Today 7.2, 7.3, 7.4

$\boxed{L23}$

Today 7.2, 7.3, 7.4

$\boxed{L23}$

Elastic potential energy

Today 7.2, 7.3, 7.4

$\boxed{L23}$

Conservative

&

Nonconservative

Forces

Today 7.2, 7.3, 7.4

$\boxed{L23}$

Force & potential energy

Today 7.2, 7.3, 7.4

$\boxed{L23}$

Wednesday 8.1

Today 7.2, 7.3, 7.4

$\boxed{L23}$

Wednesday $\underline{8.1}$

Momentum

&

impulse

We are going to look a bit closer at a spring system

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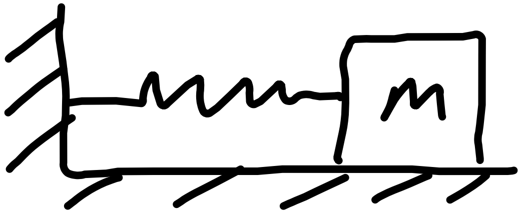

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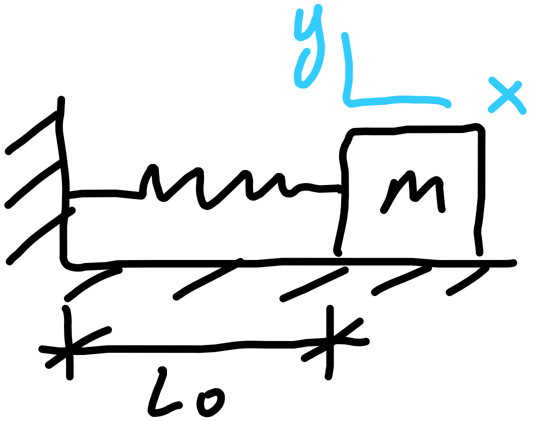

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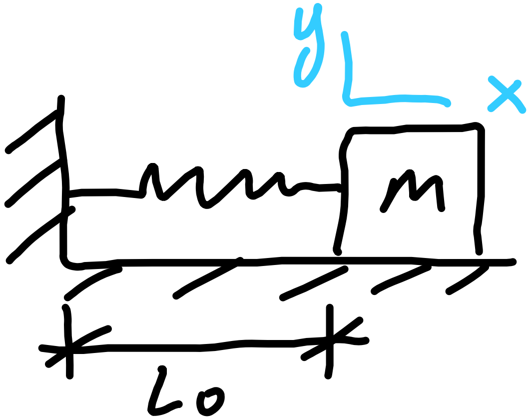

$\vec{F}_{S\to B} = -kx\hat{i}$ Force on block

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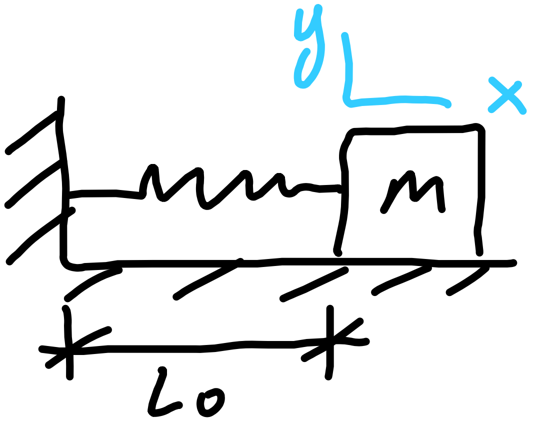

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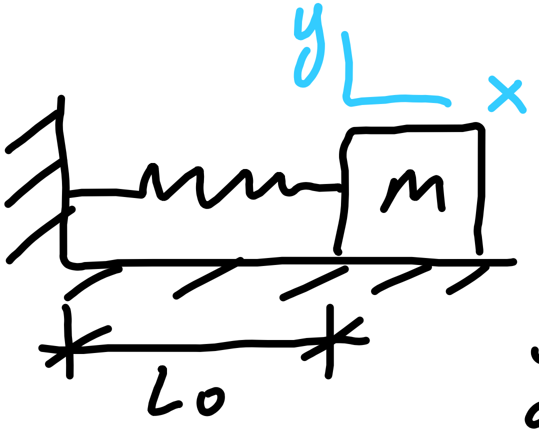

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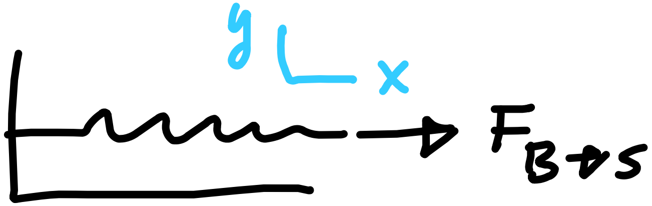
Look at spring wall system



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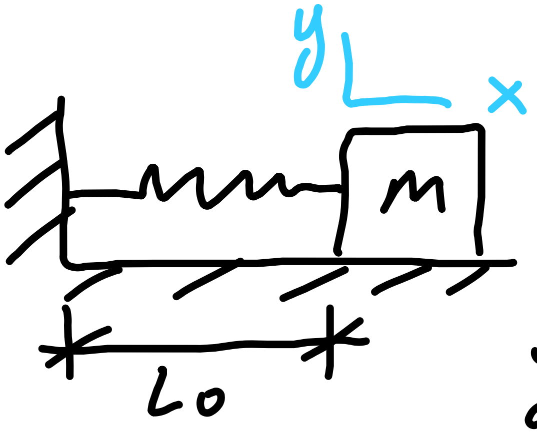

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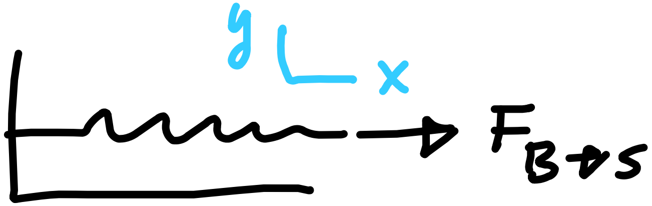

