$\boxed{L20}$

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$\boxed{\text{1-d case}}$

$$\sum F = ma$$

1-D case $\sum F = ma$ But $a = v\frac{\partial v}{\partial x}$

1-d case $\Sigma F = ma$ But $a = v\frac{dv}{dx}$

So $\Sigma F = m\,v\frac{dv}{dx}$

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so $\Sigma \int F dx = m \int v \frac{dv}{dx} dx$

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so $\Sigma\int F dx = m\int v \frac{dv}{dx} dx \Rightarrow \Sigma\int F dx = m\int v dv$

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So $\Sigma\int F dx = m\int v \frac{dv}{dx} dx \Rightarrow \Sigma\int_{x_1}^{x_2} F dx = m\int_{v_1}^{v_2} v dv$

$\Rightarrow \Sigma\int_{x_1}^{x_2} F dx = \frac{1}{2} m v^2 \Big|_{v_1}^{v_2}$

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1-D case

$\sum F = ma$ But $a = v\frac{dv}{dx}$

So $\sum\int F\,dx = m\int v\frac{dv}{dx}\,dx \Rightarrow \sum\int_{x_1}^{x_2} F\,dx = m\int_{v_1}^{v_2} v\,dv$

$$\Rightarrow \sum\int_{x_1}^{x_2} F\,dx = \frac{1}{2}mv^2\Big|_{v_1}^{v_2}$$

$$\Rightarrow \underbrace{\sum\int_{x_1}^{x_2} F\,dx}_{W_{1\to 2}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$\boxed{\text{1-D case}}$ $\quad \sum F = ma \quad$ But $\quad a = v\frac{dv}{dx}$

So $\quad \sum\int F\,dx = m\int v\frac{dv}{dx}\,dx \Rightarrow \sum\int_{x_1}^{x_2} F\,dx = m\int_{v_1}^{v_2} v\,dv$

$$\Rightarrow \sum\int_{x_1}^{x_2} F\,dx = \frac{1}{2}mv^2\Big|_{v_1}^{v_2}$$

$$\Rightarrow \underbrace{\sum\int_{x_1}^{x_2} F\,dx}_{W_{1\to 2}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

We will now derive the same result in a different way

Work for varying force

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Let

$$W = F_1 \Delta X_1 + F_2 \Delta X_2 + \cdots$$

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Force
during
displacement
of Δx_1

Work for varying force

Let

$$W = F_1 \Delta X_1 + \underbrace{F_2}_{\downarrow} \Delta X_2 + \cdots$$

force during displacement of ΔX_2

Work for varying force

Let

$$W = F_1 \Delta X_1 + F_2 \Delta X_2 + \cdots$$

$$\Rightarrow \quad W = \sum_{i=1}^{N} F_i \cdot \Delta X_i$$

Work for varying force

Let

$$W = F_1 \Delta X_1 + F_2 \Delta X_2 + \cdots$$

$$\Rightarrow \quad W = \sum_{i=1}^{N} F_i \Delta X_i$$

Take the limit

as $\Delta X \to 0$:

Work for varying force

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$$W = F_1 \Delta X_1 + F_2 \Delta X_2 + \cdots$$

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as $\Delta x \to 0$:

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So

$$W_{1\to 2} = \int_{x_1}^{x_2} F \, dx$$

(a) A particle moves from x_1 to x_2 in response to a changing force in the x-direction.

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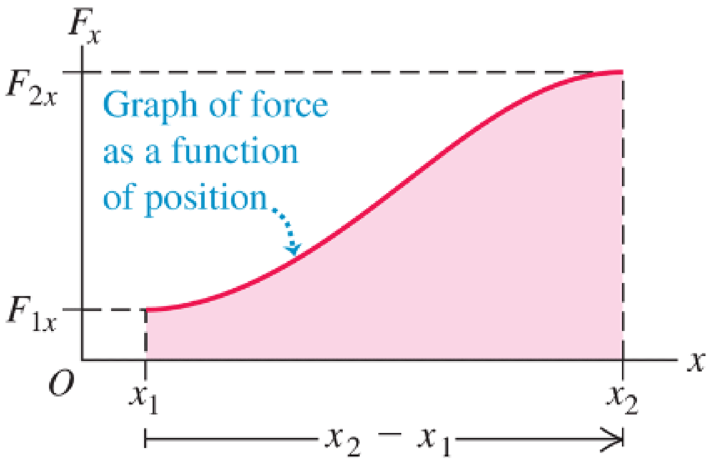
(b) The force F_x varies with position x ...



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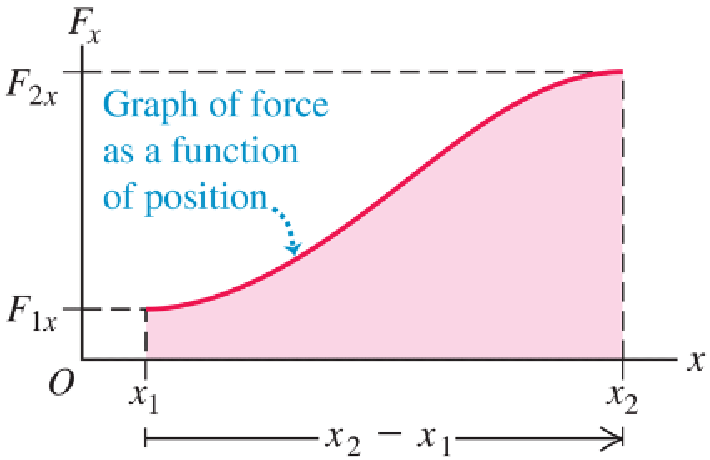
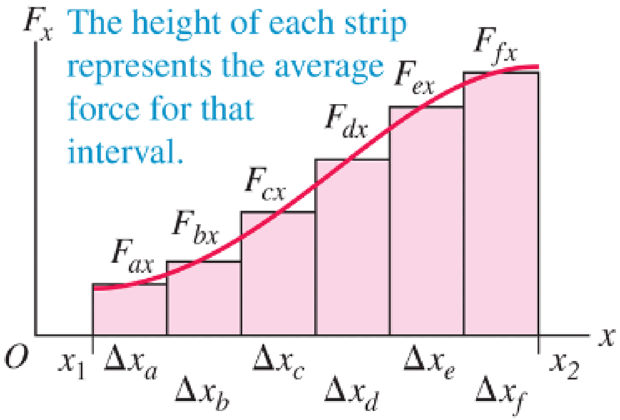


(c) ... but over a short displacement Δx, the force is essentially constant.



$$W = \int_{x_1}^{x_2} F_x \, dx$$

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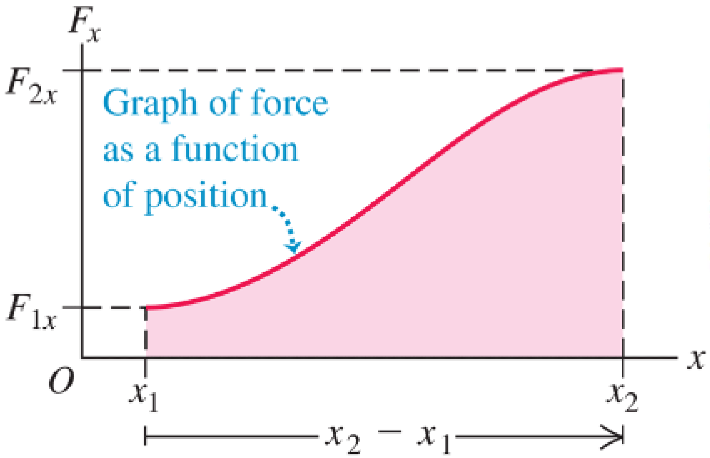

Work done on a particle by a **varying** ***x*****-component of force** F_x during **straight-line displacement** along x-axis

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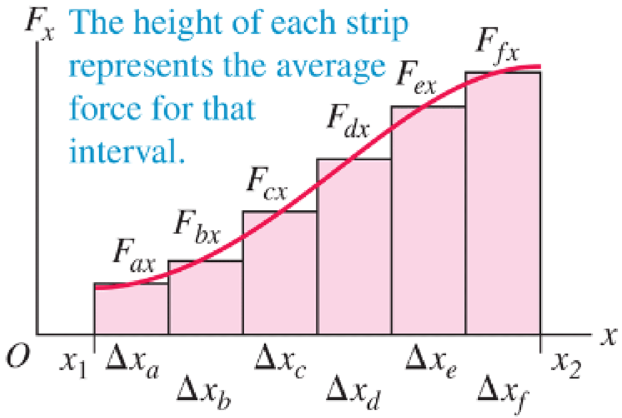

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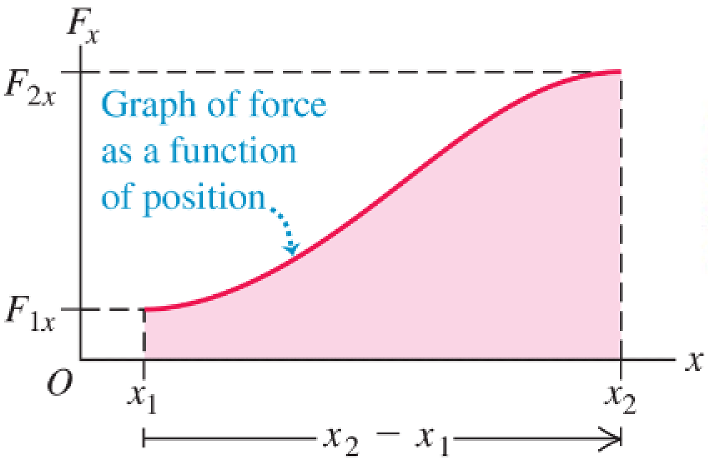
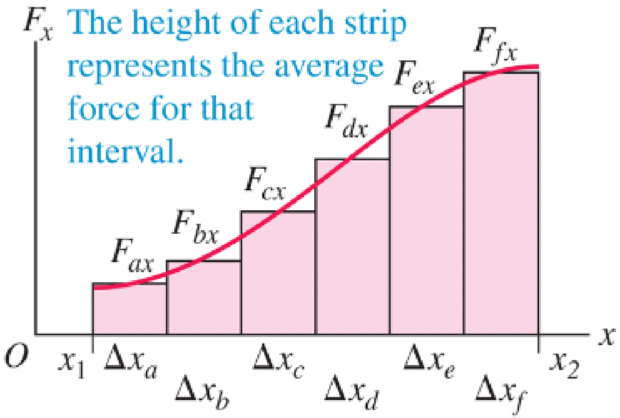

Graph of force as a function of position

Work done on a particle by a **varying x-component of force F_x** during **straight-line displacement** along x-axis

$$W = \int_{x_1}^{x_2} F_x \, dx$$

Lower limit = initial position

(c) ... but over a short displacement Δx, the force is essentially constant.


The height of each strip represents the average force for that interval.

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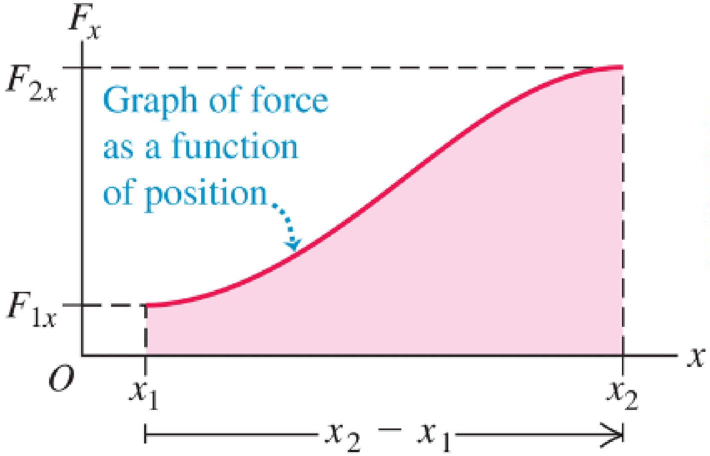

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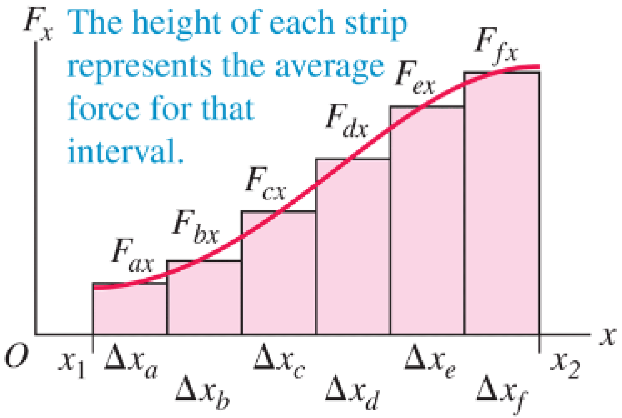

Work done on a particle by a **varying x-component of force F_x** during **straight-line displacement** along x-axis

$$W = \int_{x_1}^{x_2} F_x \, dx$$

Upper limit = final position

Lower limit = initial position

(a) A particle moves from x_1 to x_2 in response to a changing force in the x-direction.

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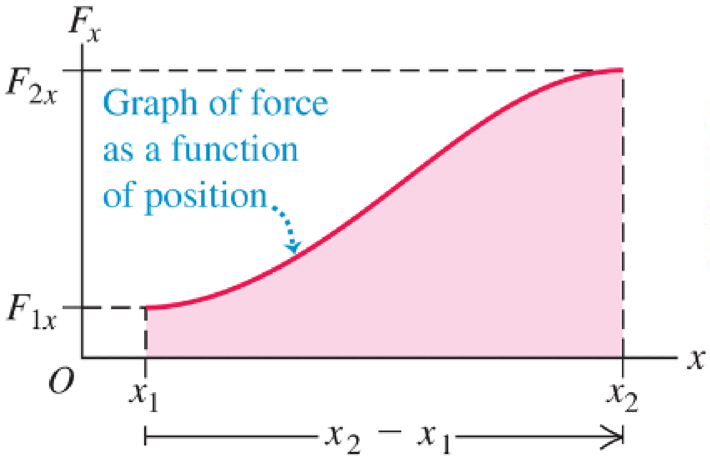

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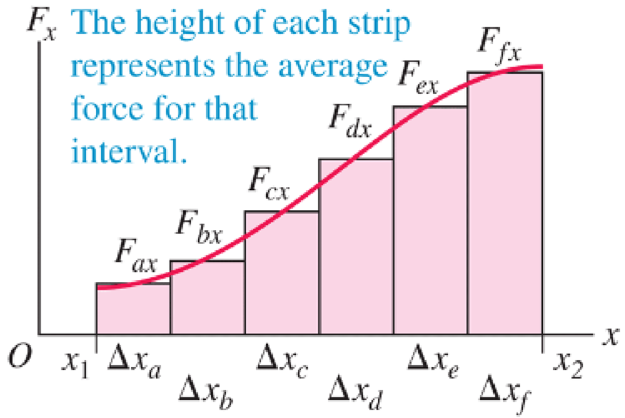

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$$W = \int_{x_1}^{x_2} F_x \, dx$$

Upper limit = final position

Integral of x-component of force

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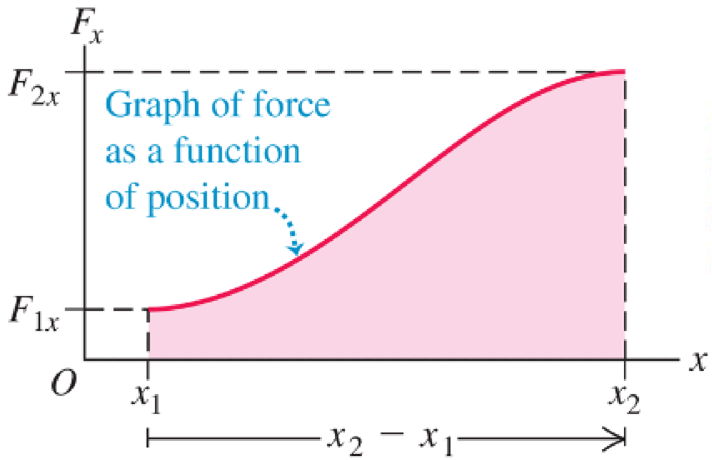

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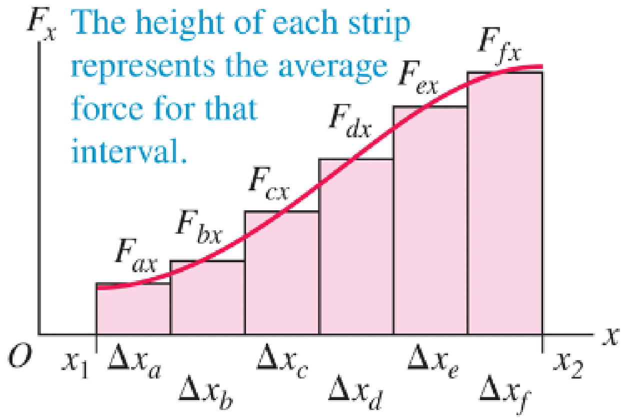

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If F_x = constant

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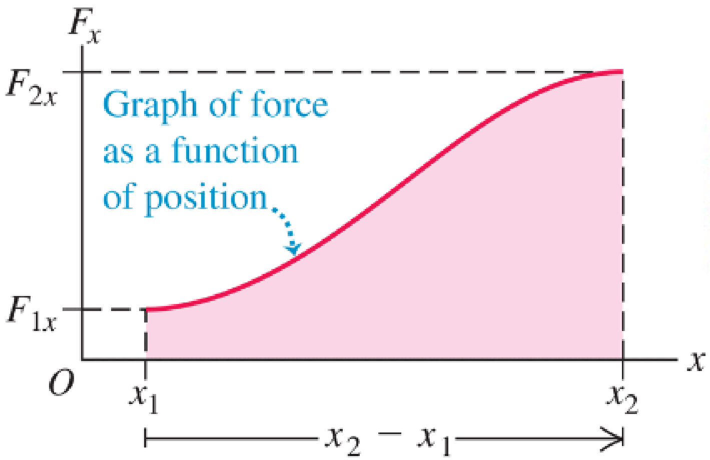

Graph of force as a function of position.

