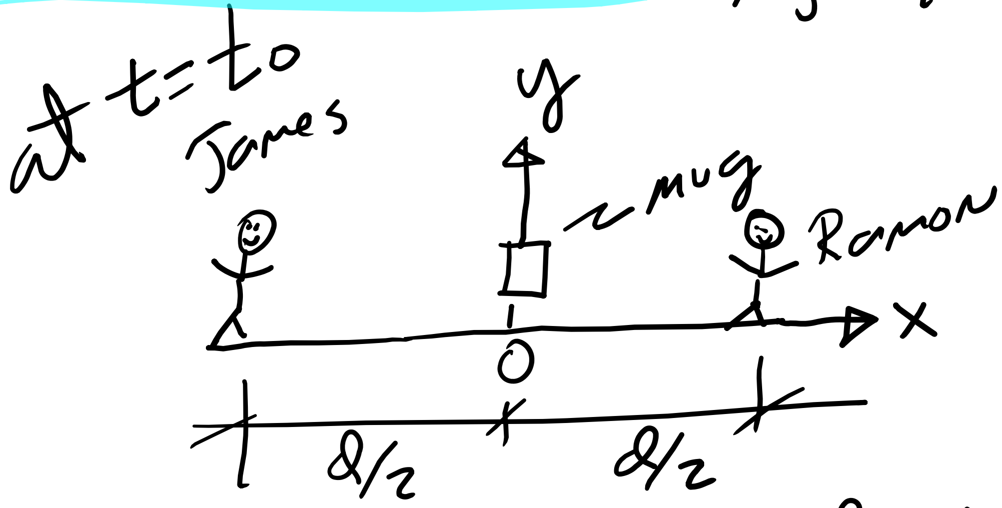

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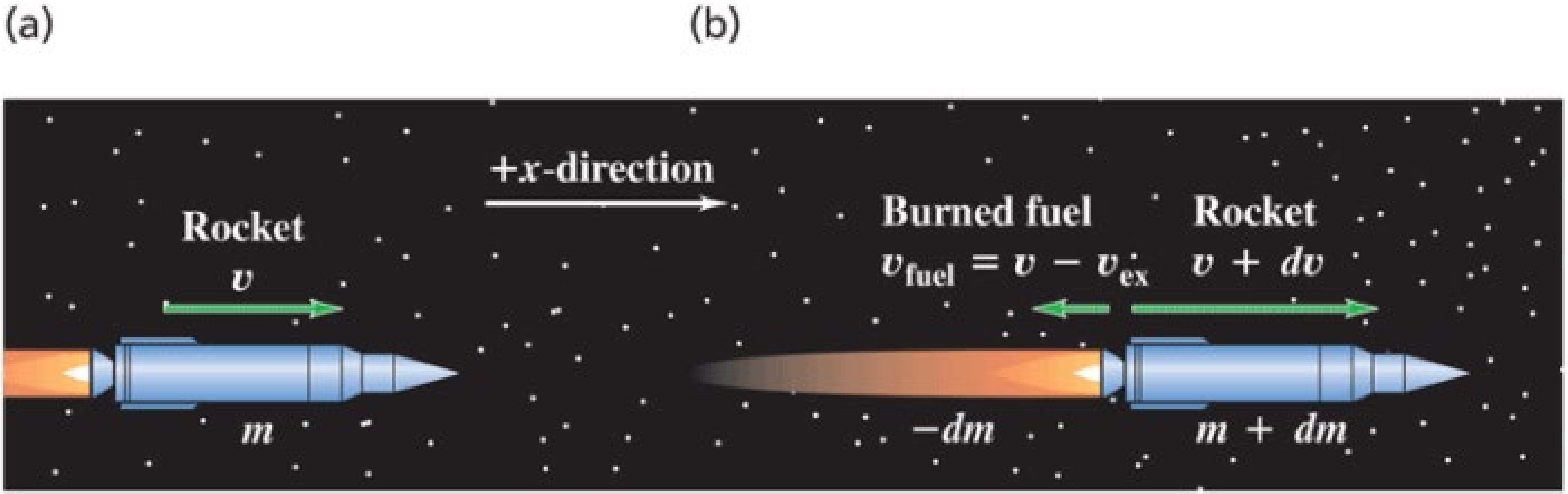
Rocket propulsion