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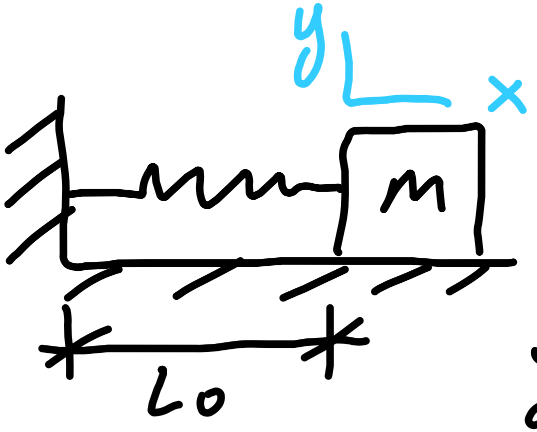

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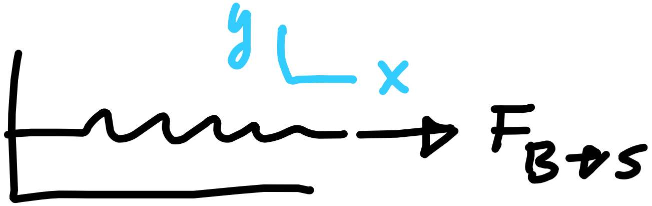

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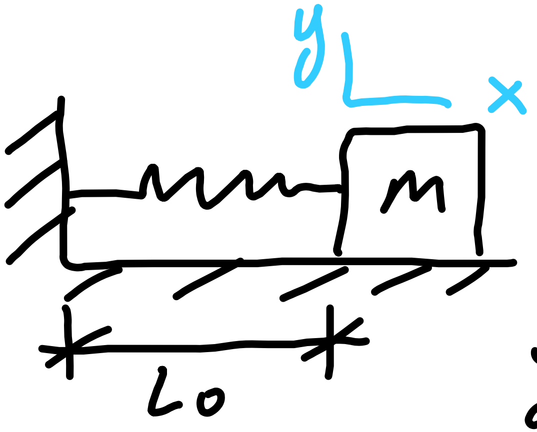

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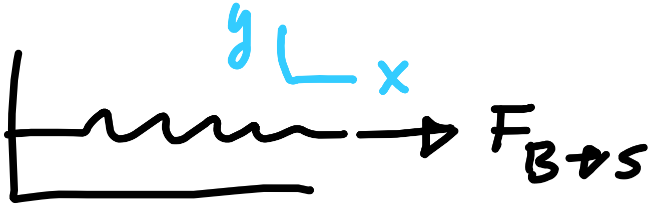

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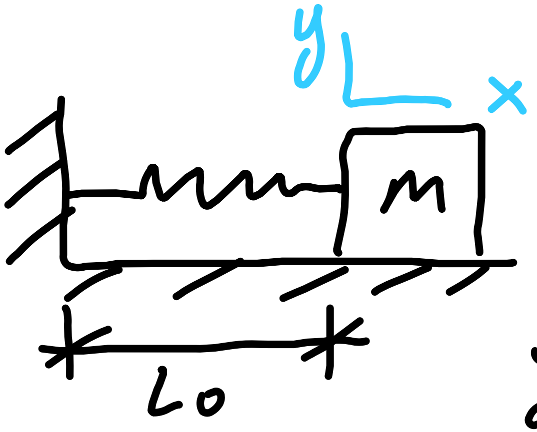

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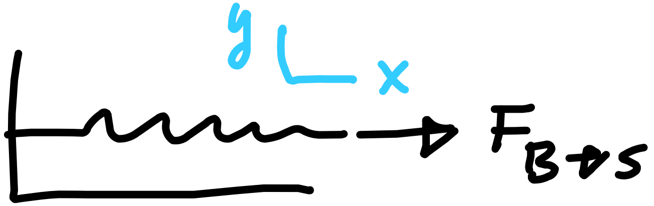

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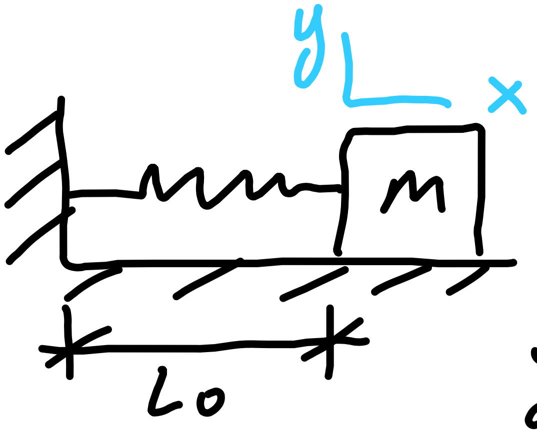

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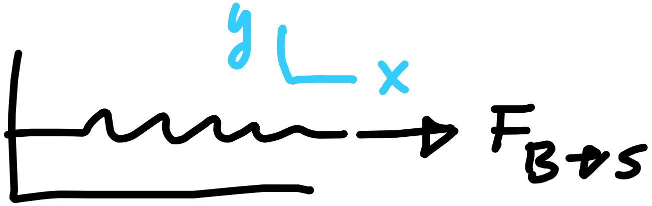

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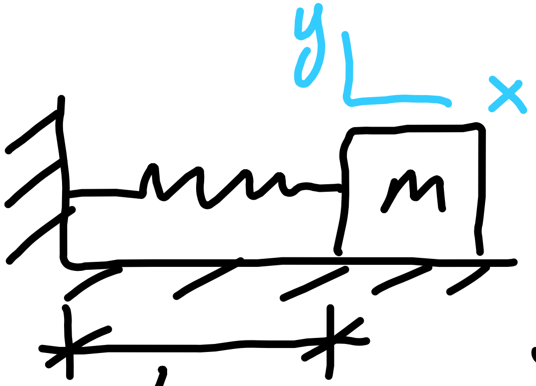

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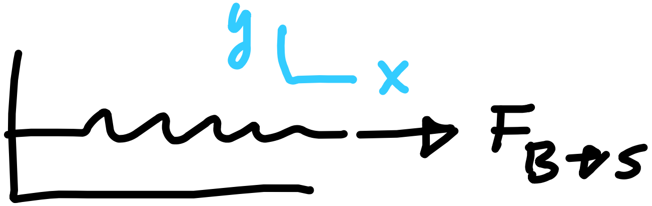