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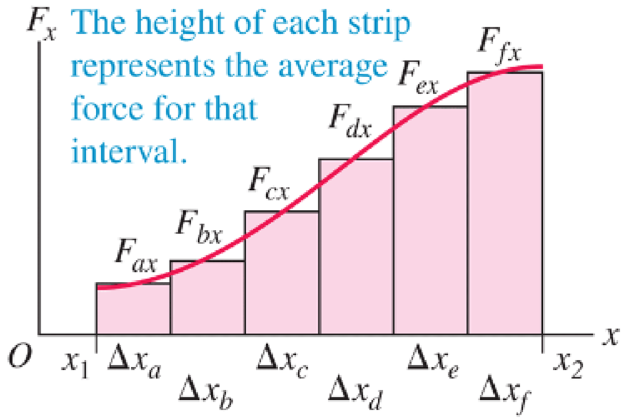

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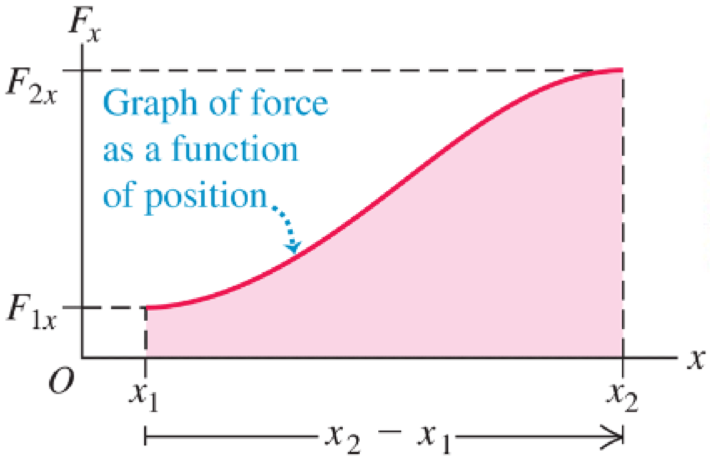

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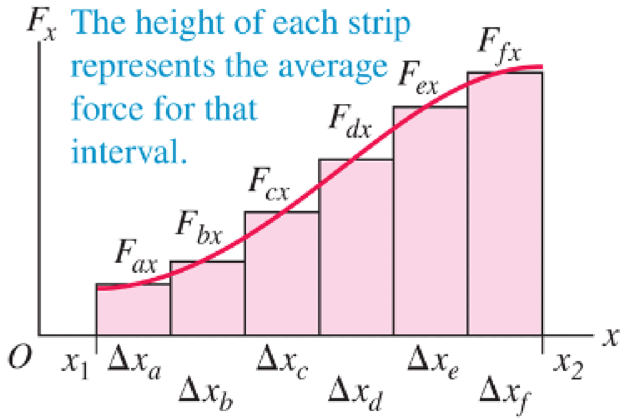

Work done on a particle by a **varying x-component of force F_x** during **straight-line displacement** along x-axis

$$W = \int_{x_1}^{x_2} F_x\,dx$$

Upper limit = final position

Integral of x-component of force

Lower limit = initial position

If $F_x = \text{constant}$

$$W = \int_{x_1}^{x_2} F_x\,dx = F_x \int_{x_1}^{x_2} dx$$

$$= F_x (x_2 - x_1)$$

(a) A particle moves from x_1 to x_2 in response to a changing force in the x-direction.

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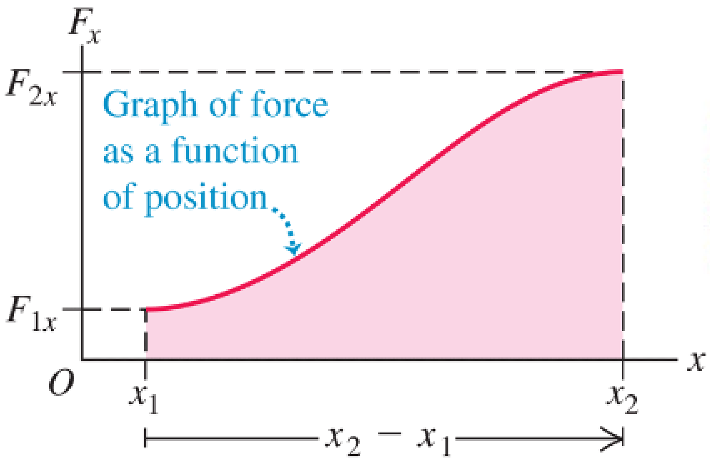

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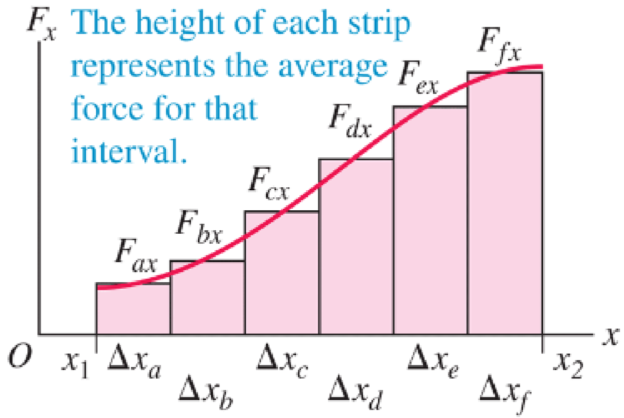

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$$= Fs$$

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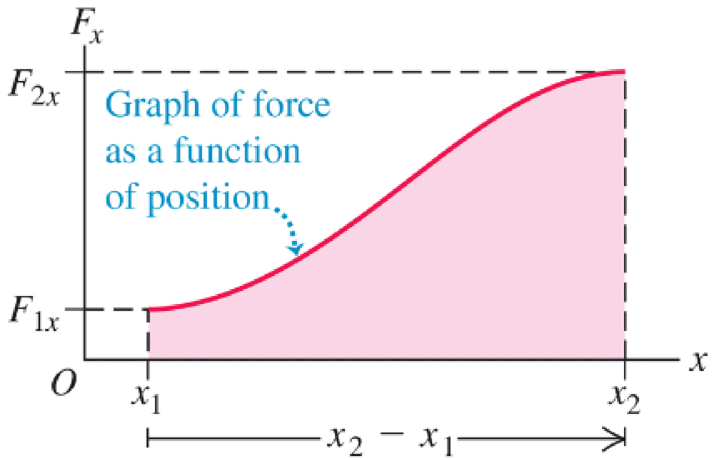

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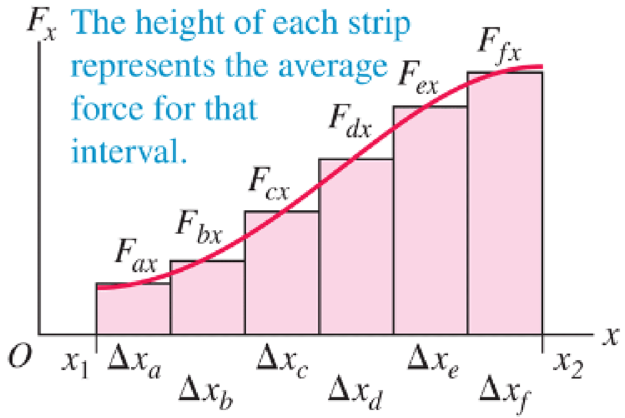
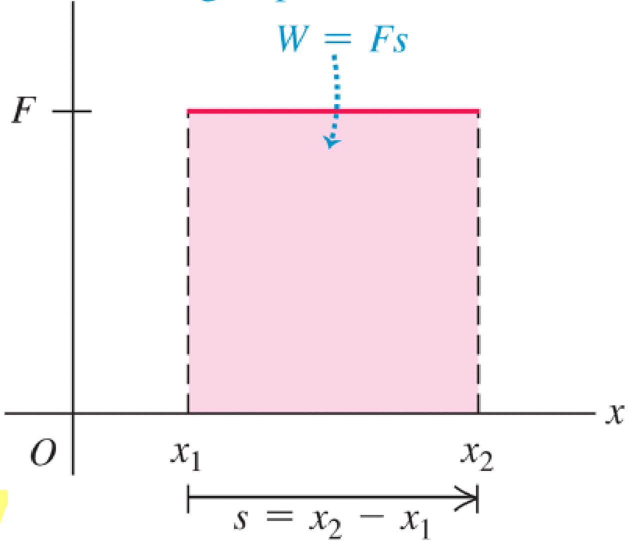


The rectangular area under the graph represents the work done by the constant force of magnitude F during displacement s:



If F_x = constant

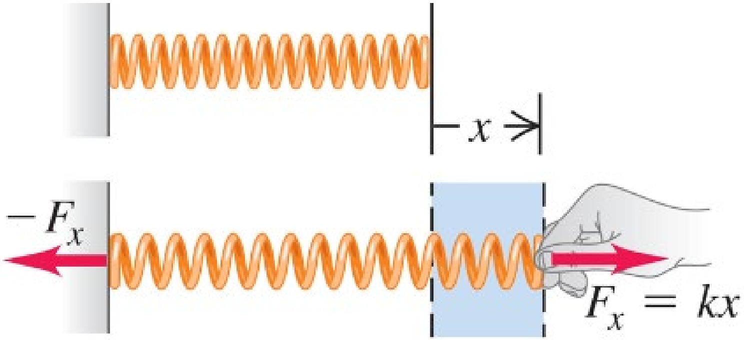
$$W = \int_{x_1}^{x_2} F_x \, dx = F_x \int_{x_1}^{x_2} dx$$

$$= F_x (x_2 - x_1)$$

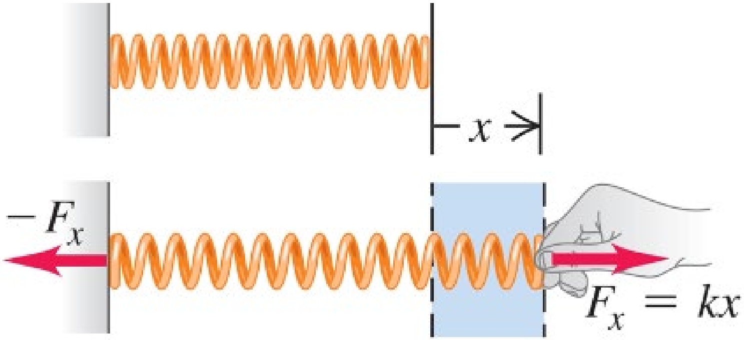
$$= Fs$$

Work to compress or stretch spring

Work to compress or stretch spring



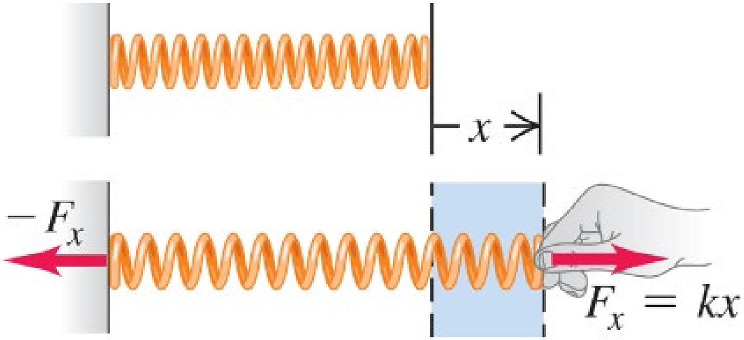
Work to compress or stretch spring



$$F = Kx$$

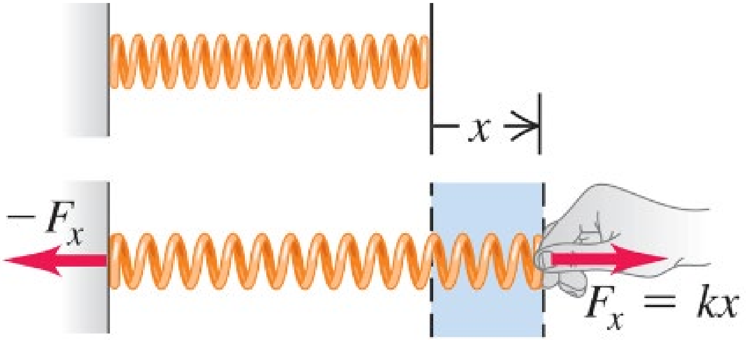
↳ required to stretch

Work to compress or stretch spring



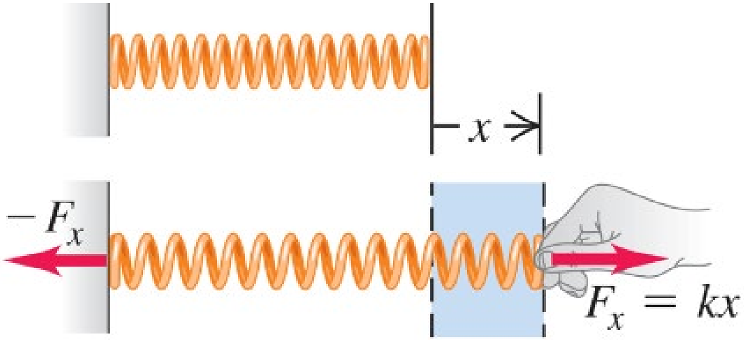
$F = Kx$ so $W = \int F\,dx$

Work to compress or stretch spring



$$F = Kx \text{ so } W = \int F\,dx = \int_{x_1}^{x_2} Kx\,dx$$

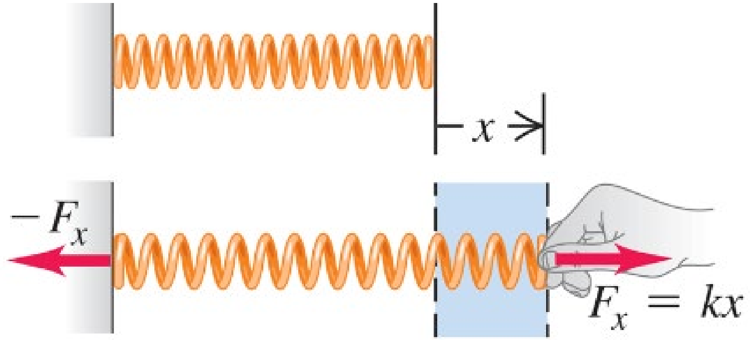
Work to compress or stretch spring



$F = kx$ so $w = \int F\,dx = \int_{x_1}^{x_2} kx\,dx$

$\Rightarrow \quad W = \frac{1}{2}kx^2 \Big|_{x_1}^{x_2}$

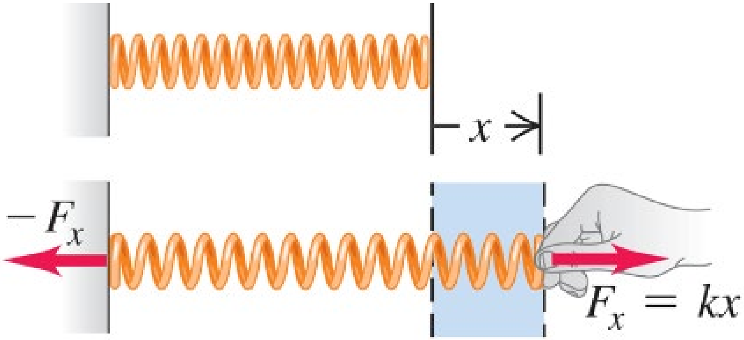
Work to compress or stretch spring



$$F = Kx \quad \text{so} \quad W = \int F\,dx = \int_{x_1}^{x_2} kx\,dx$$

$$\Rightarrow \quad W = \frac{1}{2}kx^2 \Big|_{x_1}^{x_2} \Rightarrow W = \frac{1}{2}k(x_2^2 - x_1^2)$$

Work to compress or stretch spring



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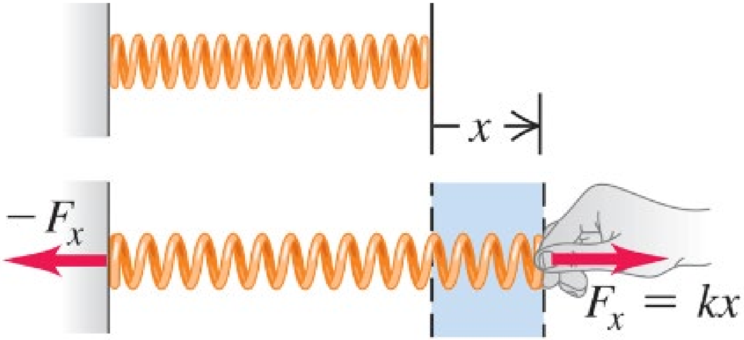
If $x_1 = 0$

Work to compress or stretch spring

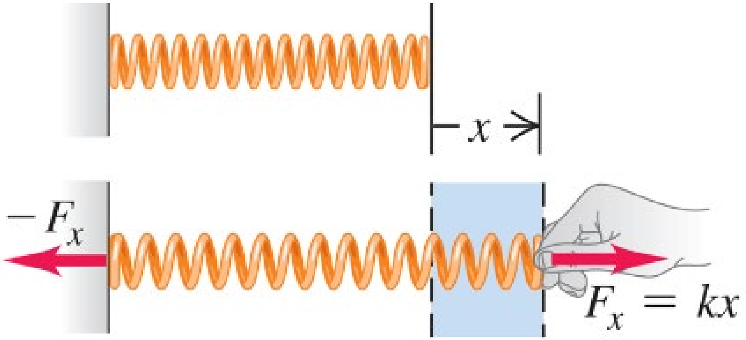


$$F = kx \text{ so } W = \int F\,dx = \int_{x_1}^{x_2} kx\,dx$$

$$\Rightarrow \quad W = \frac{1}{2}kx^2 \Big|_{x_1}^{x_2} \Rightarrow W = \frac{1}{2}k(x_2^2 - x_1^2)$$