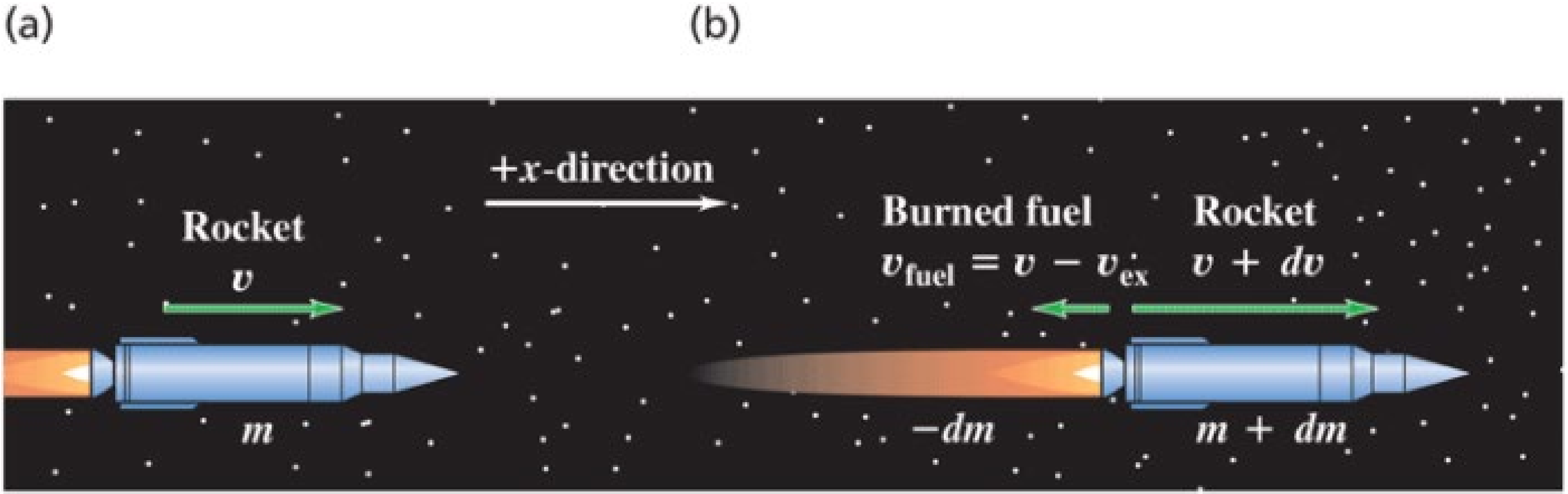
Rocket propulsion



At time t, the rocket has mass m and x-component of velocity v.

At time $t + dt$, the rocket has mass $m + dm$ (where dm is inherently *negative*) and x-component of velocity $v + dv$. The burned fuel has x-component of velocity $v_{\text{fuel}} = v - v_{\text{ex}}$ and mass $-dm$. (The minus sign is needed to make *$-dm$ positive* because dm is negative.)