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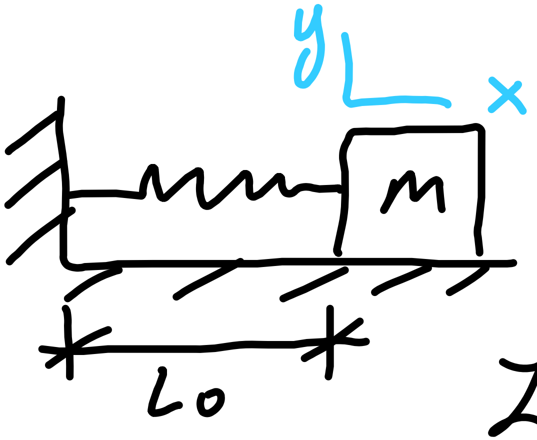

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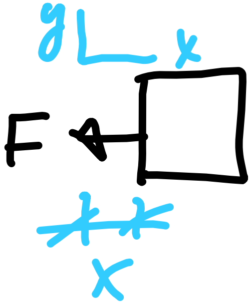

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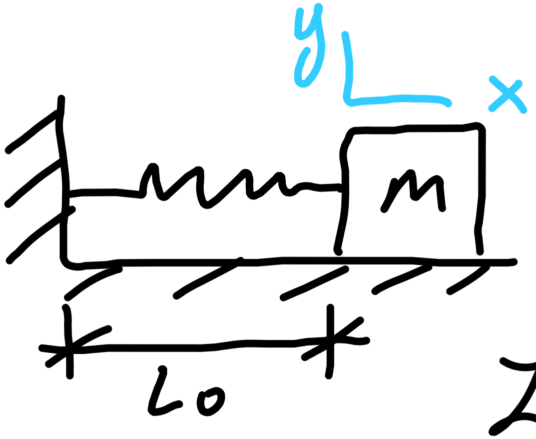

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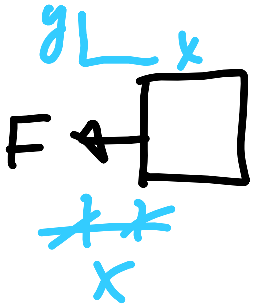

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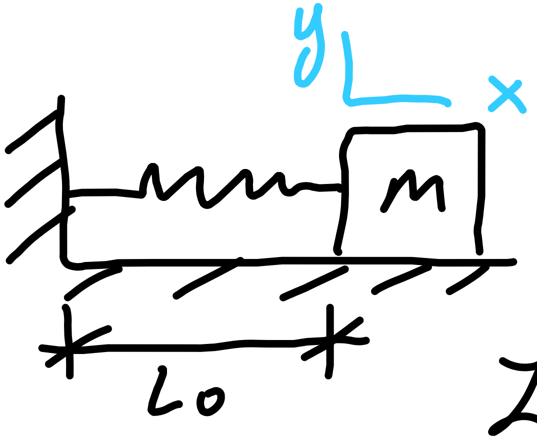

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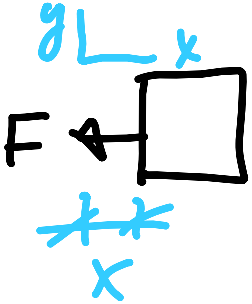

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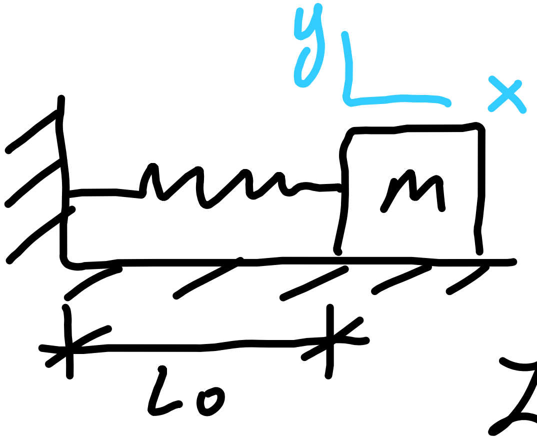

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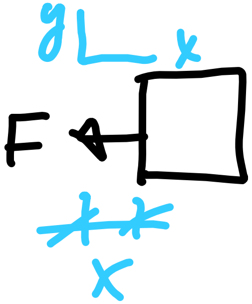

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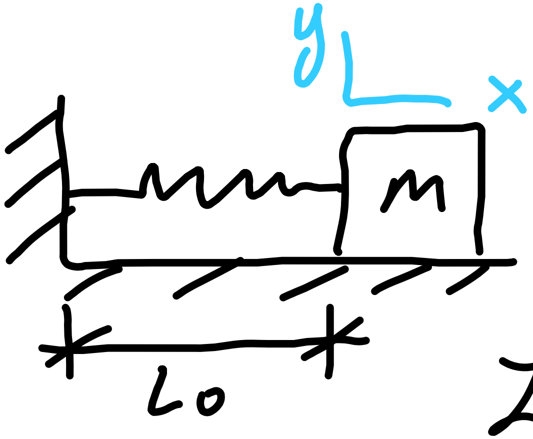

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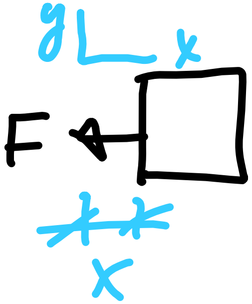

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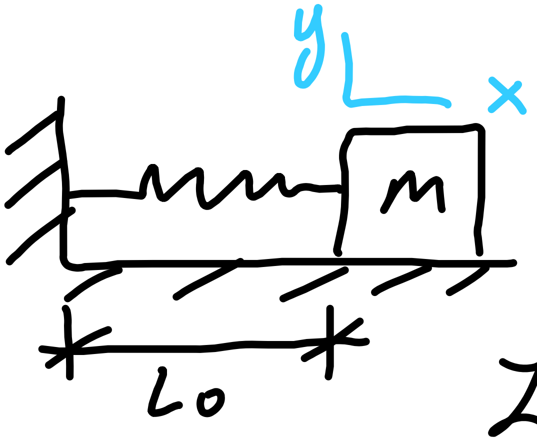