If $x_1 = 0$ [Not stretched or compressed]

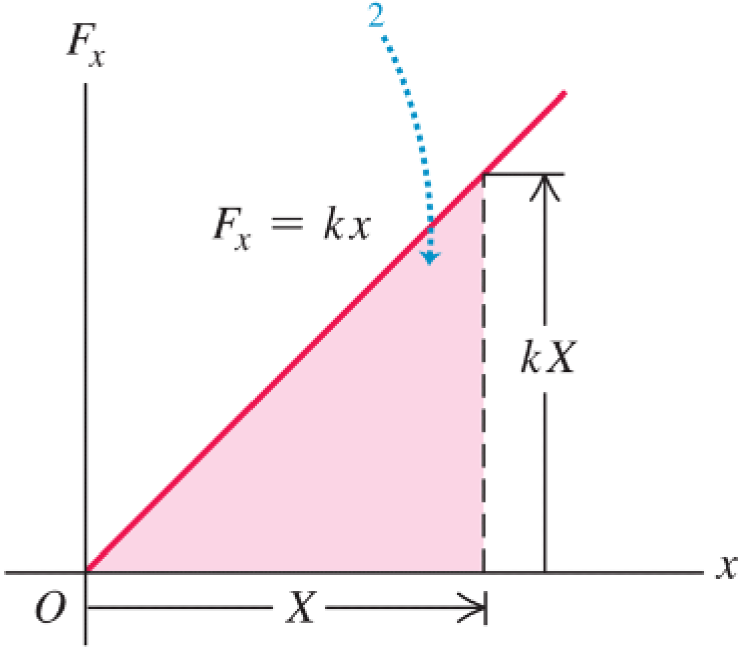
Work to compress or stretch spring



$F = Kx$ so $W = \int F\,dx = \int_{x_1}^{x_2} Kx\,dx$

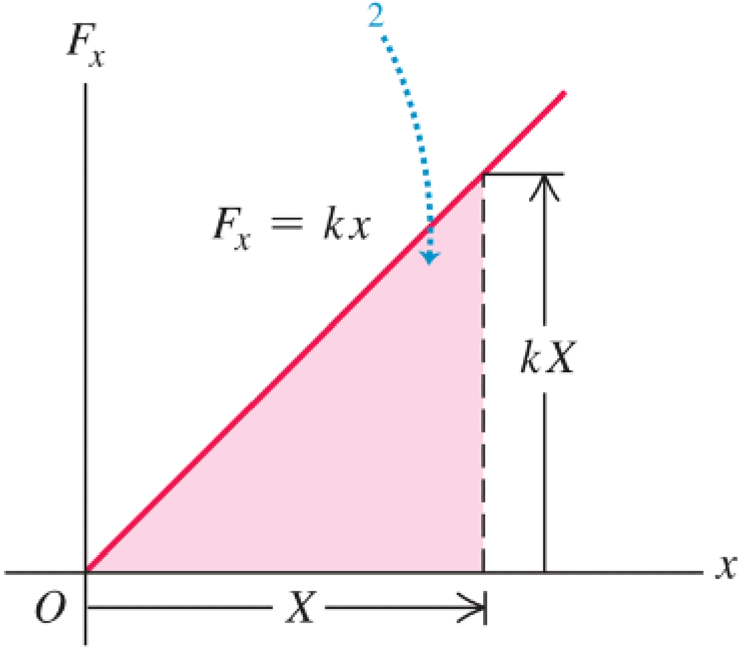
$\Rightarrow$ $W = \frac{1}{2}Kx^2 \Big|_{x_1}^{x_2}$ $\Rightarrow$ $W = \frac{1}{2}K(x_2^2 - x_1^2)$

If $x_1 = 0$ $W = \frac{1}{2}Kx^2$



The area under the graph represents the work done on the spring as the spring is stretched from $x = 0$ to a maximum value X:

$$W = \frac{1}{2}kX^2$$



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Area of triangle $= \frac{1}{2}$(base)(height)

The area under the graph represents the work done on the spring as the spring is stretched from $x = 0$ to a maximum value X:

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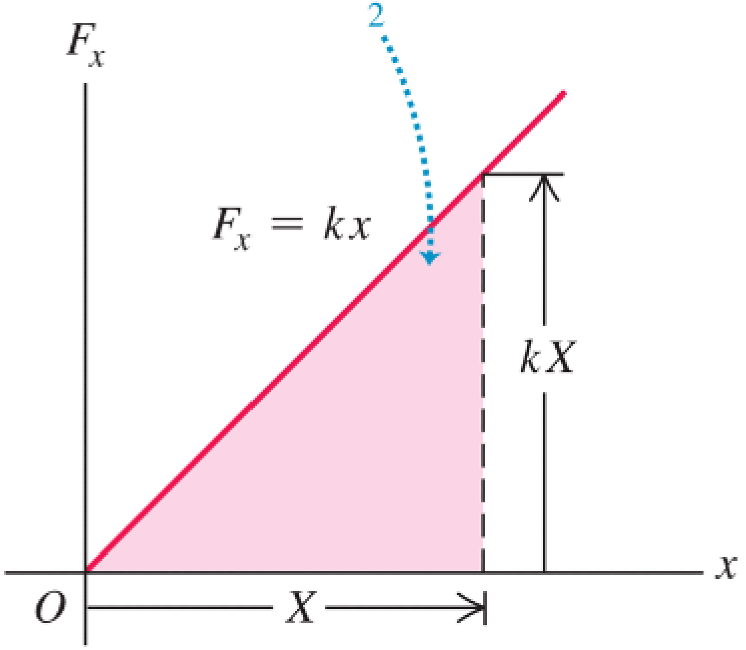
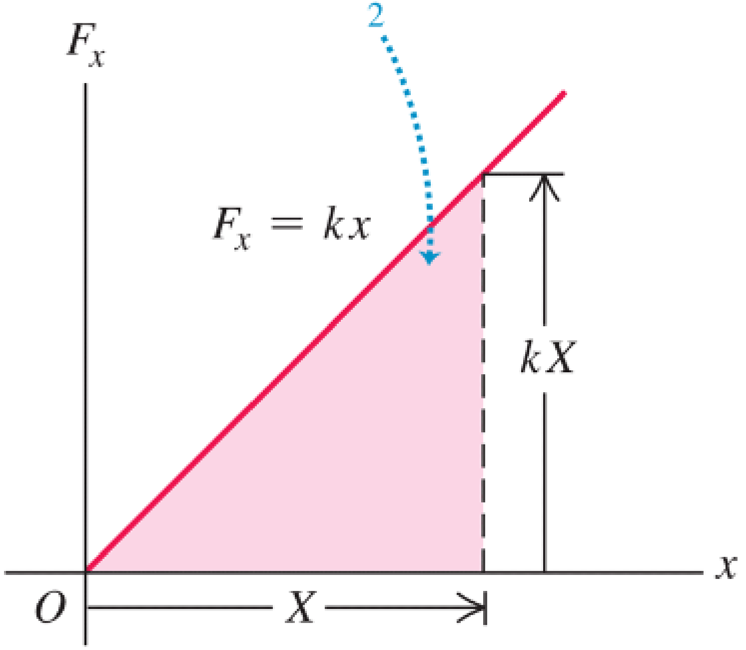


Area of triangle $= \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}(X)(kX)$

The area under the graph represents the work done on the spring as the spring is stretched from $x = 0$ to a maximum value X:

$$W = \frac{1}{2}kX^2$$



Area of triangle $= \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}(X)(kX)$

$= \frac{1}{2}kX^2$

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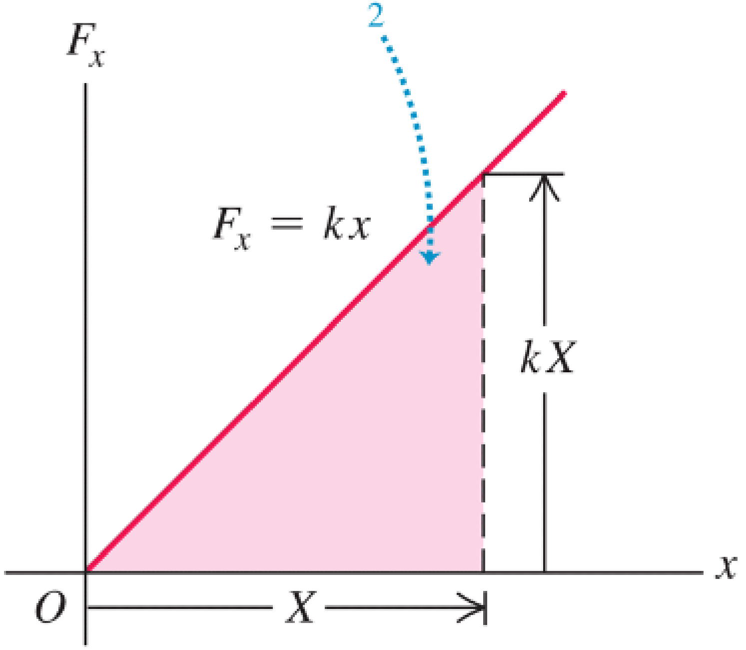

Area of triangle $= \frac{1}{2}(\text{base})(\text{height})$

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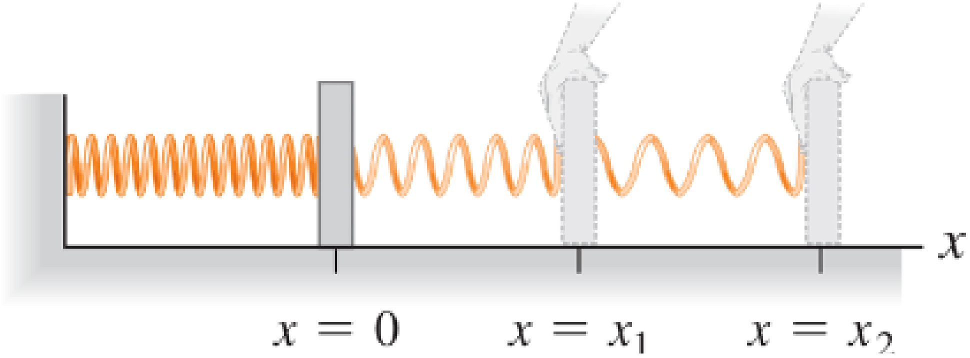
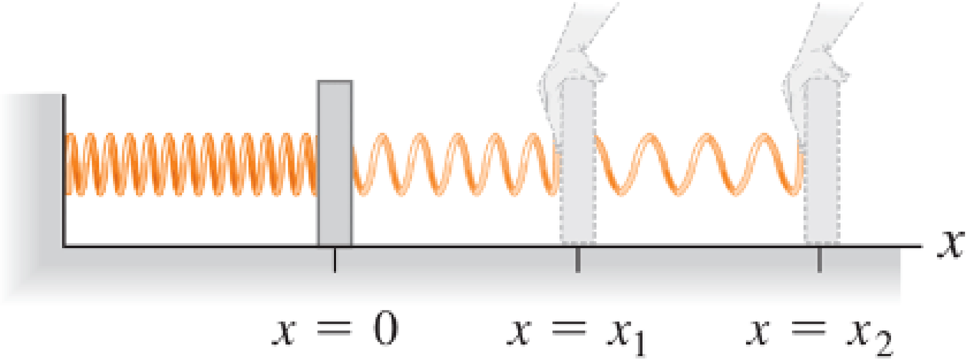
(a) Stretching a spring from elongation x_1 to elongation x_2

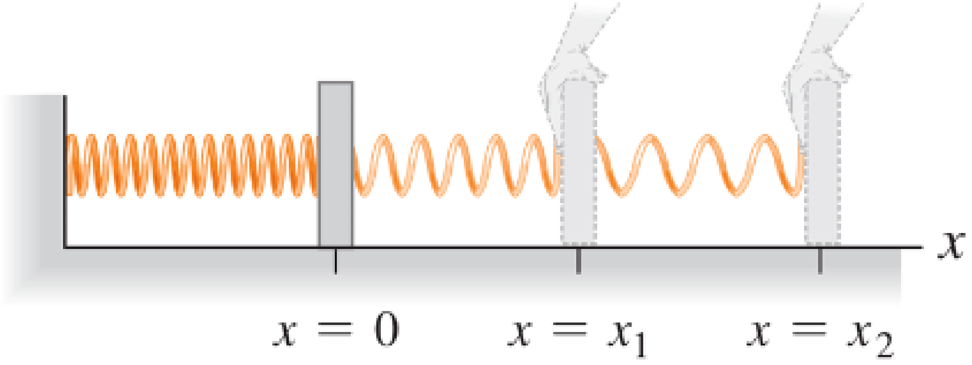


(a) Stretching a spring from elongation x_1 to elongation x_2



$$W = \frac{1}{2}K\left(X_2^2 - X_1^2\right)$$

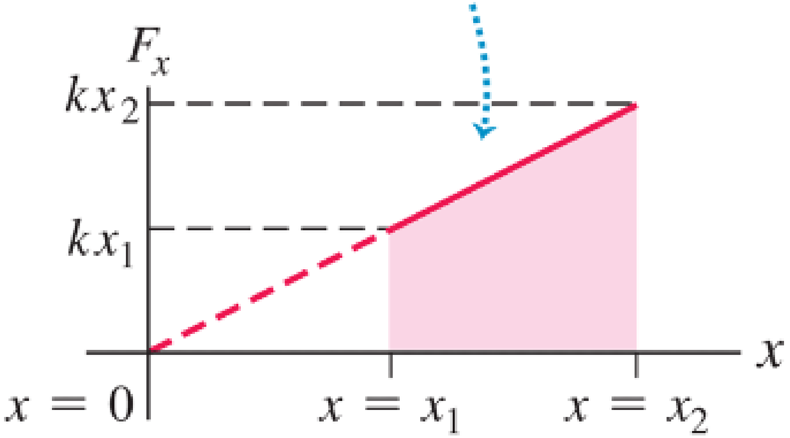
(a) Stretching a spring from elongation x_1 to elongation x_2



$$W = \frac{1}{2}K(X_2^2 - X_1^2)$$

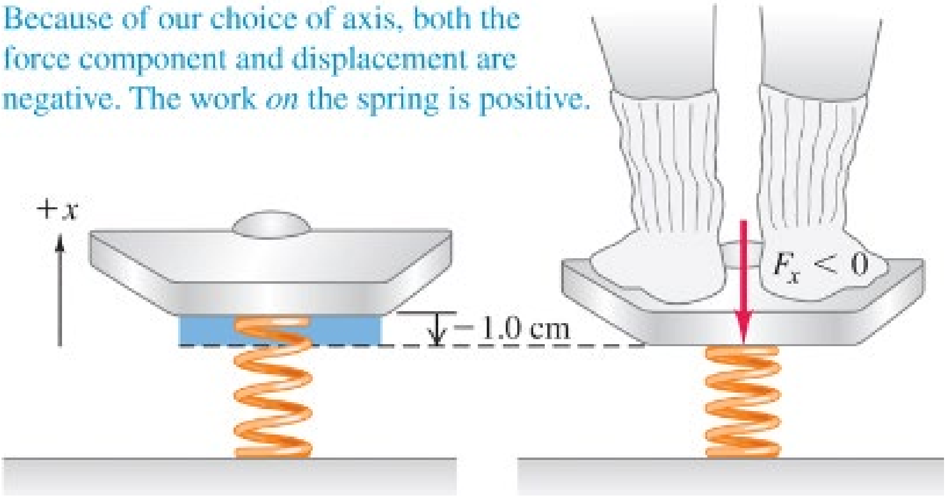
(b) Force-versus-distance graph

The trapezoidal area under the graph represents the work done on the spring to stretch it from $x = x_1$ to $x = x_2$: $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$.



Graphical

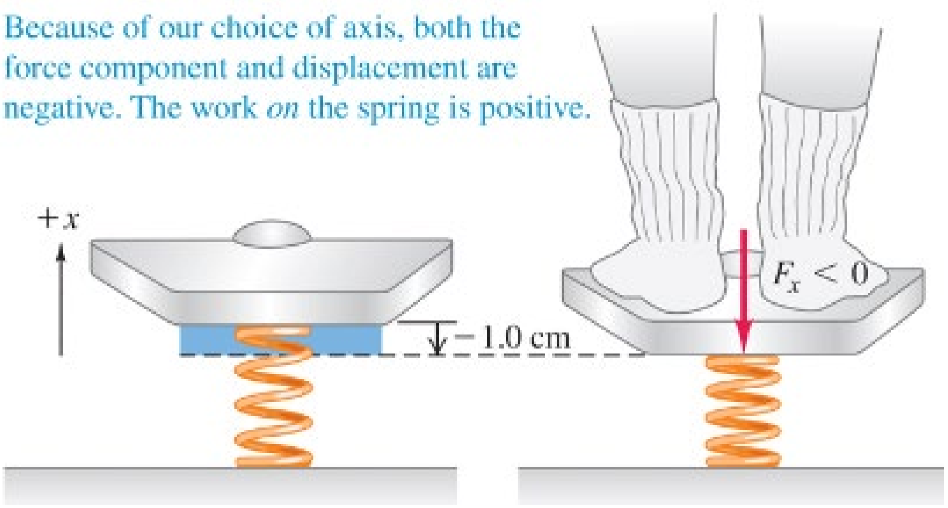
A woman weighing 600 N steps on a bathroom scale that contains a stiff spring (Fig. 6.21). In equilibrium, the spring is compressed 1.0 cm under her weight. Find the force constant of the spring and the total work done on it during the compression.