Rocket propulsion



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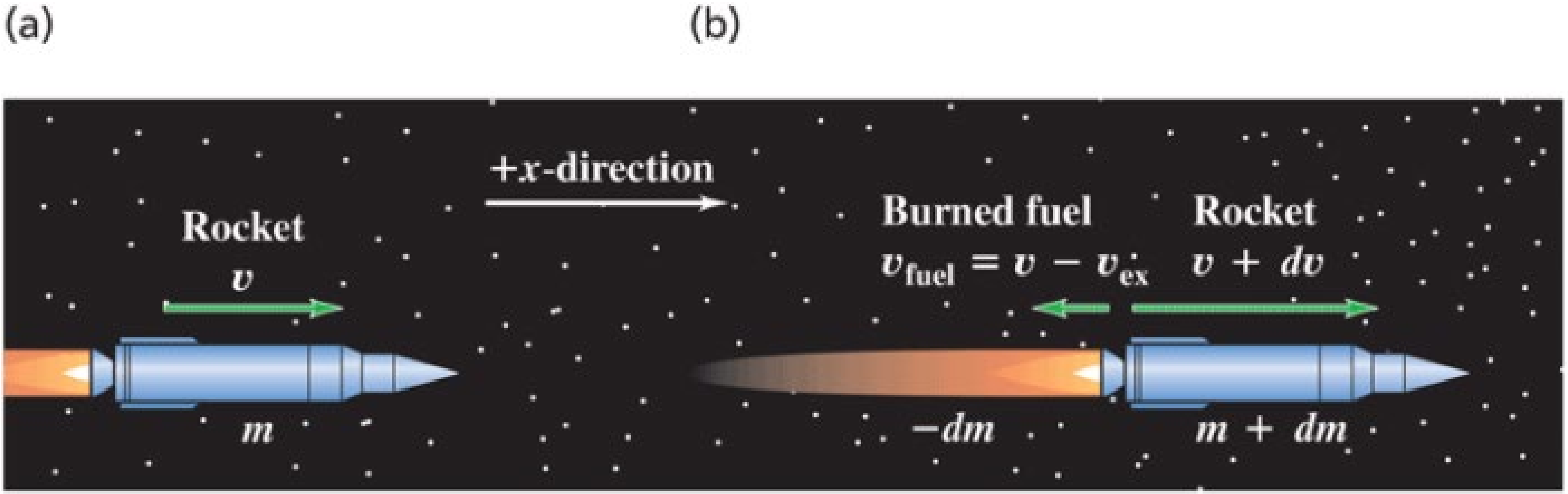
$$P_R = (m+dm)(V+dV)$$

Rocket propulsion



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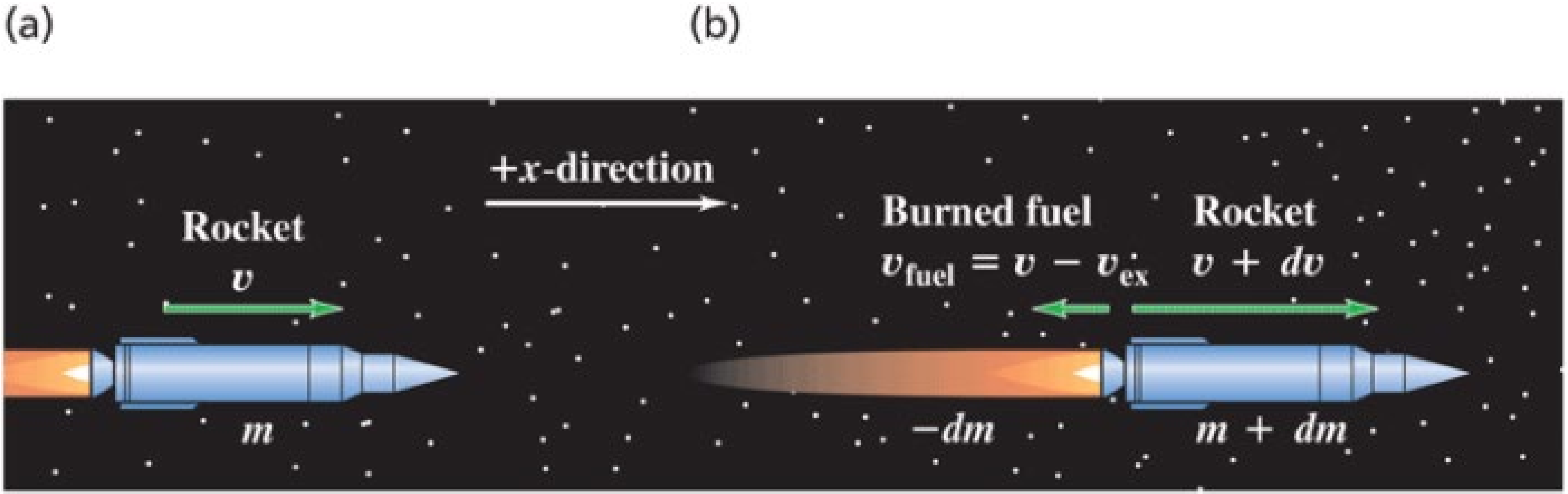