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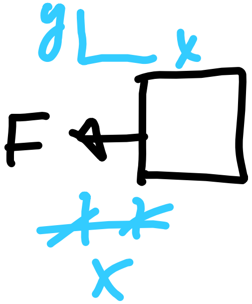

$w_B = -\int kx\,dx = -\frac{1}{2}kx^2$

so $w_B = K_2 - K_1$

But $\delta U = -W$ so $-(U_2 - U_1) = K_2 - K_1$

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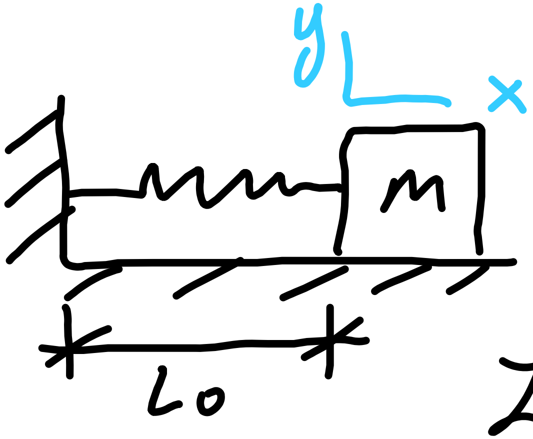

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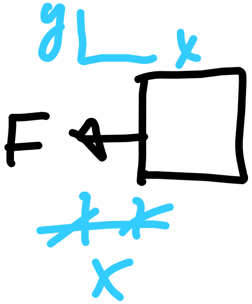

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$\Rightarrow \quad U_1 + K_1 = U_2 + K_2$

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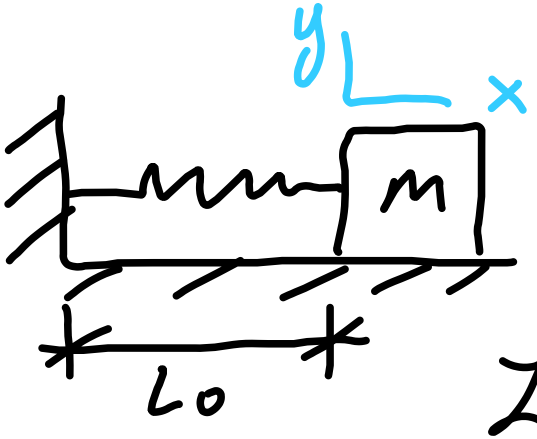

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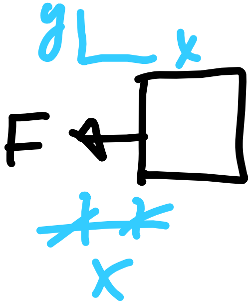

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But $\delta U = -W$ so $-(U_2 - U_1) = K_2 - K_1$

$\Rightarrow$ $U_1 + K_1 = U_2 + K_2$, where

$U = \frac{1}{2}kx^2$

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Typically we can neglect the force on spring & work performed on spring. Instead, it is the force by spring & work performed by spring that are useful

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4) When starting & ending endpoints are the same, total work is zero

You are rearranging your furniture and wish to move a 40.0 kg futon 2.50 m across the room. A heavy coffee table, which you don't want to move, blocks this straight-line path. Instead, you slide the futon along a dogleg path; the doglegs are 2.00 m and 1.50 m long. How much more work must you do to push the futon along the dogleg path than along the straight-line path? The coefficient of kinetic friction is $\mu_k = 0.200$.

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$F_F = mg\mu_k$

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$m = 40\,\text{kg},\ L = 2.5\,\text{m},\ L_1 = 2\,\text{m},\ L_2 = 1.5\,\text{m},\ \mu_k = 0.2$

$F_F = mg\mu_k \quad \& \quad W_L = mg\mu_k L \quad \&$

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$$[W_{L_1} + W_{L_2}] - W_L = mg\mu_K[L_1 + L_2 - L]$$
$$= 40 * 9.8 * 0.2 * [3.5 - 2.5]\,J$$

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$F_F = mg\mu_k$ & $W_L = mg\mu_K L$ &

$W_{L_1} + W_{L_2} = mg\mu_K(L_1+L_2)$ So

$$[W_{L_1}+W_{L_2}] - W_L = mg\mu_K[L_1+L_2-L]$$

$$= 40*9.8*0.2*[3.5-2.5]\,J$$

$$= 78.4\,J$$

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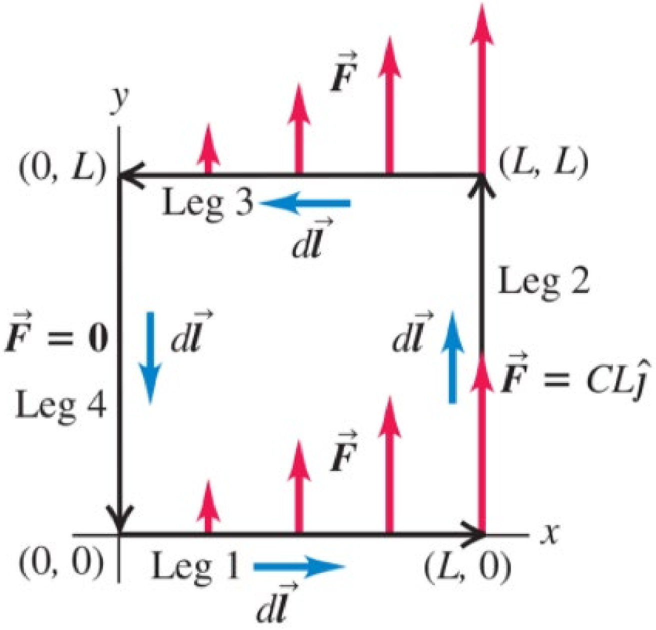
Path Dependent so Nonconservative Force

In a region of space the force on an electron is $\vec{F} = Cx\hat{\boldsymbol{j}}$, where C is a positive constant. The electron moves around a square loop in the xy-plane (Fig. 7.20). Calculate the work done on the electron by force $\vec{F}$ during a counterclockwise trip around the square. Is this force conservative or nonconservative?