A woman weighing 600 N steps on a bathroom scale that contains a stiff spring (Fig. 6.21). In equilibrium, the spring is compressed 1.0 cm under her weight. Find the force constant of the spring and the total work done on it during the compression.

$mg = 600N$

$\Delta x = 1cm = 0.01m$



Because of our choice of axis, both the force component and displacement are negative. The work *on* the spring is positive.

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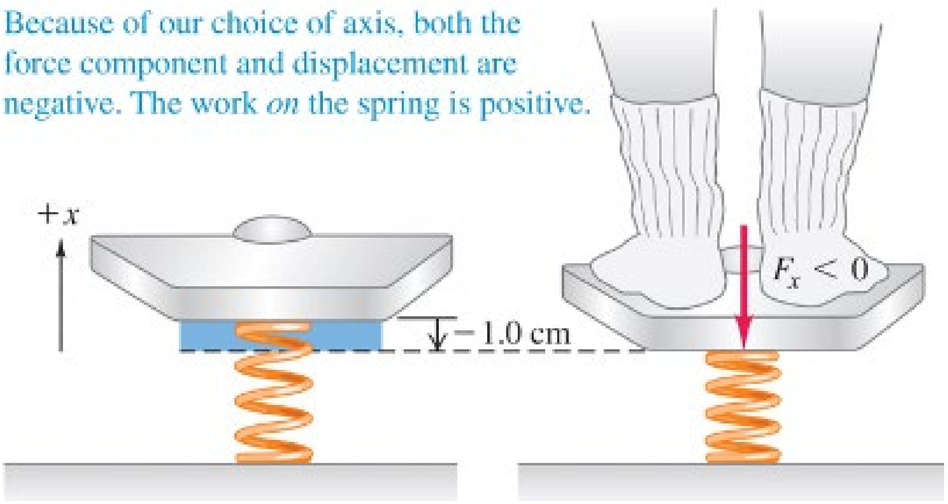
Find k: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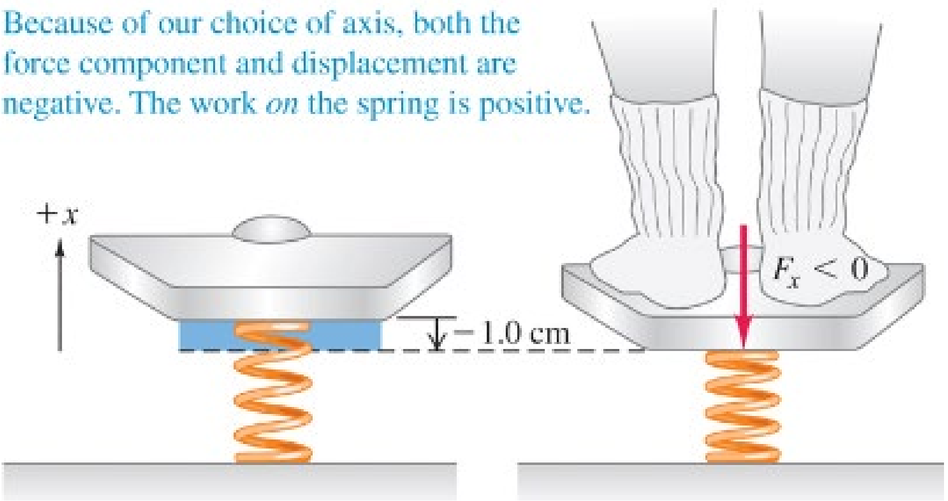
Find k:

$F = ma$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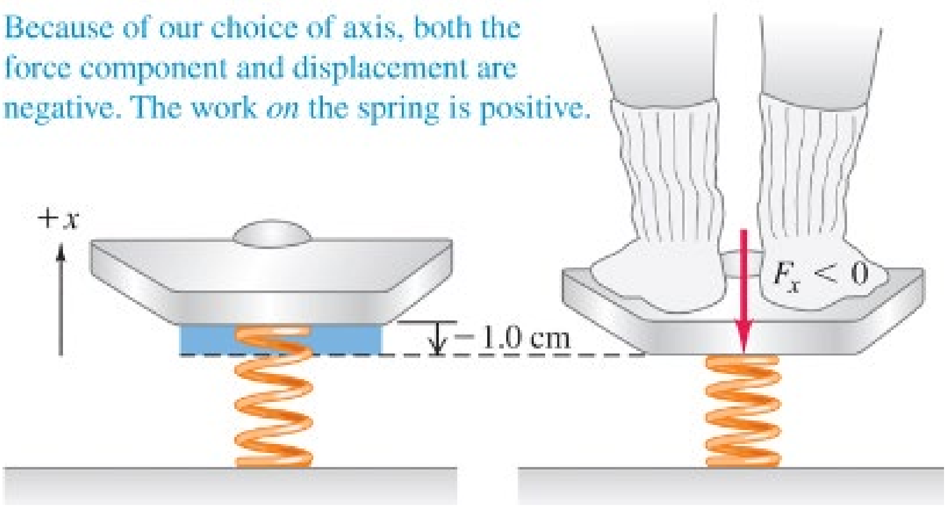
$\Delta x = 1cm = 0.01m$

Find k:

$F = ma \Rightarrow kx = mg$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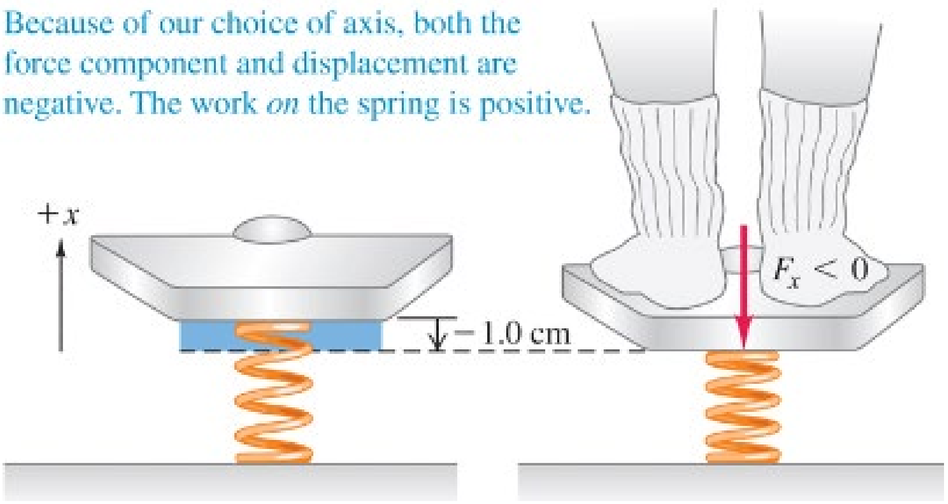

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$$k = \frac{mg}{\Delta x}$$

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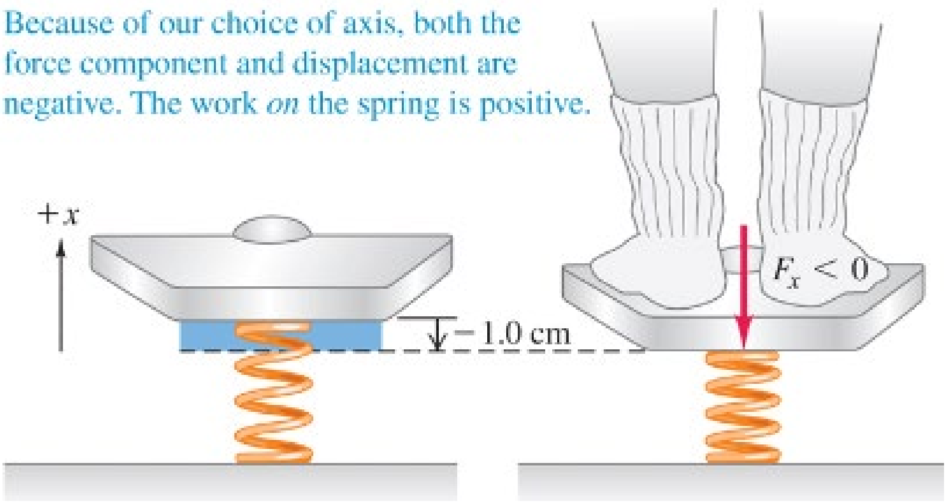

Because of our choice of axis, both the force component and displacement are negative. The work *on* the spring is positive.

Find k:

$F = ma \Rightarrow kx = mg$

$$k = \frac{mg}{\Delta x} = \frac{600N}{0.01m}$$

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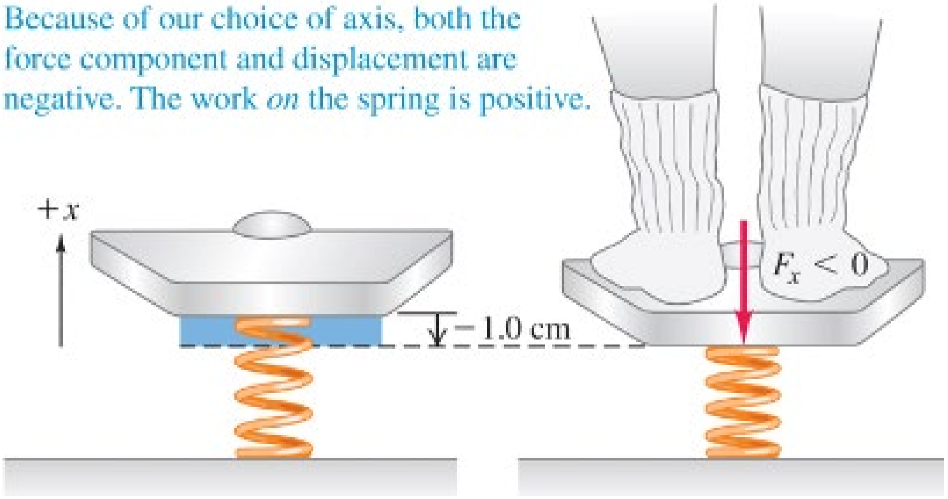
Find k:

$F = ma \Rightarrow kx = mg$

$k = \frac{mg}{\Delta x} = \frac{600N}{0.01m} = 60kN$



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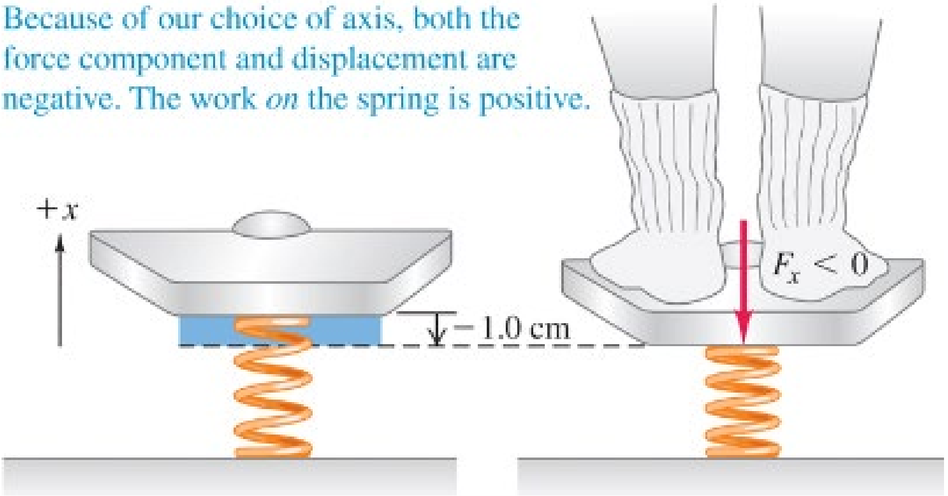
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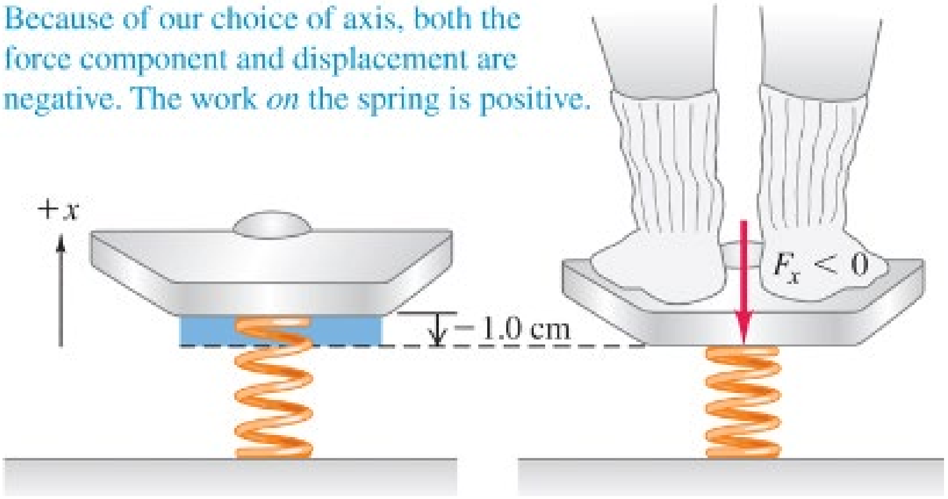

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Find W:

$\Rightarrow \quad W = \frac{1}{2} k x^2$

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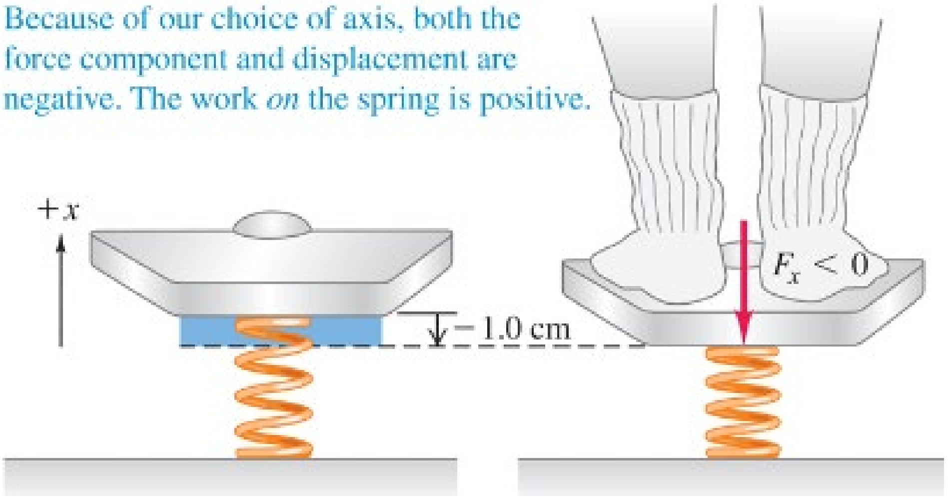
$\Delta x = 1\text{cm} = 0.01\text{m}$



Because of our choice of axis, both the force component and displacement are negative. The work *on* the spring is positive.

Find k:

$F = ma \Rightarrow kx = mg$

$K = \frac{mg}{\Delta x} = \frac{600\text{N}}{0.01\text{m}} = 60\text{KN}$

Find W:

$\Rightarrow W = \frac{1}{2} K x^2 = \left(\frac{60\text{KN}}{2}\right) * \left(\frac{1\text{m}}{100}\right)^2$

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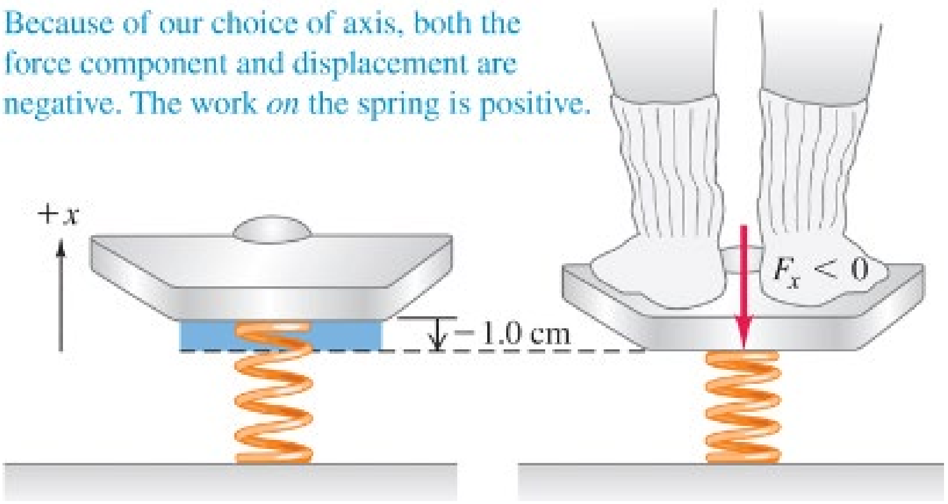
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Find k:

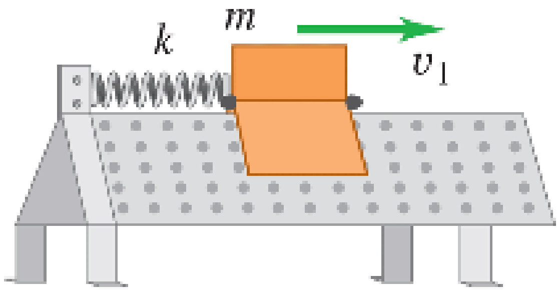
$F = ma \Rightarrow kx = mg$

$K = \frac{mg}{\Delta x} = \frac{600N}{0.01m} = 60KN$

Find W:

$\Rightarrow \quad W = \frac{1}{2} K x^2 = \left(\frac{60KN}{2}\right) * \left(\frac{1m}{100}\right)^2 = 3J$

An air-track glider of mass 0.100 kg is attached to the end of a horizontal air track by a spring with force constant $20.0\,\mathrm{N/m}$ (Fig. 6.22a). Initially the spring is unstretched and the glider is moving at $1.50\,\mathrm{m/s}$ to the right. Find the maximum distance d that the glider moves to the right (a) if the air track is turned on, so that there is no friction, and (b) if the air is turned off, so that there is kinetic friction with coefficient $\mu_k = 0.47$.