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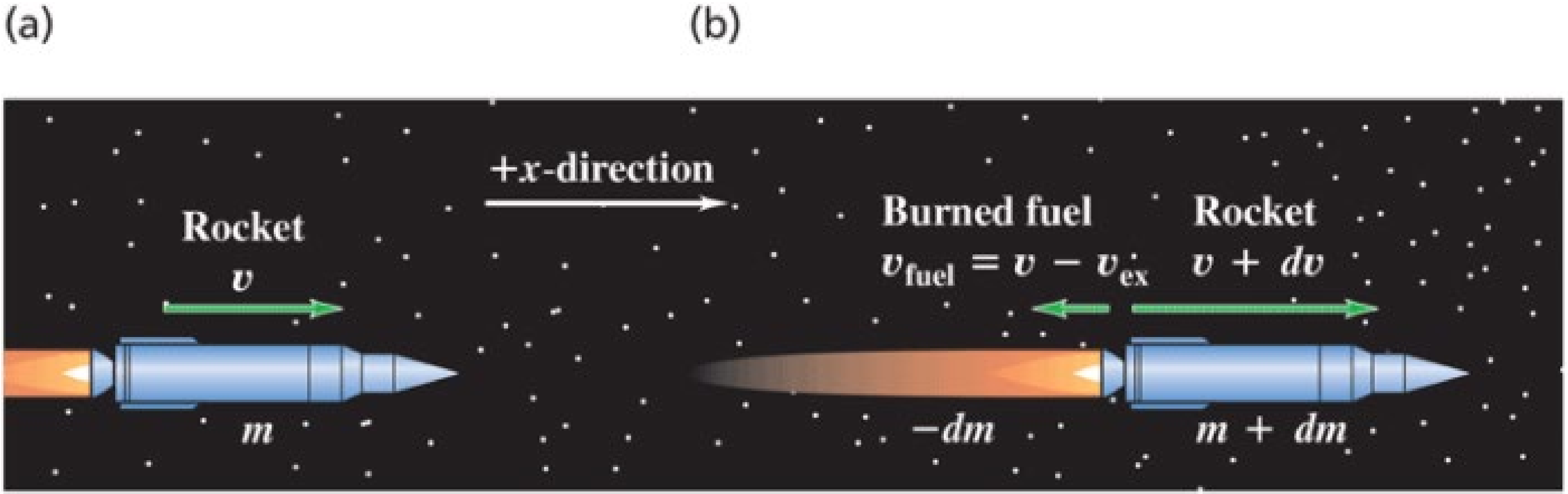
so $P = P_R + P_f$

Rocket propulsion



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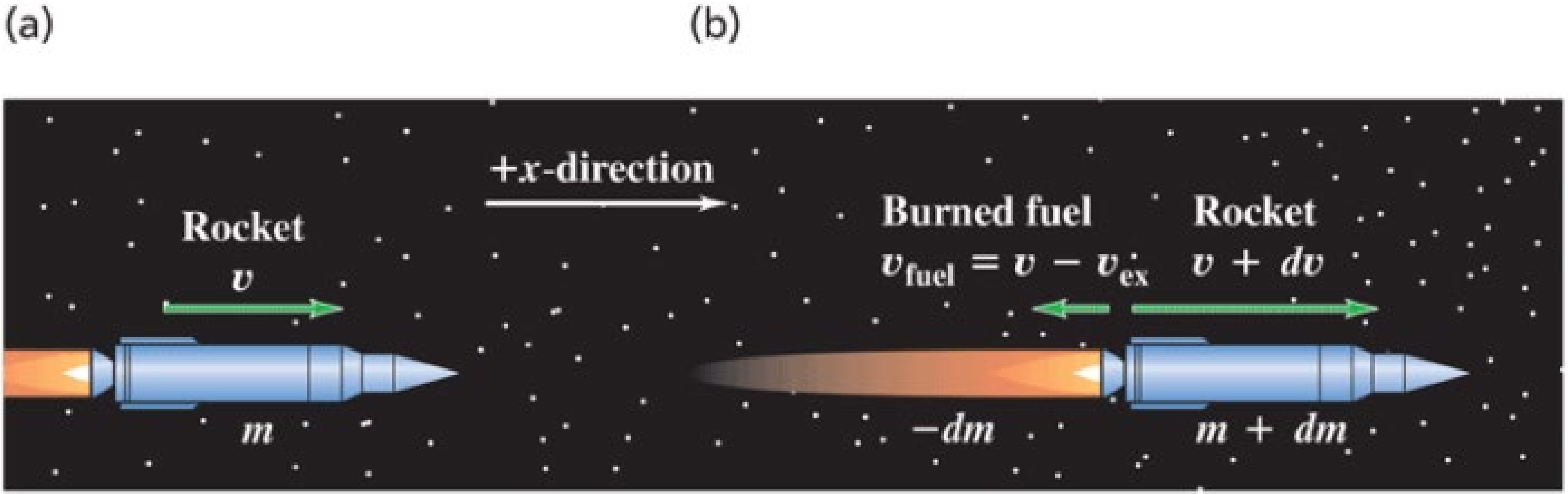
$$P_R = (m+dm)(v+dv), \quad P_f = -dm(v-v_{ex})$$