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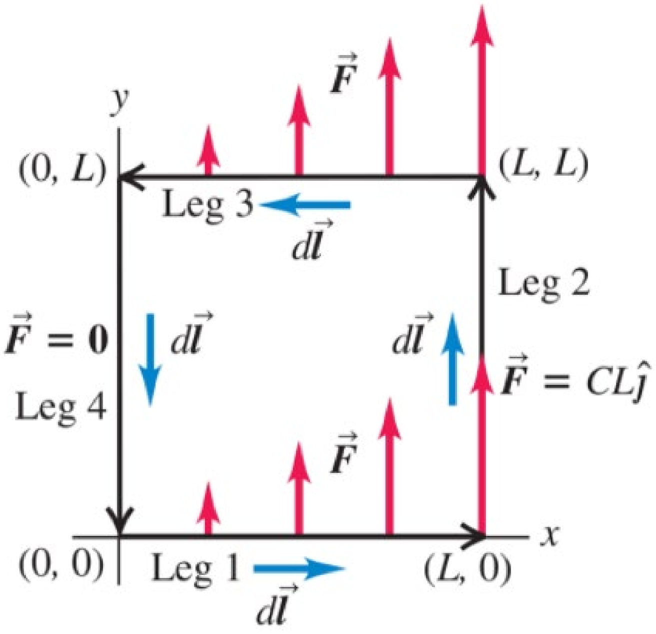

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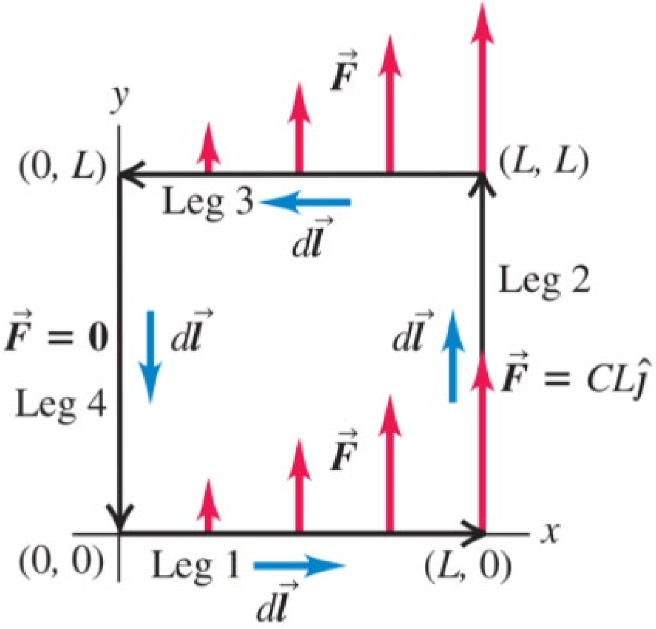

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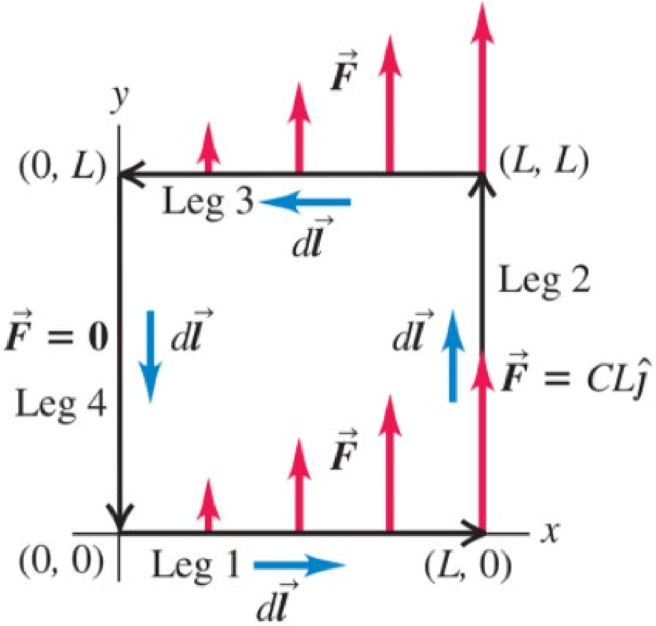

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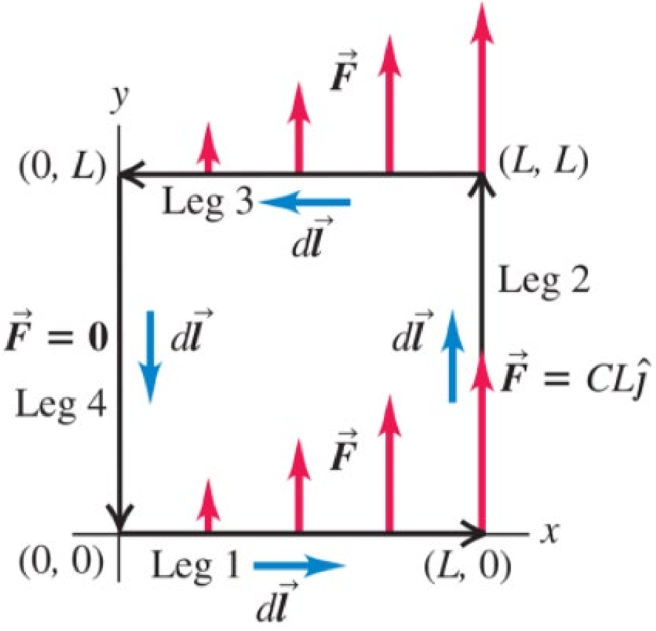