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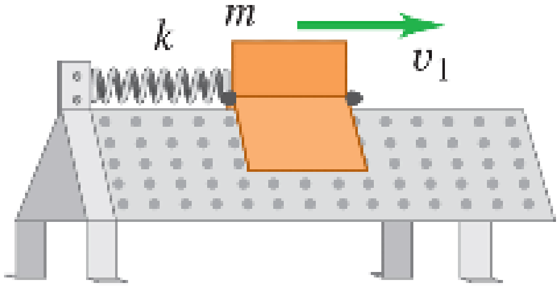
$m = 0.1\,kg,\quad k = 20\,N/m$



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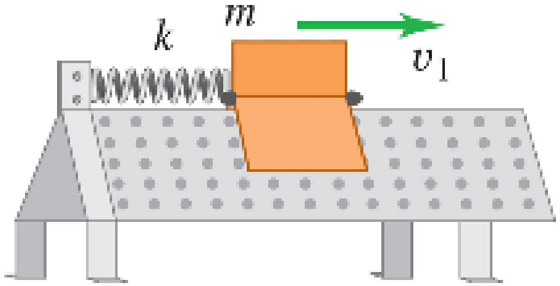
$m = 0{,}1\,kg$, $K = 20\,N/m$

$X_1 = 0$ & $V_1 = 1{,}5\,m/s$



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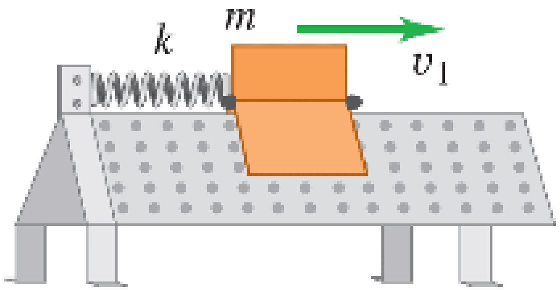
$m = 0.1\,kg$, $K = 20\,N/m$

$X_1 = 0$ & $V_1 = 1.5\,m/s$

Find d is $\mu_k = 0$:



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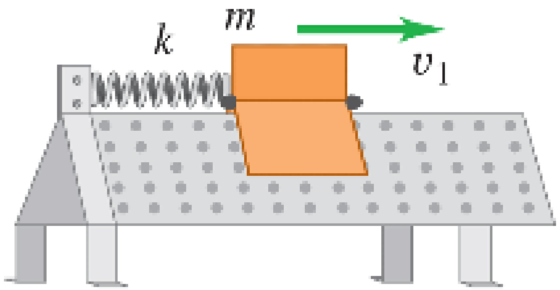
Find d is $\mu_k = 0$:

$$W_{1\to 2} = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2$$



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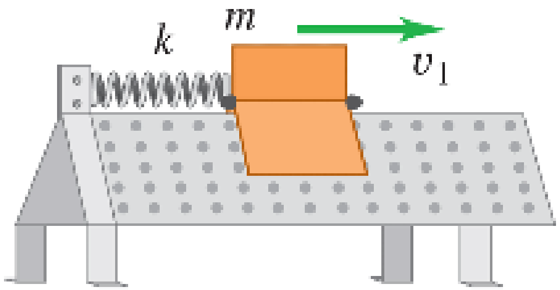
$$W_{1\to 2} = \overset{0}{\cancel{\frac{1}{2} m V_2^2}} - \frac{1}{2} m V_1^2$$



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$X_1 = 0$ & $V_1 = 1.5\,m/s$

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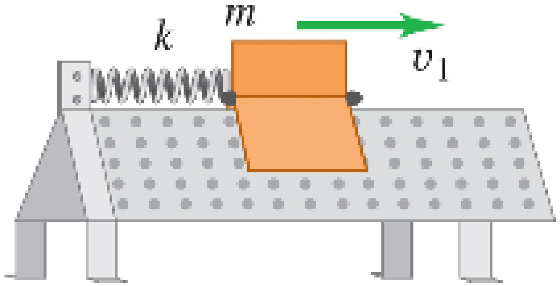


$$W_{1\to 2} = \overset{0}{\cancel{\tfrac{1}{2} m V_2^2}} - \tfrac{1}{2} m V_1^2 \Rightarrow -\tfrac{1}{2} K d^2 = -\tfrac{1}{2} m V_1^2$$

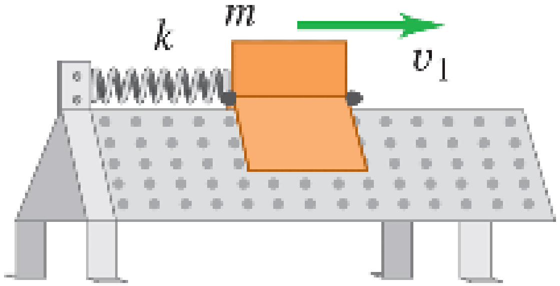
Note work done on spring > 0

An air-track glider of mass 0.100 kg is attached to the end of a horizontal air track by a spring with force constant $20.0\,\mathrm{N/m}$ (Fig. 6.22a). Initially the spring is unstretched and the glider is moving at $1.50\,\mathrm{m/s}$ to the right. Find the maximum distance d that the glider moves to the right (a) if the air track is turned on, so that there is no friction, and (b) if the air is turned off, so that there is kinetic friction with coefficient $\mu_k = 0.47$.

$m = 0.1\,\mathrm{kg}$, $K = 20\,\mathrm{N/m}$

$X_1 = 0$ & $V_1 = 1.5\,\mathrm{m/s}$

Find d is $\mu_k = 0$:



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Note work done on spring > 0

But work done on object < 0

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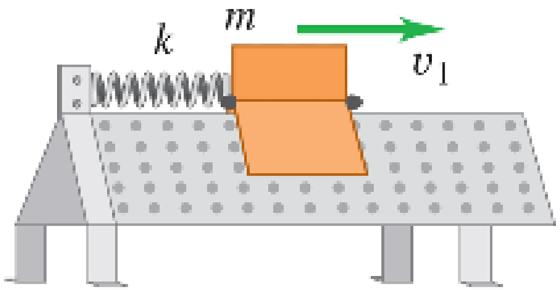

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$$\Rightarrow \quad d^2 = \frac{m}{K} V_1^2$$

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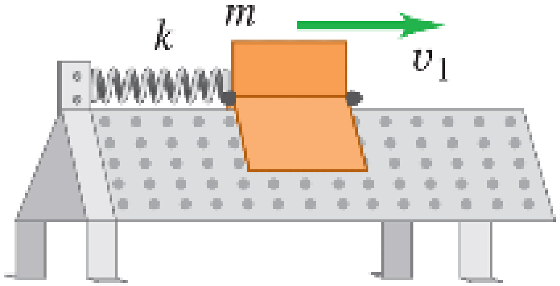

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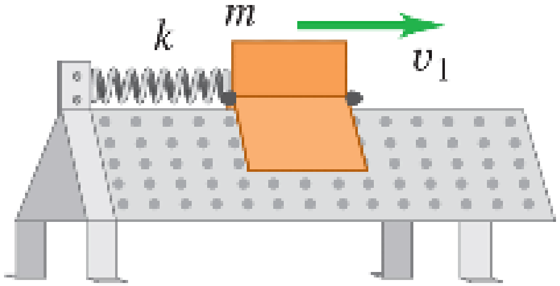

$$W_{1\to 2} = \overset{0}{\cancel{\frac{1}{2} m V_2^2}} - \frac{1}{2} m V_1^2 \Rightarrow -\frac{1}{2} K d^2 = -\frac{1}{2} m V_1^2$$

$$\Rightarrow d^2 = \frac{m}{K} V_1^2 \Rightarrow d = V_1 \sqrt{\frac{m}{K}}$$

$$\Rightarrow d = (1.5\,m/s)\left[\frac{0.1\,kg}{20\,kg/s^2}\right]^{1/2} = (1.5\,m)\sqrt{\frac{1}{200}}$$

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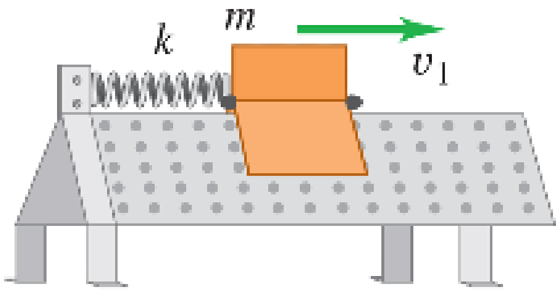

Find d is $\mu_k = 0$:

$$W_{1\to 2} = \cancel{\tfrac{1}{2} m V_2^2}^{\,0} - \tfrac{1}{2} m V_1^2 \Rightarrow -\tfrac{1}{2} K d^2 = -\tfrac{1}{2} m V_1^2$$

$$\Rightarrow \quad d^2 = \frac{m}{K} V_1^2 \Rightarrow d = V_1 \sqrt{\frac{m}{K}}$$

$$\Rightarrow \quad d = (1.5\,m/s)\left[\frac{0.1\,kg}{20\,kg/s^2}\right]^{1/2} = (1.5\,m)\sqrt{\frac{1}{200}}$$

$$= 0.106\,m$$

Work-Energy for motion along a curve

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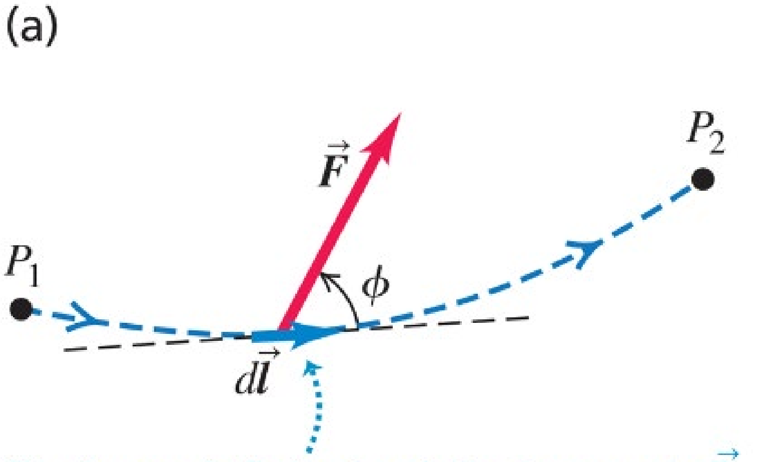

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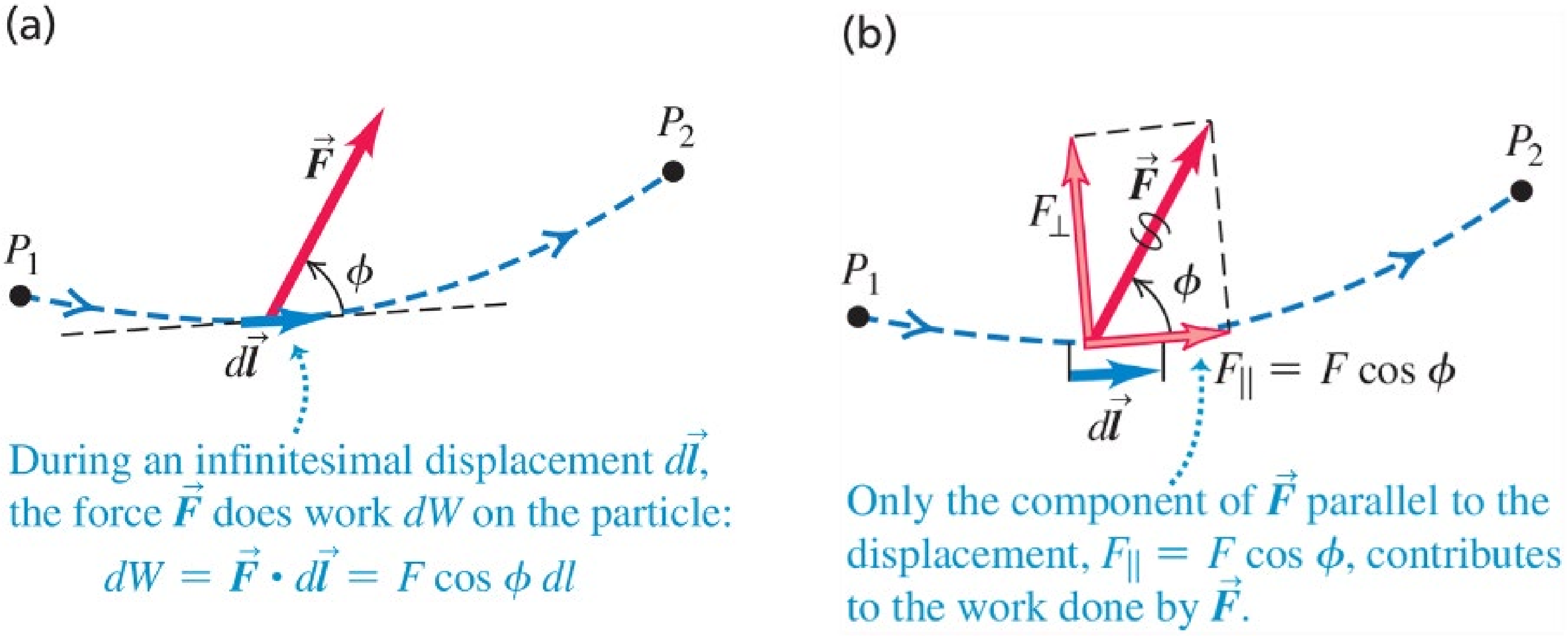

During an infinitesimal displacement $d\vec{l}$, the force $\vec{F}$ does work dW on the particle:

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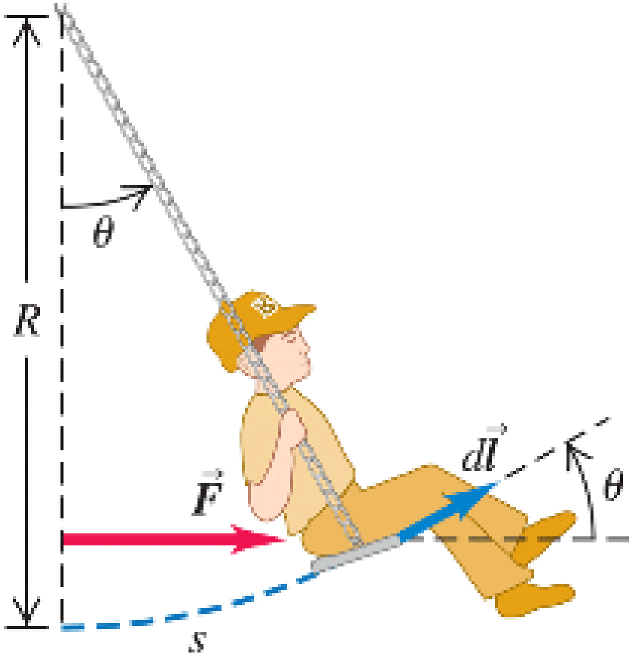
Only the component of $\vec{F}$ parallel to the displacement, $F_{\parallel} = F \cos\phi$, contributes to the work done by $\vec{F}$.

At a family picnic you are appointed to push your obnoxious cousin Throckmorton in a swing (Fig. 6.24a). His weight is w, the length of the chains is R, and you push Throcky until the chains make an angle θ_0 with the vertical. To do this, you exert a varying horizontal force $\vec{F}$ that starts at zero and gradually increases just enough that Throcky and the swing move very slowly and remain very nearly in equilibrium throughout the process. (a) What is the total work done on Throcky by all forces? (b) What is the work done by the tension T in the chains? (c) What is the work you do by exerting force $\vec{F}$? (Ignore the weight of the chains and seat.)