so $$P = P_R + P_f = mv + m\,dv + dm\,v + dm\,dv - dm\,v + dm\,v_{ex}$$

Rocket propulsion



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so $$P = P_R + P_f = mV + mdV + dmV + dmdV - dmV + dmV_{ex}$$

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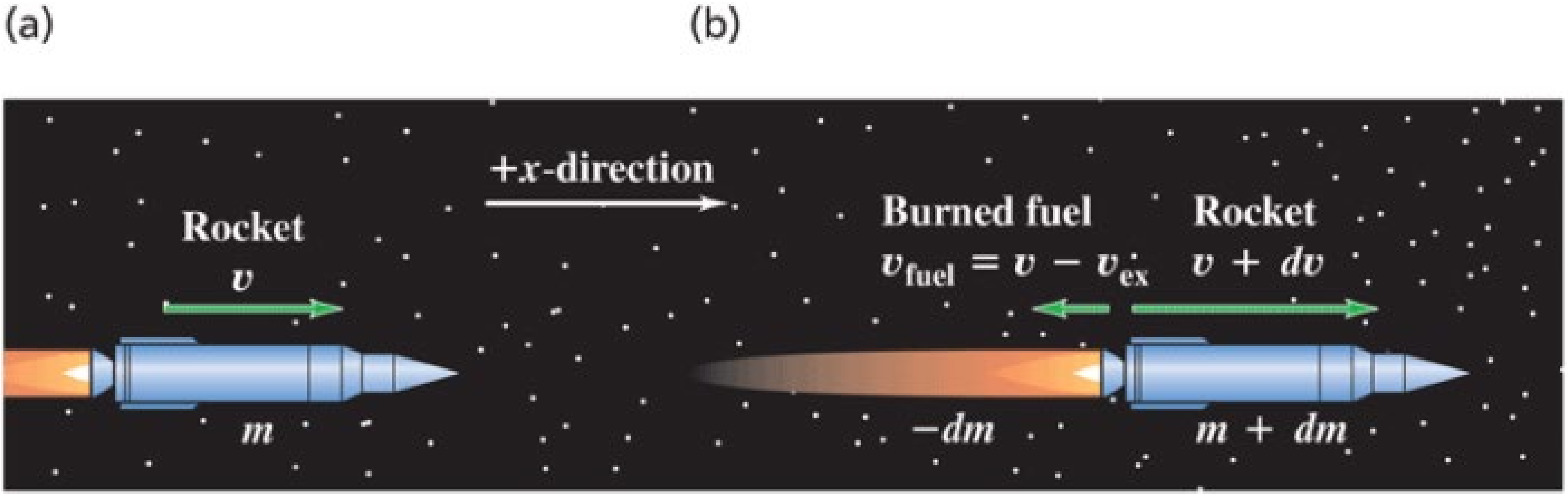
Rocket propulsion



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Note: $dm\,dV \rightarrow$ too small to matter