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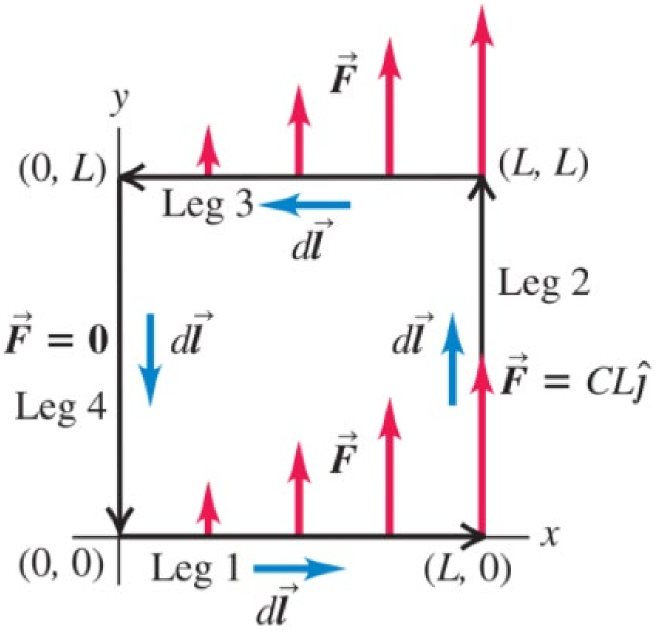

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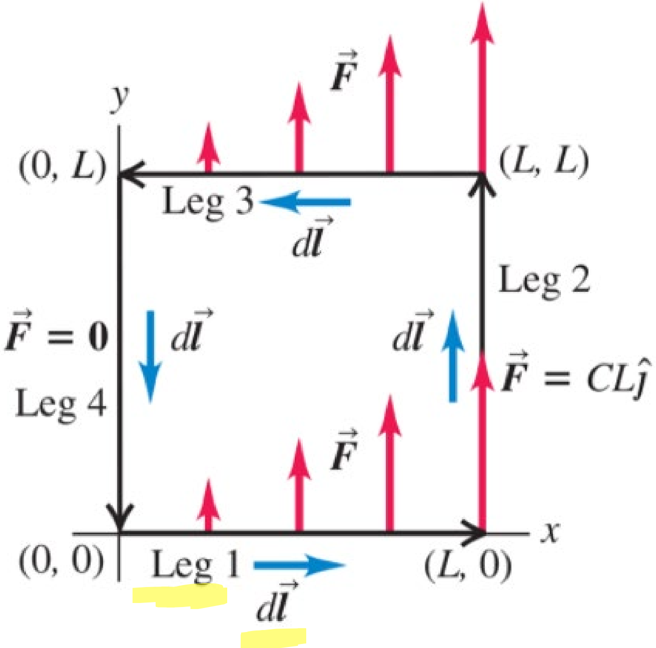

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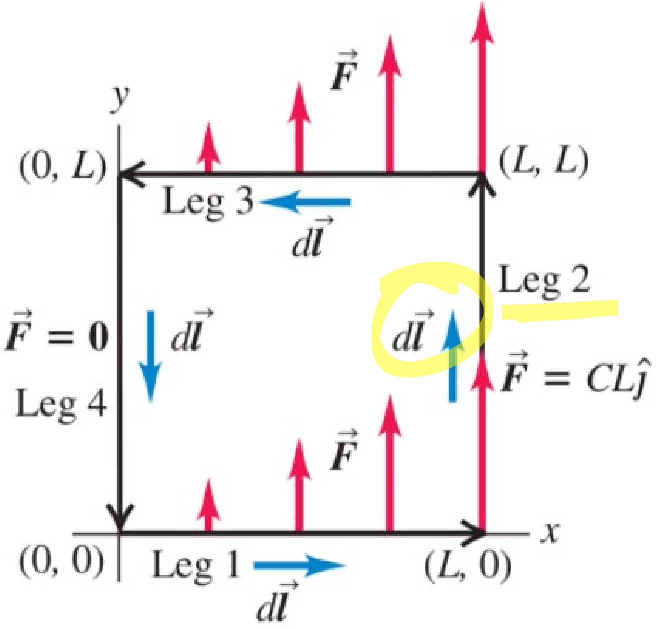

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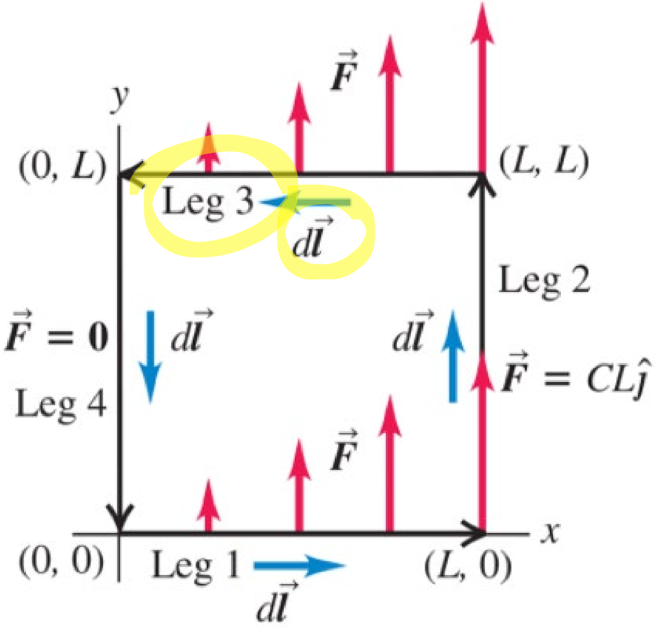

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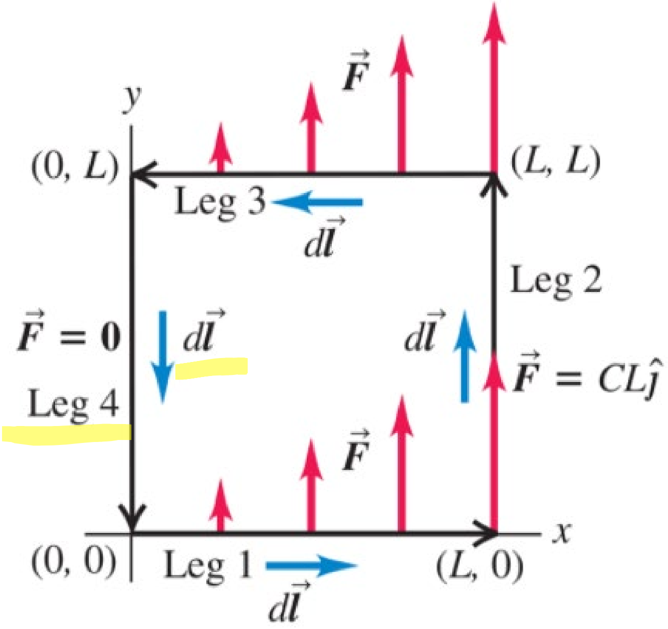