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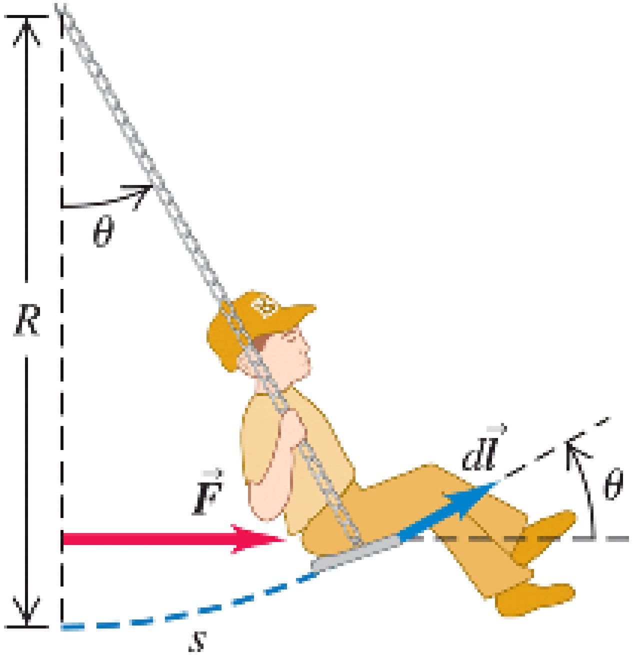

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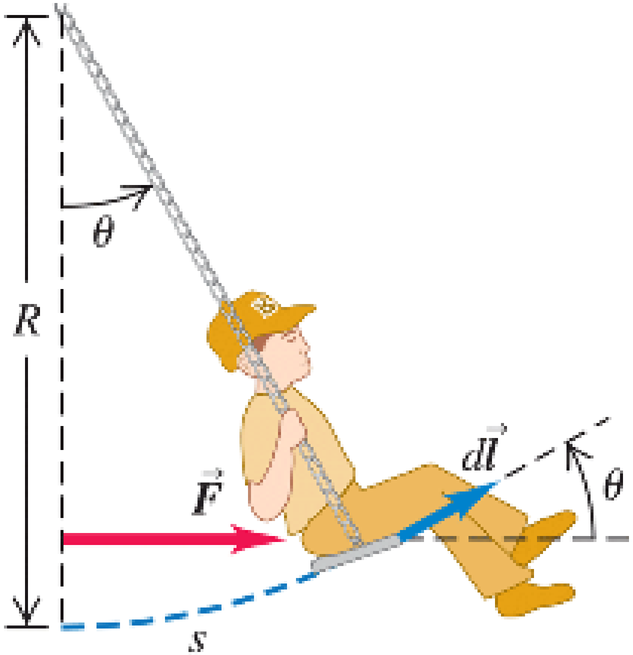

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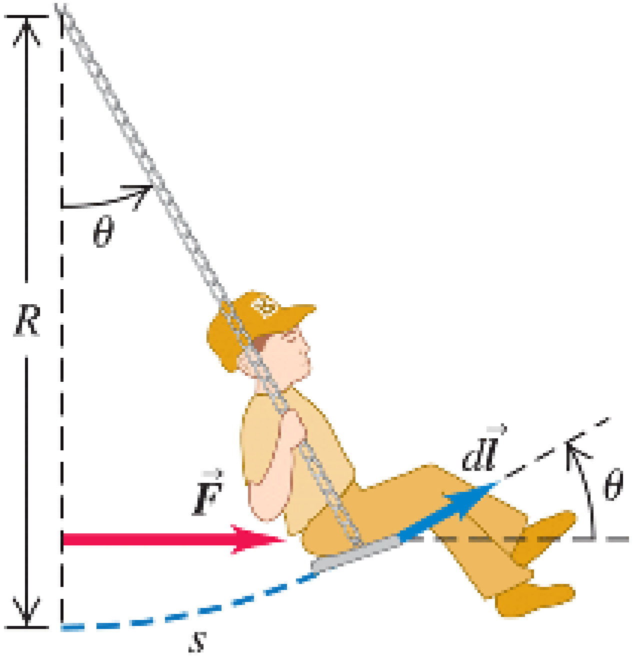

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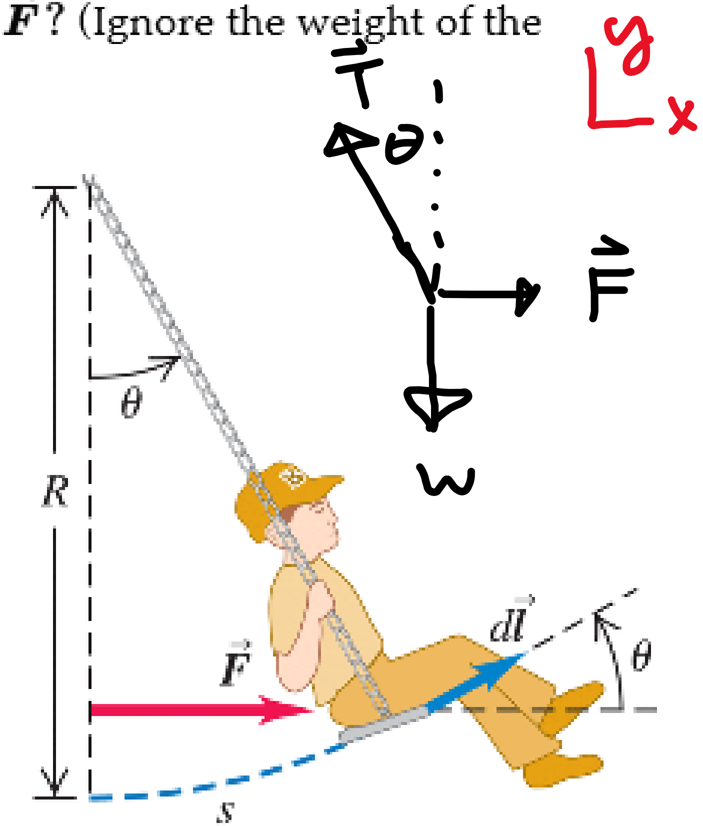

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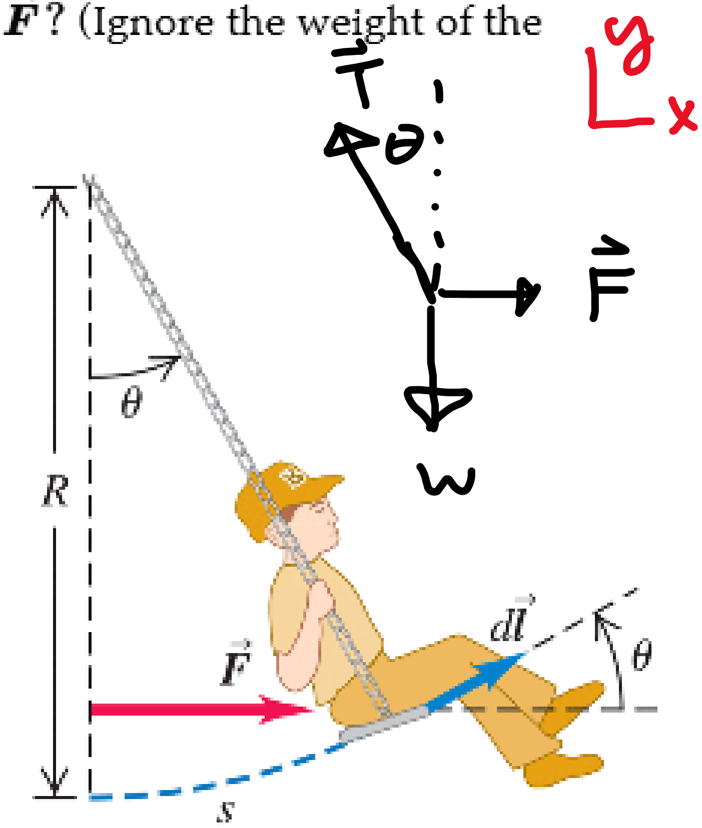

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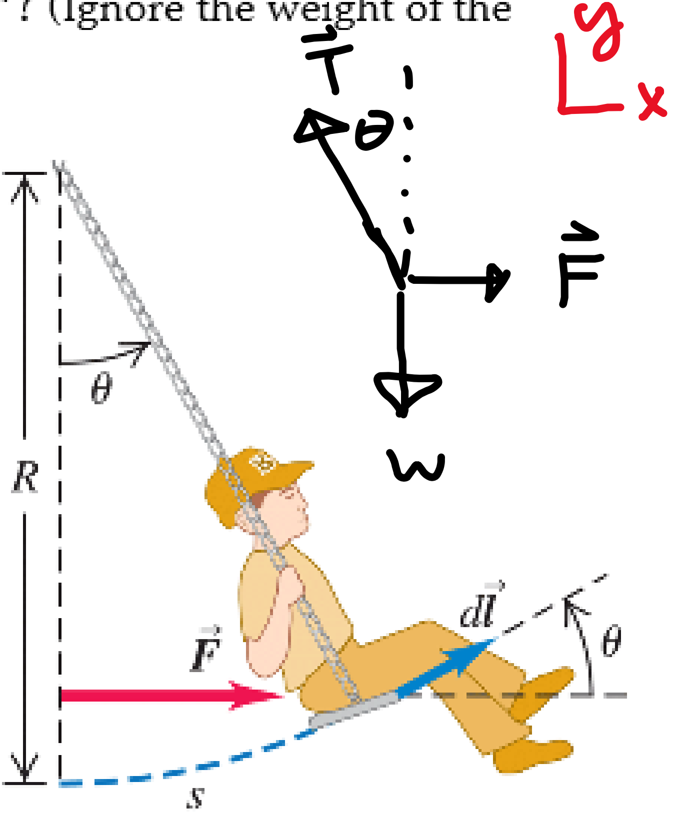

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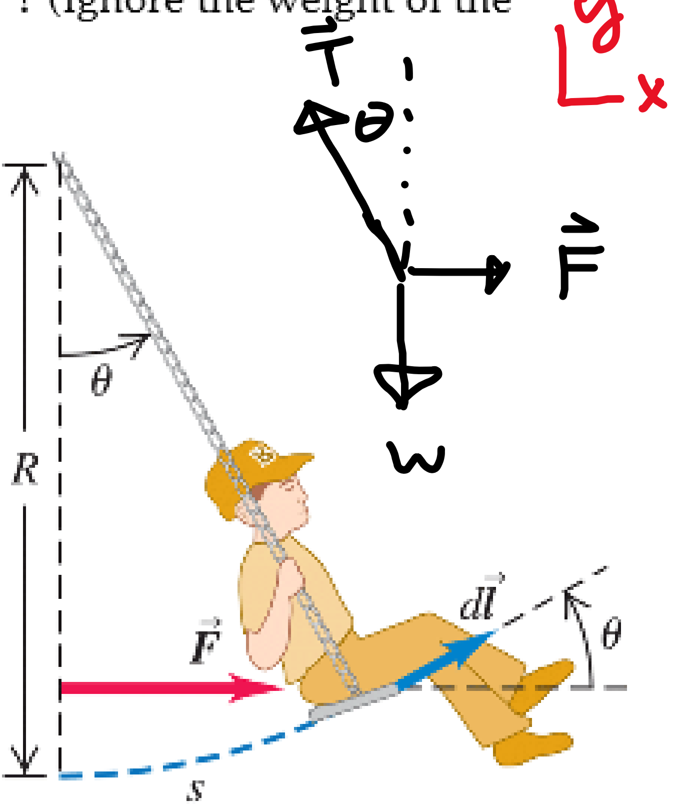

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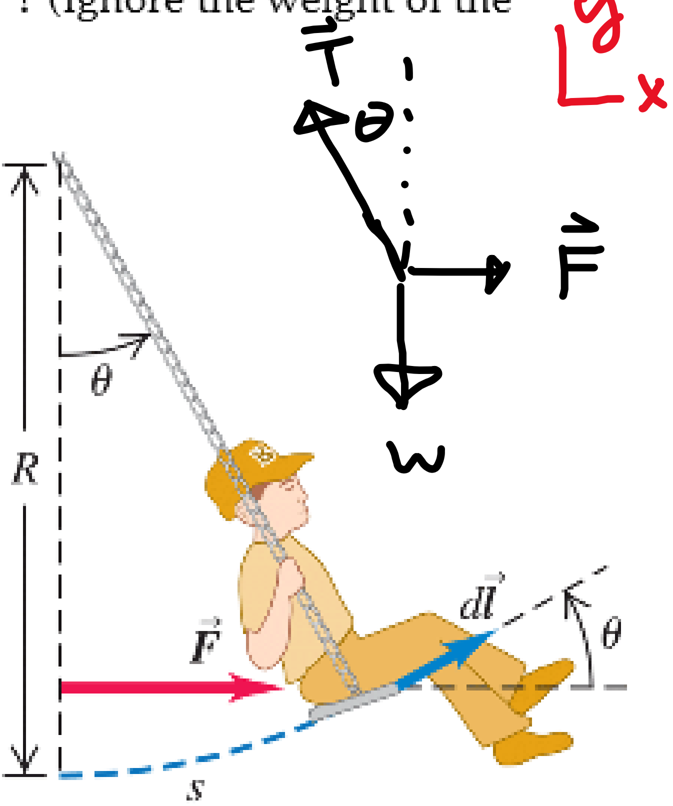

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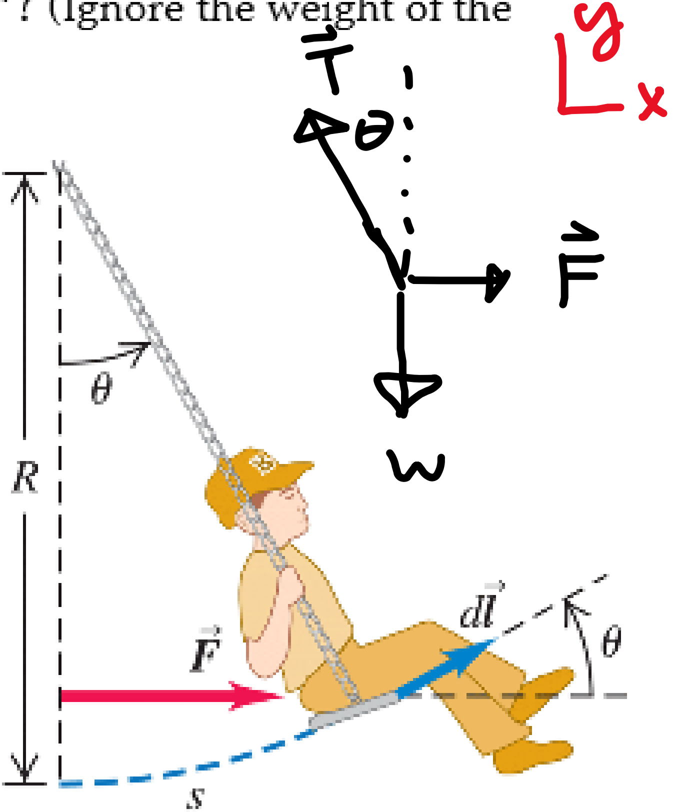

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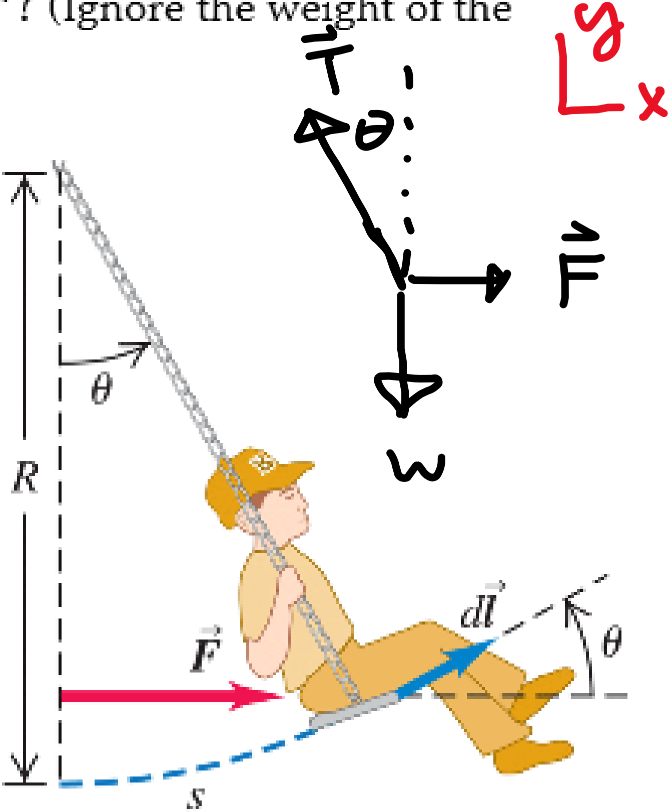
Note $\vec{T}$ is ⊥ to motion



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$w, R, \theta_0, \vec{F}$

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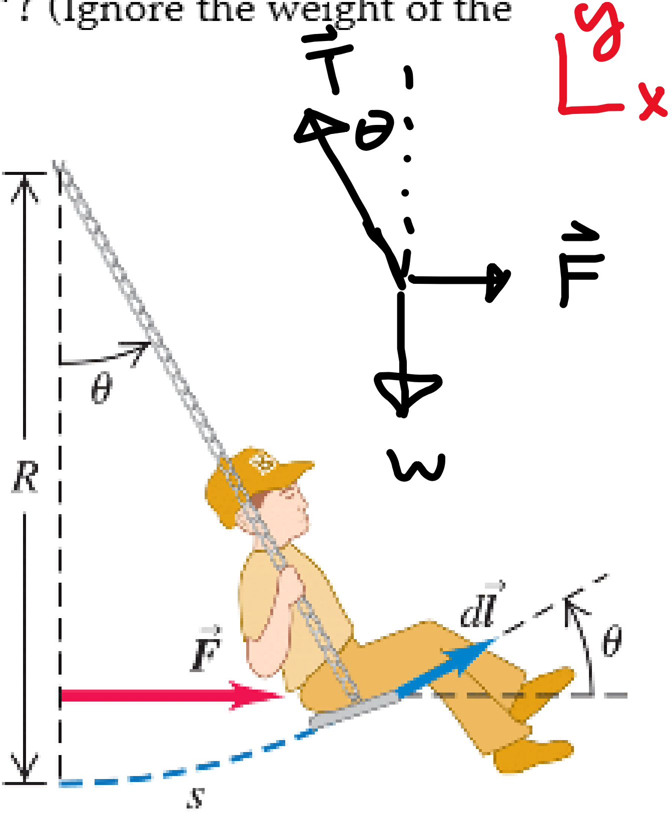
$\Sigma F_x = 0$ & $\Sigma F_y = 0$