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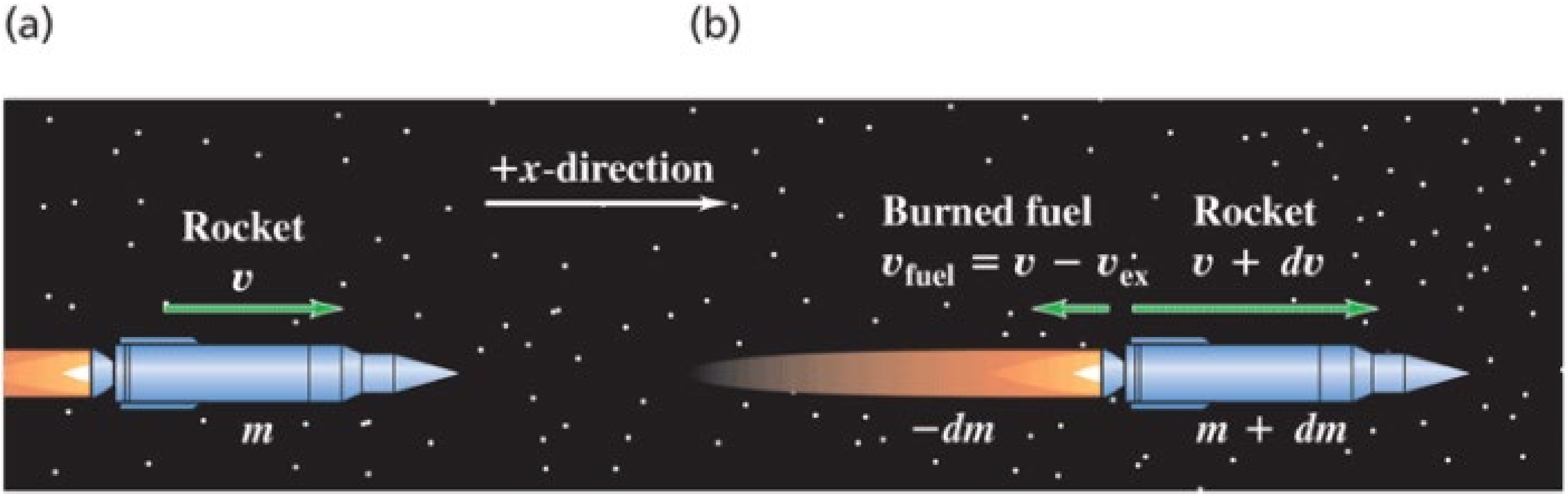
Rocket propulsion



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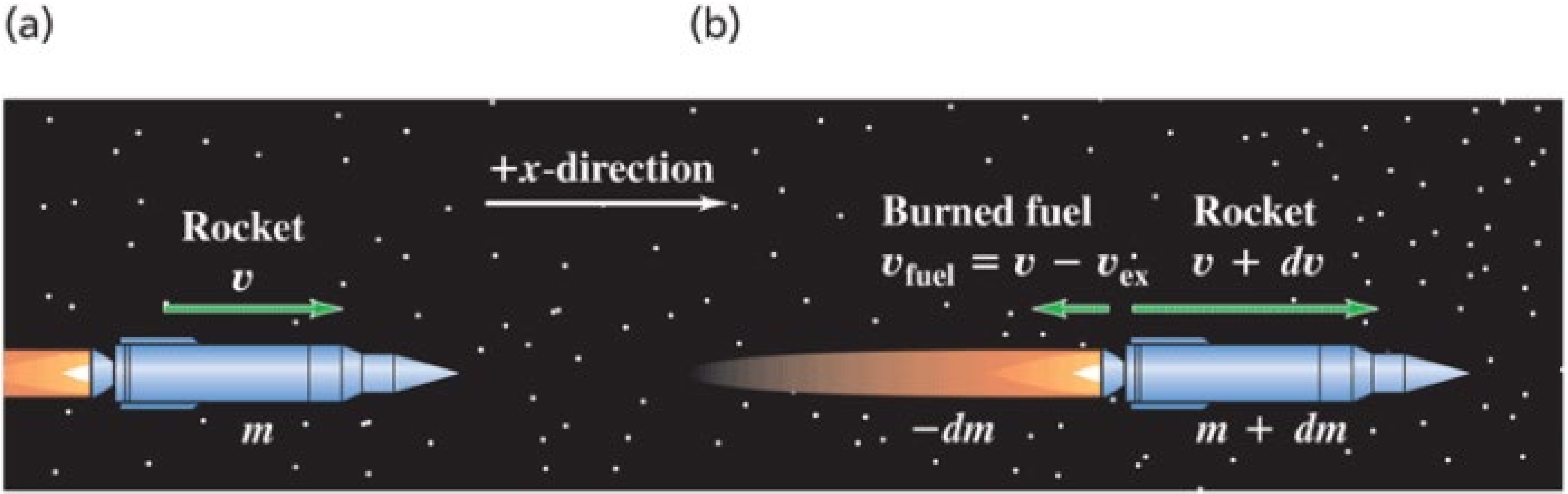
$\Rightarrow P = mV + m\,dV + dm\,V_{ex}$ & $P =$ const.