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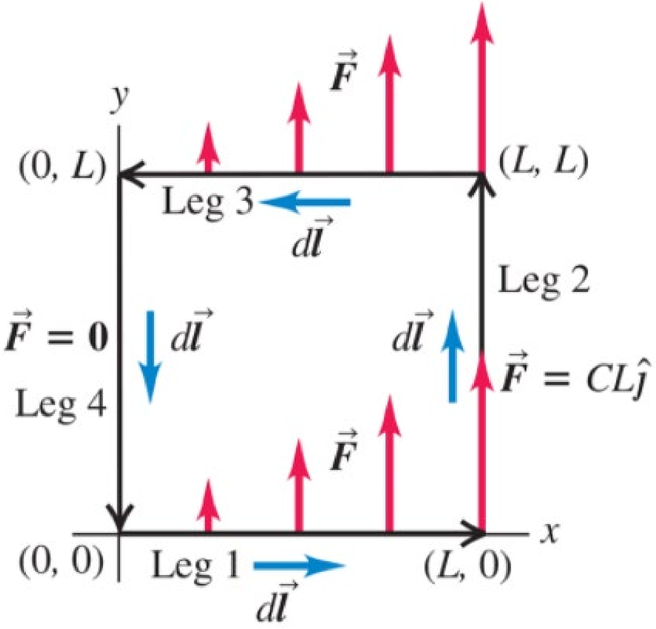

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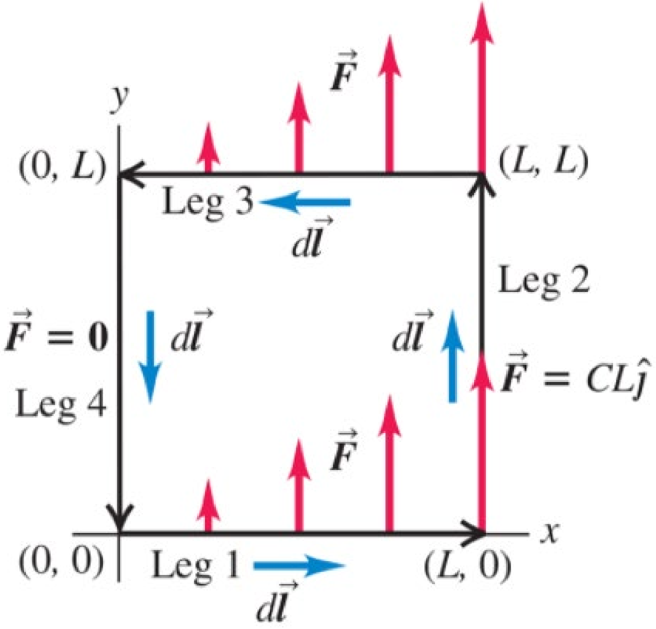

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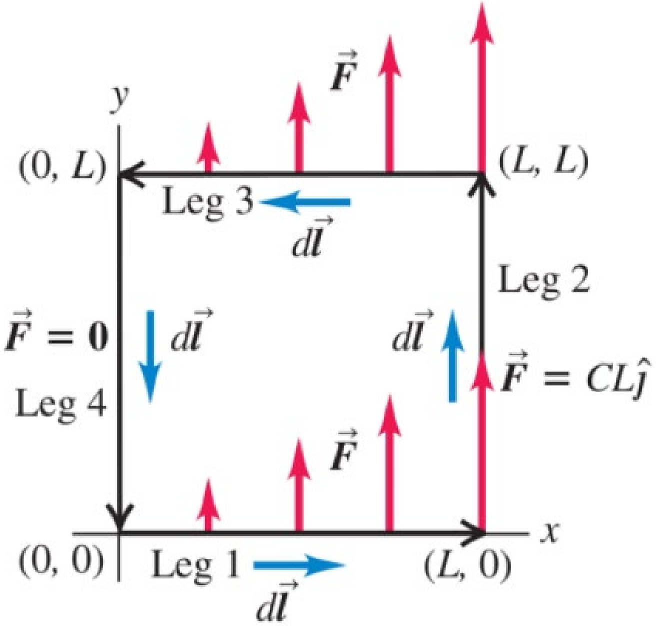

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$$W = \int_0^L CL\,dy + \int_L^0 C0\,dy$$ since $F(x=0) = C0$

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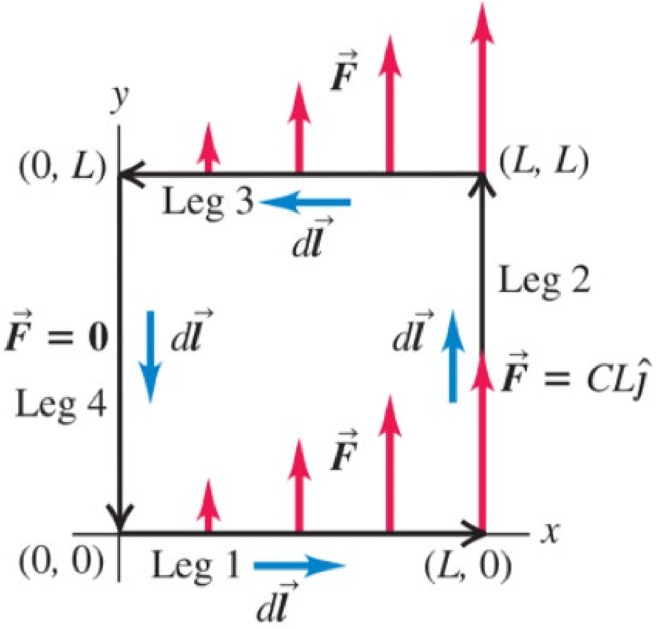
So $W = CL^2$