$\Rightarrow F = T\sin\theta$ & $w = T\cos\theta$

$\Rightarrow F = w\tan\theta$

Note $\vec{T}$ is $\perp$ to motion $\Rightarrow$ No work due to T



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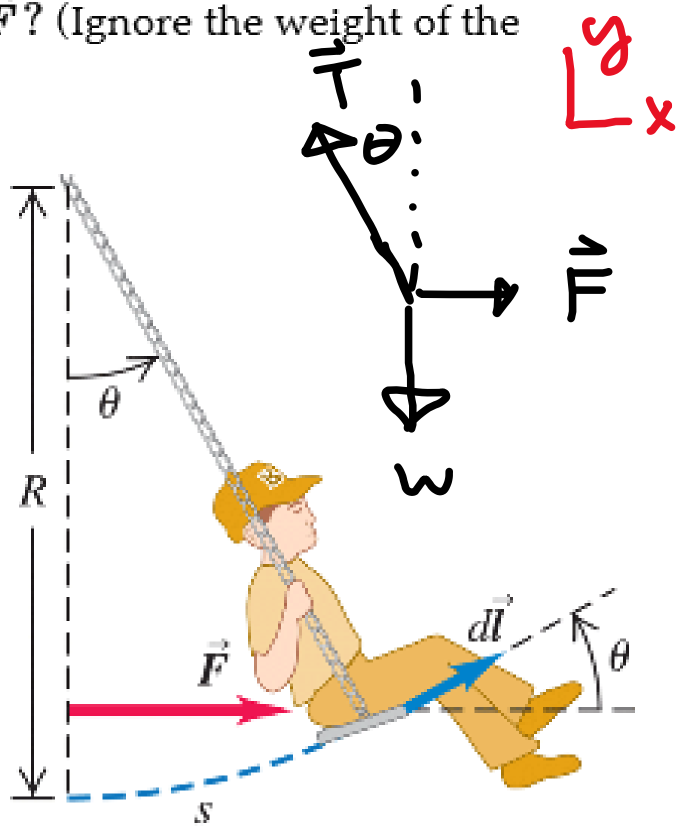
$\Rightarrow \quad F = w\tan\theta$

$d\vec{l} = (R\,d\theta)(\hat{i}\cos\theta + \hat{j}\sin\theta)$



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$w, R, \theta_0, \vec{F}$

Find W_{TOT} :

$$W_{TOT} = \int w \tan\theta \cos\theta \, R \, d\theta +$$

$$\Rightarrow \quad F = T\sin\theta \;\; \& \;\; w = T\cos\theta$$

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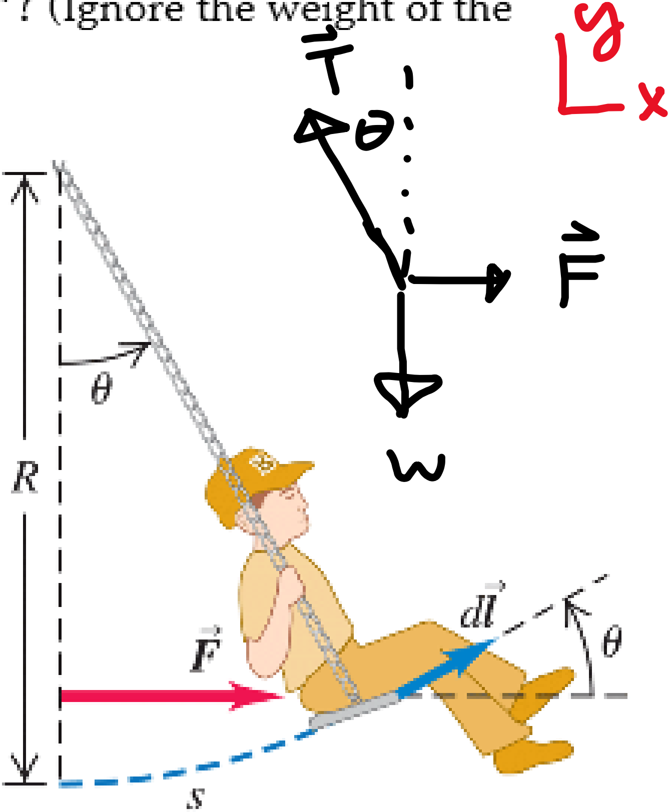

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$w, R, \theta_0, \vec{F}$

Find W_{TOT}:

$$W_{TOT} = \int w\tan\theta\cos\theta R\,d\theta + \left[-\int w\sin\theta R\theta\,d\theta\right]$$

$$\Rightarrow \quad F = T\sin\theta \;\&\; w = T\cos\theta$$

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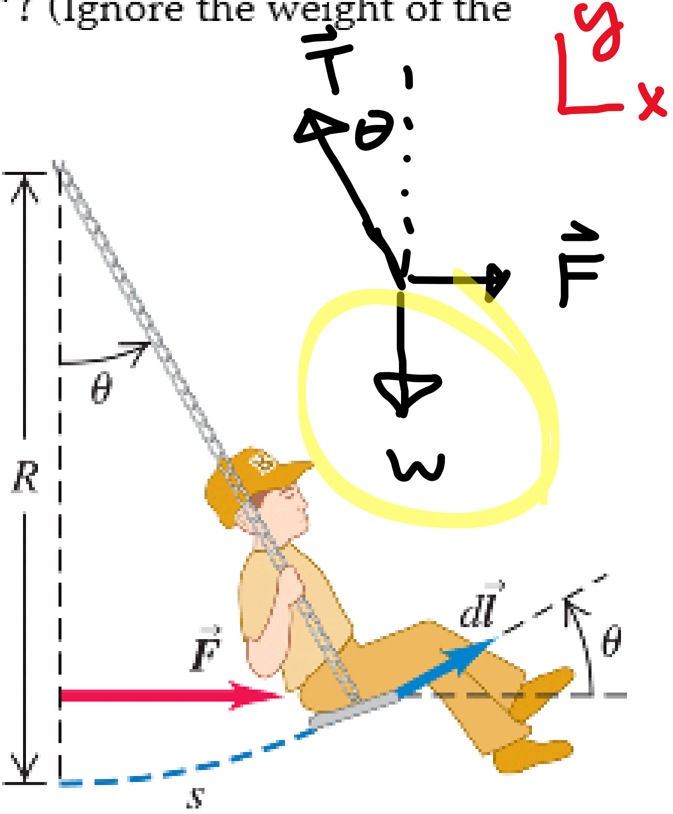
$$d\vec{l} = (R\,d\theta)(\hat{i}\cos\theta + \hat{j}\sin\theta)$$



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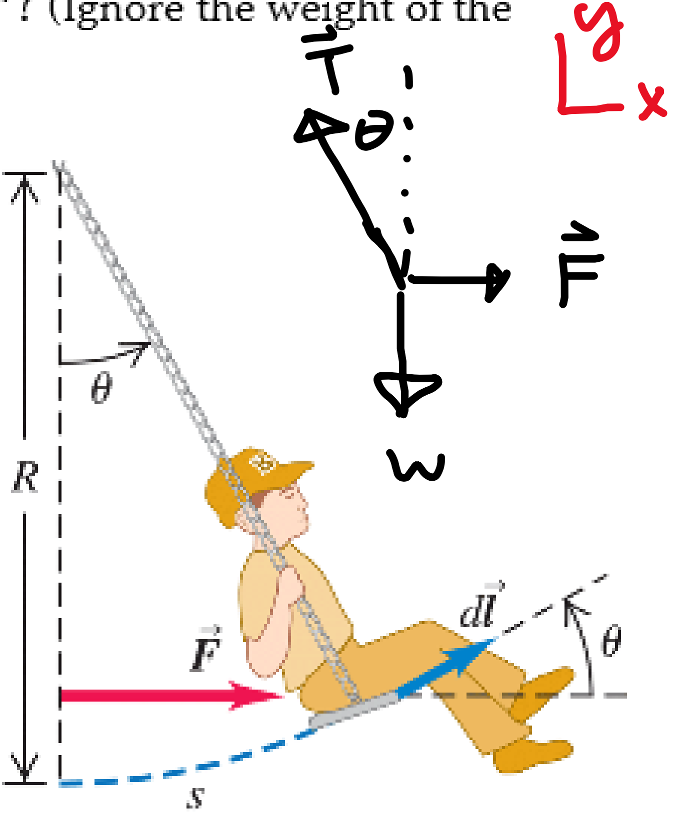
$$= W_F + W_w$$



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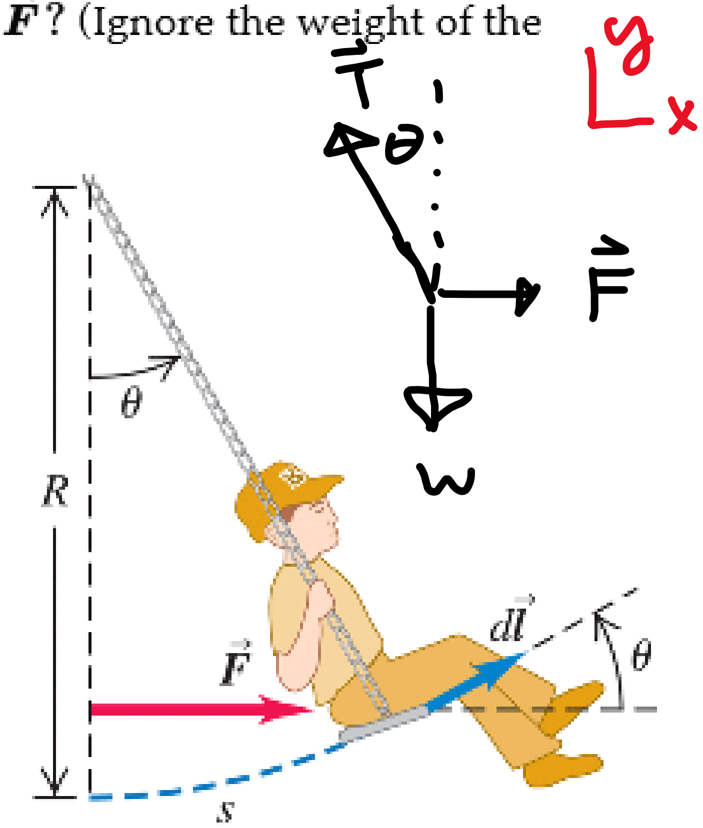
$$W_F = wR\int \sin\theta d\theta$$



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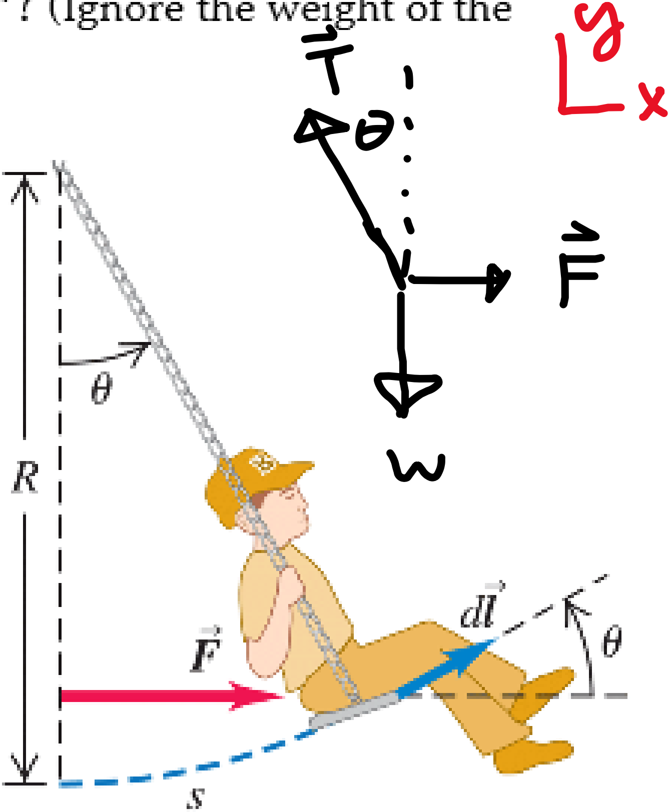

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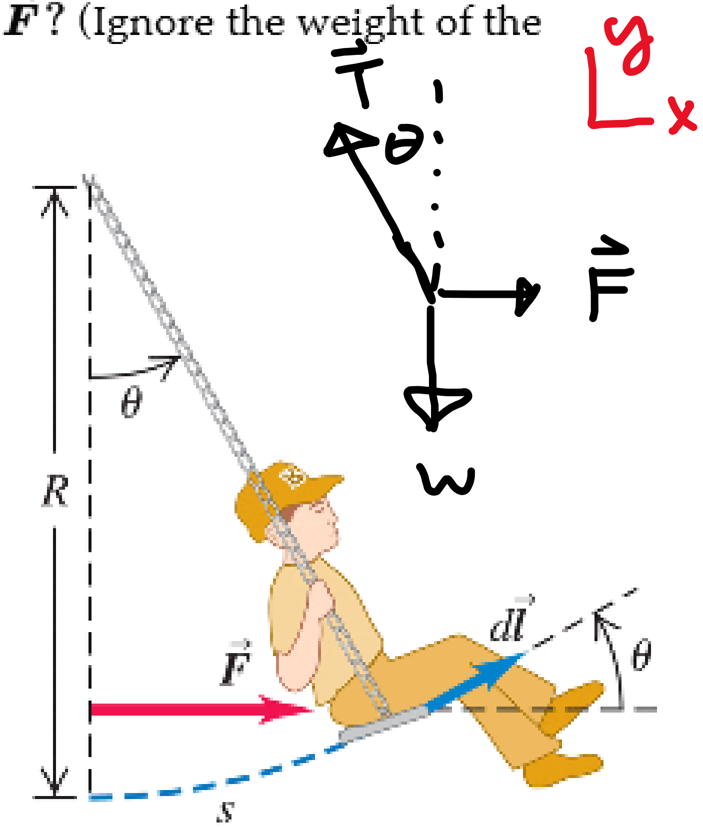
Equal & opposite



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$w, R, \Theta_0, \vec{F}$

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$$W_F = wR\int \sin\theta\, d\theta \;\&$$

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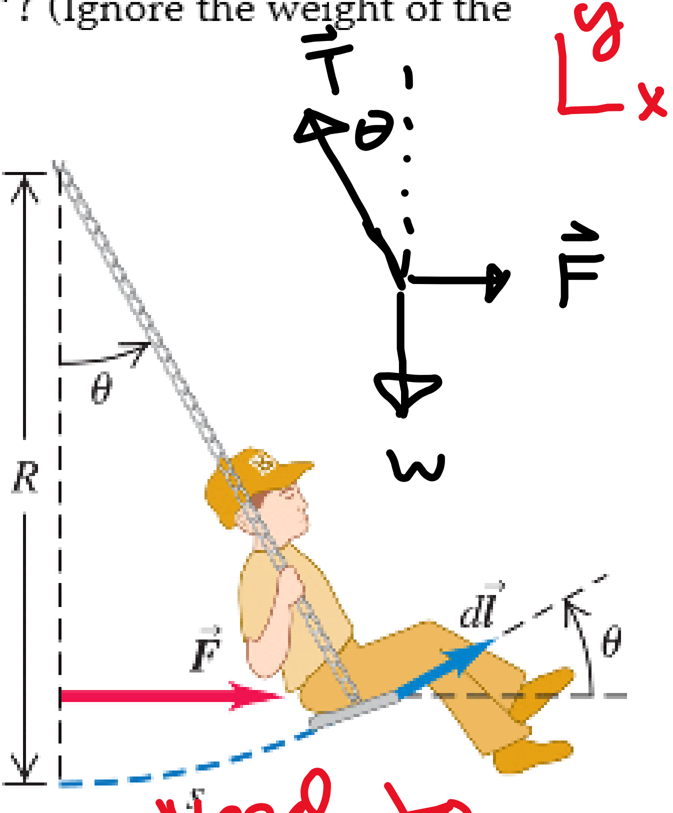
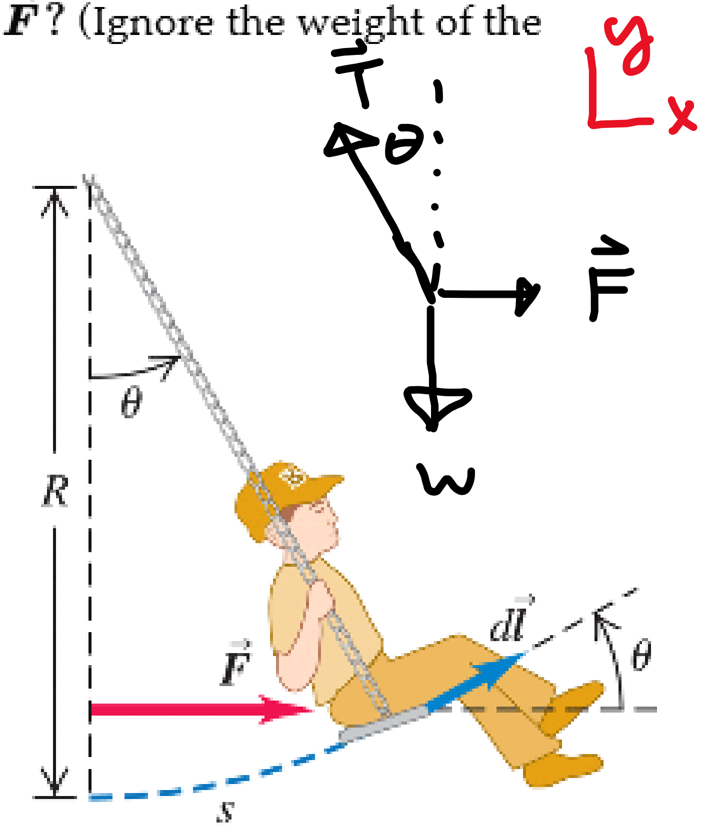