Rocket propulsion



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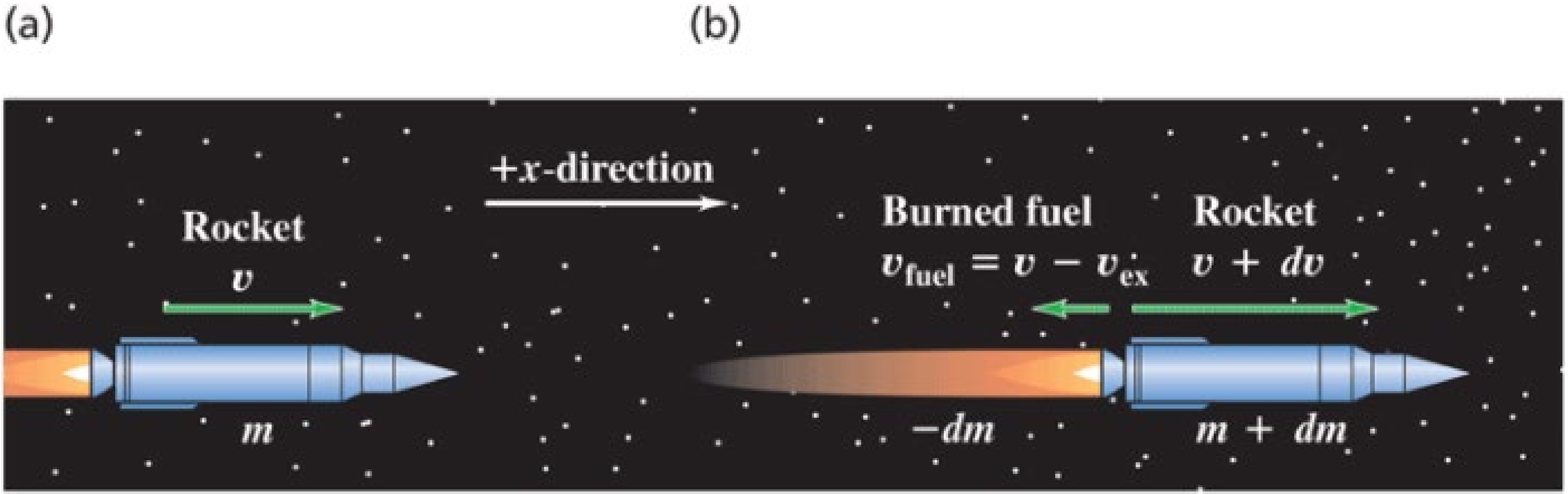
Rocket propulsion