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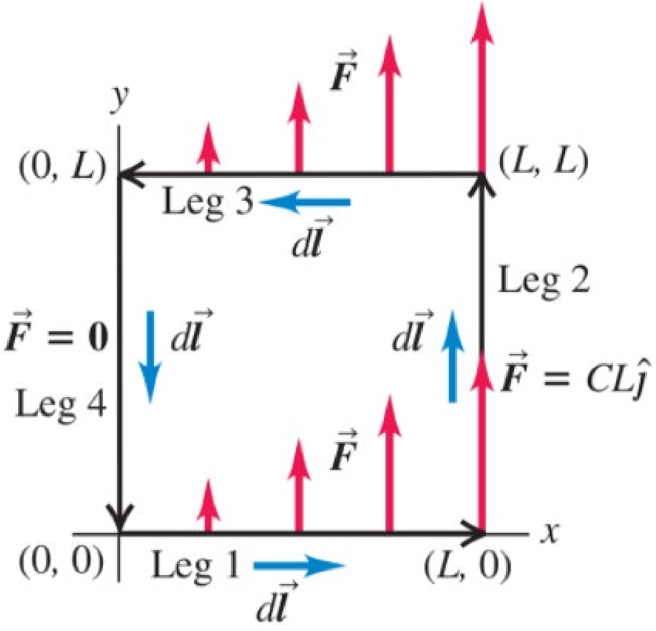
& nonconservative since work for round trip $\neq 0$



Internal energy

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While driving, a tire will deform &
flex

Internal energy

While diving, a tire will deform & flex ⇒ heat generation & raised temperature

Internal energy

While driving, a tire will deform & flex ⇒ heat generation & raised temperature ⇒ Increased internal energy

Internal energy

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$\Delta U_{int} = -W_{other}$

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So $K_1 + U_1 - \Delta U_{int} = K_2 + U_2$

Internal energy

$$\Delta U_{int} = -W_{other}$$

So $K_1 + U_1 - \Delta U_{int} = K_2 + U_2$

or $\Delta K + \Delta U + \Delta U_{int} = 0$

Force & potential energy

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$W = -\Delta U$

Force & potential energy

$W = -\Delta U$ In 1d (say x-direction)

$W = F_x \Delta x$

Force & potential energy

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$W = F_x \Delta x \Rightarrow \quad F_x \Delta x = -\Delta U$

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$W = -\Delta U$ In 1d (say x-direction)

$W = F_x \Delta x \Rightarrow F_x \Delta x = -\Delta U \Rightarrow F_x = -\frac{\Delta U}{\Delta x}$

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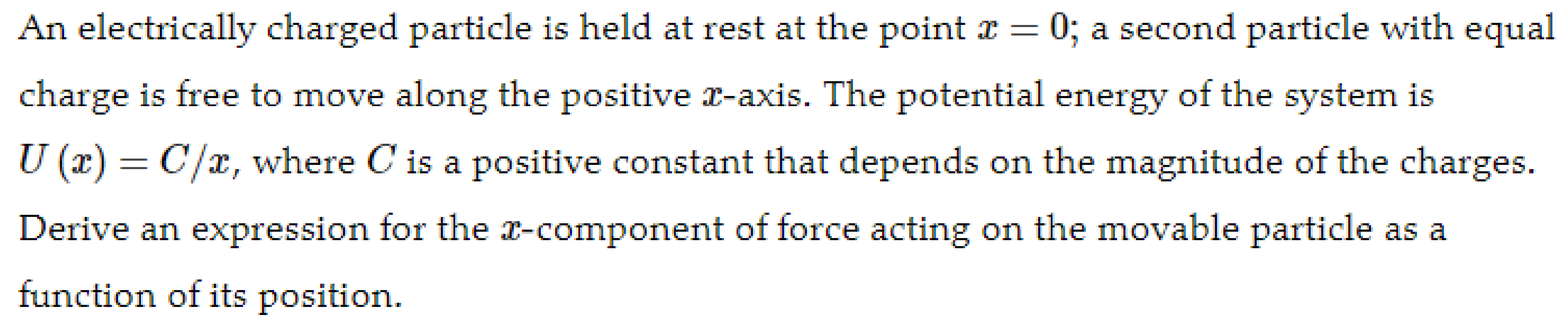

An electrically charged particle is held at rest at the point $x = 0$; a second particle with equal charge is free to move along the positive x-axis. The potential energy of the system is $U(x) = C/x$, where C is a positive constant that depends on the magnitude of the charges. Derive an expression for the x-component of force acting on the movable particle as a function of its position.

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$$U = C/x$$

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$U = C/x$

$$F_x = -\frac{\partial U}{\partial x} = -\frac{\partial}{\partial x}\left[C/x\right] = +\frac{C}{x^2}$$

3-d case

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3-d case

$F_x = -\frac{\partial U}{\partial x}$, $F_y = -\frac{\partial U}{\partial y}$

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3-d case

$F_x = -\frac{\partial u}{\partial x}$, $F_y = -\frac{\partial u}{\partial y}$, $F_z = -\frac{\partial u}{\partial z}$

$$\Rightarrow \quad \vec{F} = -\left(\frac{\partial u}{\partial x}\hat{i} + \frac{\partial u}{\partial y}\hat{j} + \frac{\partial u}{\partial z}\hat{k}\right)$$

3-d case

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Gravity

3-d case

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Gravity $U = mgy$

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$$\Rightarrow \quad \vec{F} = -\left(\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right) = -\vec{\nabla} U$$

Gravity $U = mgy$, $\vec{F} = -\vec{\nabla} U$

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A puck with coordinates x and y slides on a level, frictionless air-hockey table. It is acted on by a conservative force described by the potential-energy function

$$U(x,\ y) = \frac{1}{2}\,k\left(x^2 + y^2\right)$$

Note that $r = \sqrt{x^2 + y^2}$ is the distance on the table surface from the puck to the origin. Find a vector expression for the force acting on the puck, and find an expression for the magnitude of the force.

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