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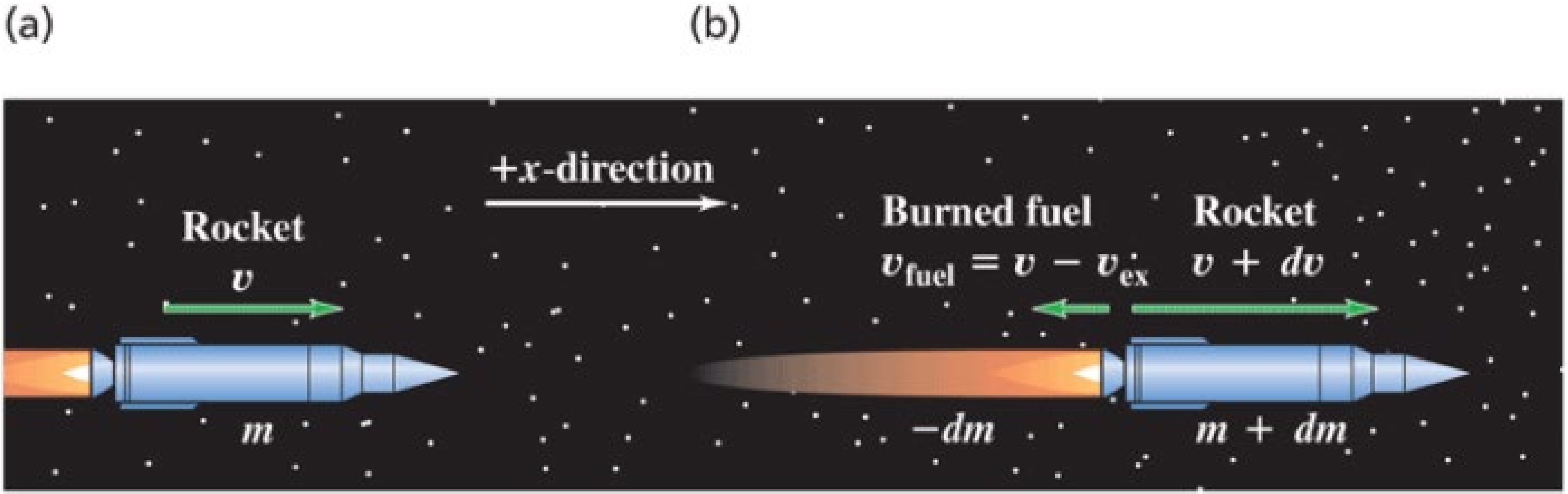
So $$P = P_R + P_f = mV + mdV + dmV + dmdV - dmV + dmV_{ex}$$

$$\Rightarrow P = mV + mdV + dmV_{ex}$$ & $P =$ const. $= mV$ since $\Sigma F_{ext} = 0$

Rocket propulsion



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Note: $dm\,dV \to$ too small to matter

$$P_R = (m+dm)(V+dV),\quad P_f = -dm(V-V_{ex})$$

so $$P = P_R + P_f = mV + m\,dV + dm\,V + dm\,dV - dm\,V + dm\,V_{ex}$$

$$\Rightarrow P = mV + m\,dV + dm\,V_{ex} \quad \text{So} \quad m\,dV = -dm\,V_{ex}$$

From previous slide

$$m\,dv = -V_{ext}\,dm$$

From previous slide

$m\,dv = -V_{ext}\,dm$ so $m\frac{dv}{dt} = -V_{ext}\frac{dm}{dt}$

From previous slide

$m\,dv = -V_{ext}\,dm$ so $m\frac{dv}{dt} = -V_{ext}\frac{dm}{dt}$

But $m\frac{dv}{dt} = ma$: Rocket

From previous slide

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From previous slide

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$$F = -V_{ext}\frac{dm}{dt}$$

From previous slide

$m\,dv = -V_{ext}\,dm$ so $m\frac{dv}{dt} = -V_{ext}\frac{dm}{dt}$

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$$\underbrace{F}_{\text{Thrust}} = -V_{ext}\frac{dm}{dt}$$

To provide enough thrust to lift its payload into space, this *Atlas V* launch vehicle ejects more than 1000 kg of burned fuel per second at speeds of nearly 4000 m/s.

The engine of a rocket in outer space, far from any planet, is turned on. The rocket ejects burned fuel at a constant rate; in the first second of firing, it ejects $\frac{1}{120}$ of its initial mass m_0 at a relative speed of 2400 m/s. What is the rocket's initial acceleration?

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Direction is opposite to exhaust

$$|a| = 20 \, m/s$$

Suppose that $\frac{3}{4}$ of the initial mass of the rocket in Example 8.15 is fuel, so the fuel is completely consumed at a constant rate in 90 s. The final mass of the rocket is $m = m_0/4$. If the rocket starts from rest in our coordinate system, find its speed at the end of this time.

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$$\Rightarrow V_F = 2400\frac{m}{s}\ln(4) \Rightarrow \boxed{V_F = 3327\, m/s}$$