Equal & opposite only need to calculate one

At a family picnic you are appointed to push your obnoxious cousin Throckmorton in a swing (Fig. 6.24a). His weight is w, the length of the chains is R, and you push Throcky until the chains make an angle θ_0 with the vertical. To do this, you exert a varying horizontal force $\vec{F}$ that starts at zero and gradually increases just enough that Throcky and the swing move very slowly and remain very nearly in equilibrium throughout the process. (a) What is the total work done on Throcky by all forces? (b) What is the work done by the tension T in the chains? (c) What is the work you do by exerting force $\vec{F}$? (Ignore the weight of the chains and seat.) $W_F = wR\int \sin\theta\, d\theta$



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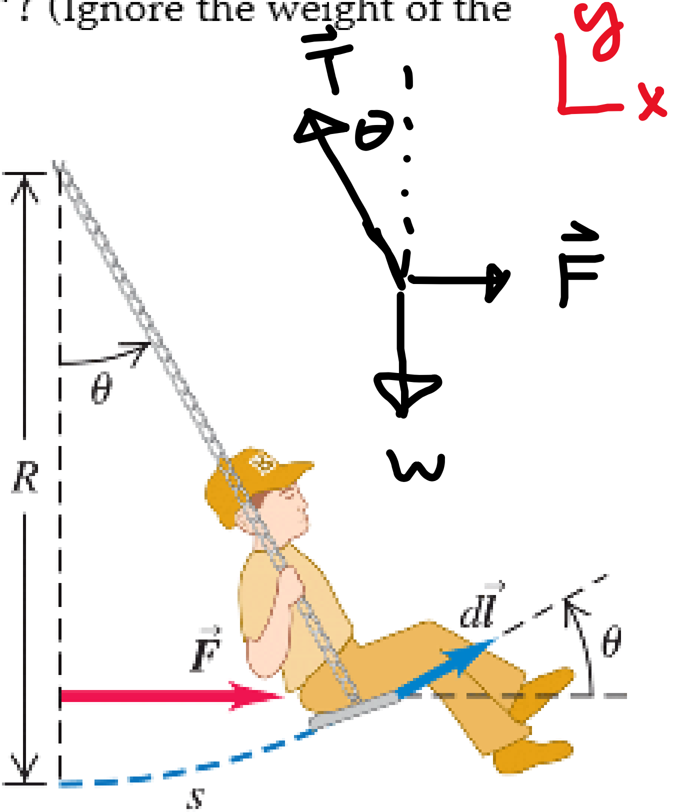
$$W_F = wR\int \sin\theta \, d\theta$$

$$= wR\left[-\cos\theta\right]\Big|_0^{\theta_0}$$



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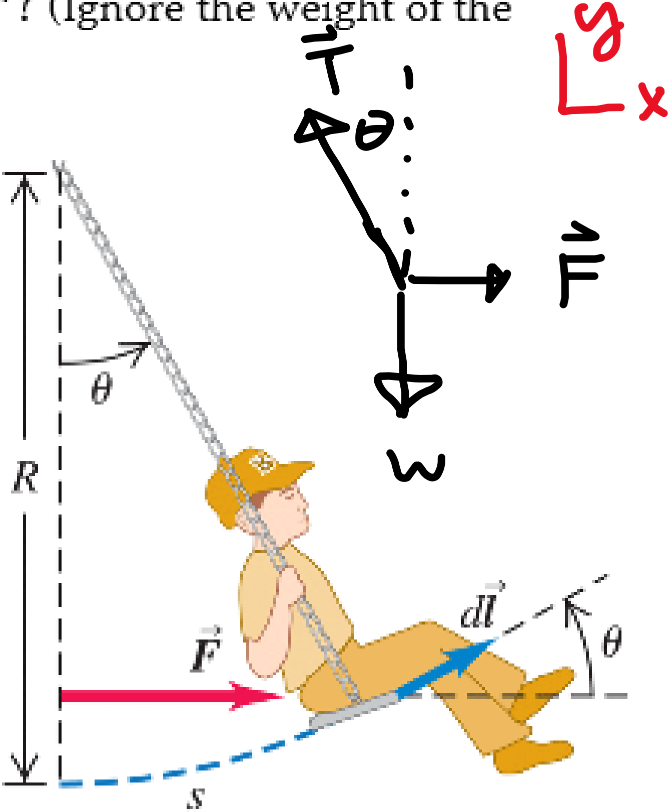
$$W_F = wR\int \sin\theta \, d\theta$$

$$= wR\left[-\cos\theta\right]\Big|_0^{\theta_0}$$

$$= wR\left[-\cos\theta_0 + 1\right]$$



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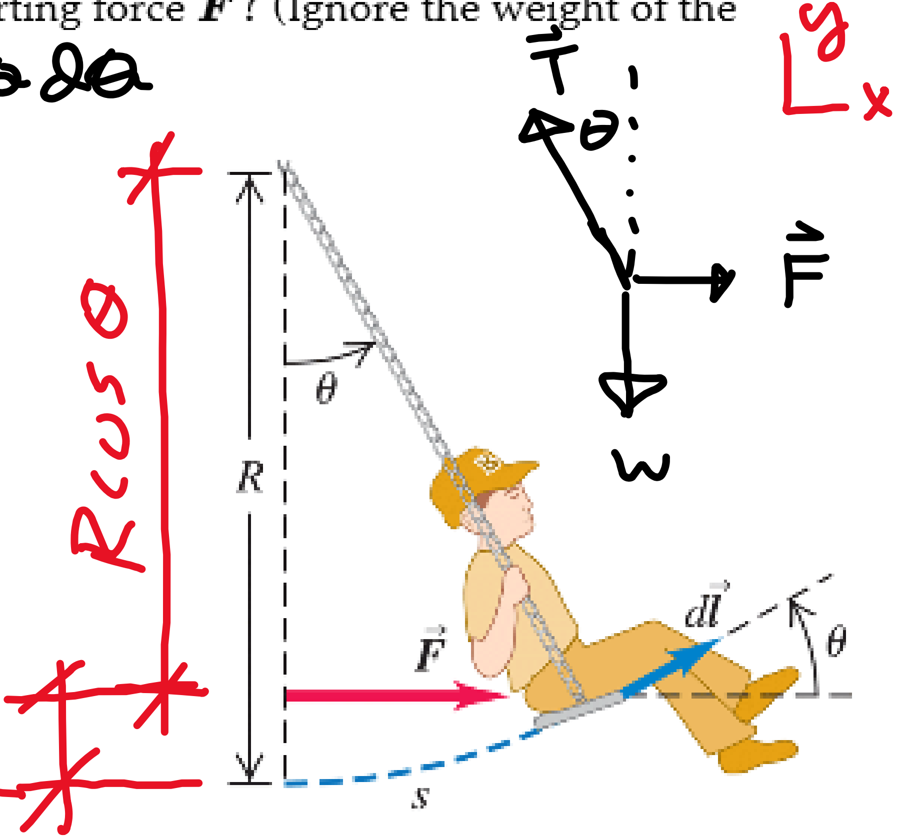

$$h = R[1-\cos\theta]$$

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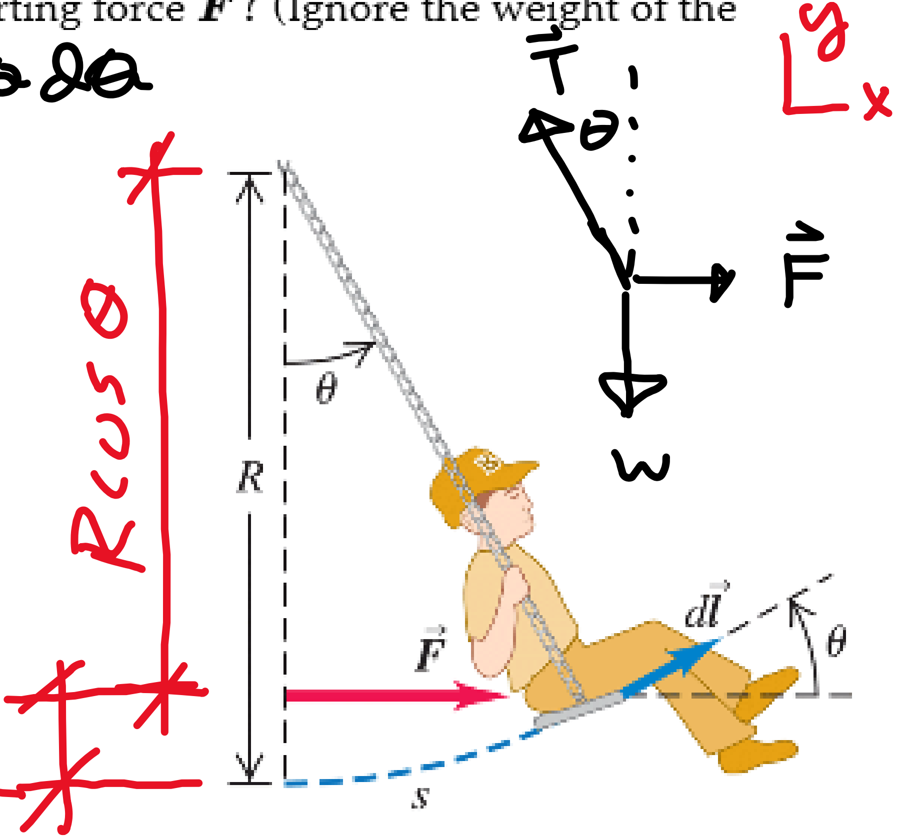
$$W_F = wR\int \sin\theta\, d\theta$$

$$= wR[-\cos\theta]\Big|_0^{\theta_0}$$

$$= wR[-\cos\theta_0 + 1]$$

$$\text{so } \boxed{W_F = -W_w = mgh}$$



$$h = R[1-\cos\theta]$$

Power

$$P_{AV} = \frac{\Delta W}{\Delta t}$$

Power

Average power during Δt

$$\xrightarrow{} P_{AV} = \frac{\Delta W}{\Delta t}$$

Power

Average power during Δt → $$P_{AV} = \frac{\Delta W}{\Delta t}$$ ← work done during Δt

Power

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← Duration

Power

Average power during Δt →

$$P_{AV} = \frac{\Delta W}{\Delta t}$$

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← Duration

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

Power

Average power during Δt → $$P_{AV} = \frac{\Delta W}{\Delta t}$$ ← work done during Δt

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$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

Power

$$P_{AV} = \frac{\Delta W}{\Delta t}$$

Instantaneous power

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Average power over infinitesimally short time interval

Power

$$P_{AV} = \frac{\Delta W}{\Delta t}$$

Instantaneous power →

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

} Time rate of doing work

↗ Average power over infinitesimally short time interval

Power

$$P_{AV} = \frac{\Delta W}{\Delta t}$$

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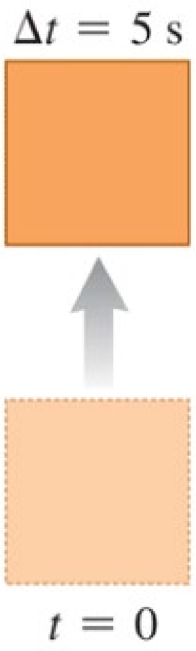
* SI unit of power ≡ Watt

& $W \equiv J/s$

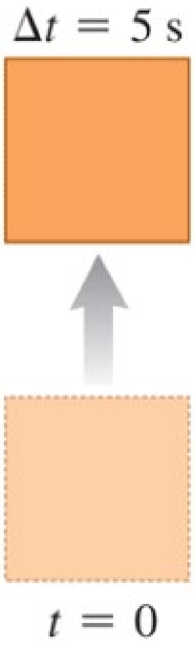


Work you do on the box to lift it in $\Delta t = 5$ s:

$$\Delta W = 100\text{ J}$$

Your average power output:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} = \frac{100\text{ J}}{5\text{ s}} = 20\text{ W}$$



Work you do on the box
to lift it in $\Delta t = 5$ s:

$$\Delta W = 100 \text{ J}$$

Your average power output:

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Work you do on the same
box to lift it the same
distance in $\Delta t = 1$ s:

$$\Delta W = 100 \text{ J}$$

Your average power output:

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{100 \text{ J}}{1 \text{ s}} = 100 \text{ W}$$

Power

* Common unit: horse power

Power

- Common unit : horse power

hp = 746W = 0.746 KW

Velocity & power

Velocity & power

$$P_{ave} = \frac{W}{\Delta t}$$

Velocity & power

$$P_{AVE} = \frac{W}{\Delta t} = \frac{F_{\parallel} \Delta s}{\Delta t}$$

Velocity & power

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& for instananeous case

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Velocity & power

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Each of the four jet engines on an Airbus A380 airliner develops a thrust (a forward force on the airliner) of 322,000 N (72,000 lb). When the airplane is flying at 250 m/s (900 km/h, or roughly 560 mi/h), what horsepower does each engine develop?

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$V = 250 km/h \Rightarrow P = (322 * 10^3 N)(250 \frac{m}{s})$

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$V = 250 km/h \Rightarrow P = (322 * 10^3 N)(250 \frac{m}{s}) = 8.05 * 10^7 W$

Velocity & power

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$$P = 8.05 * 10^7 W \left(\frac{hp}{746 W}\right)$$

Velocity & power

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& for instananeous case

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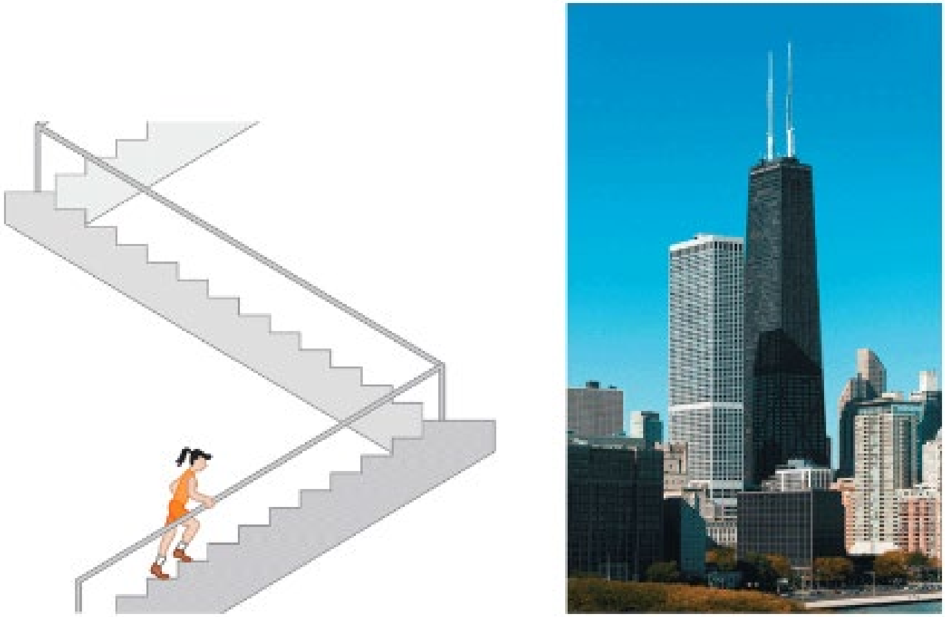
$$P = 8.05 * 10^7 W \left(\frac{hp}{746 W}\right) = 108{,}000 hp$$

A 50.0 kg marathon runner runs up the stairs to the top of Chicago's 443-m-tall Willis Tower, the second tallest building in the United States (Fig. 6.28). To lift herself to the top in 15.0 minutes, what must be her average power output?



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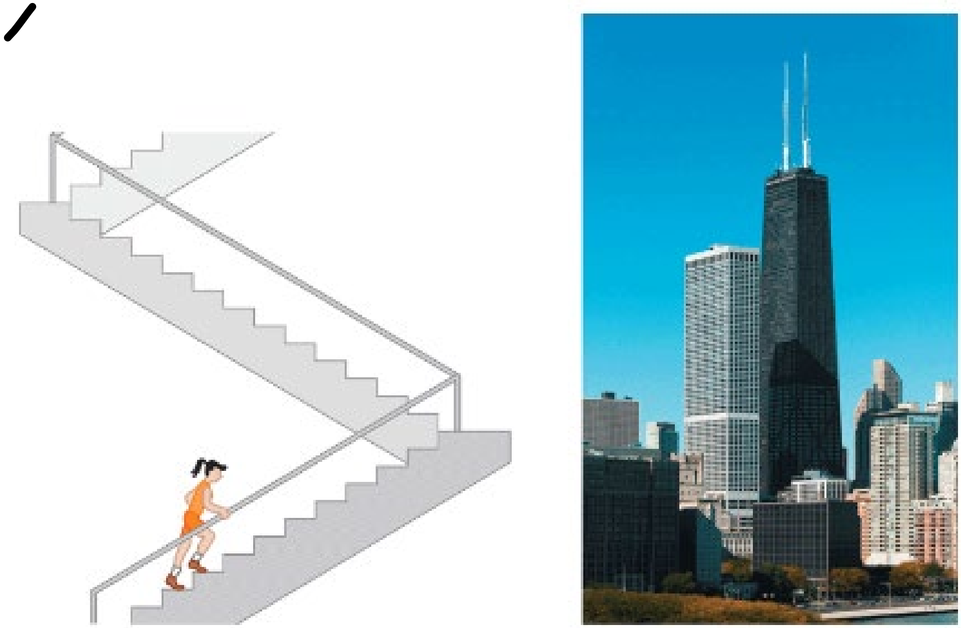
$m = 50\,kg$, $\Delta y = 443\,m$,

$t = 15\,min$



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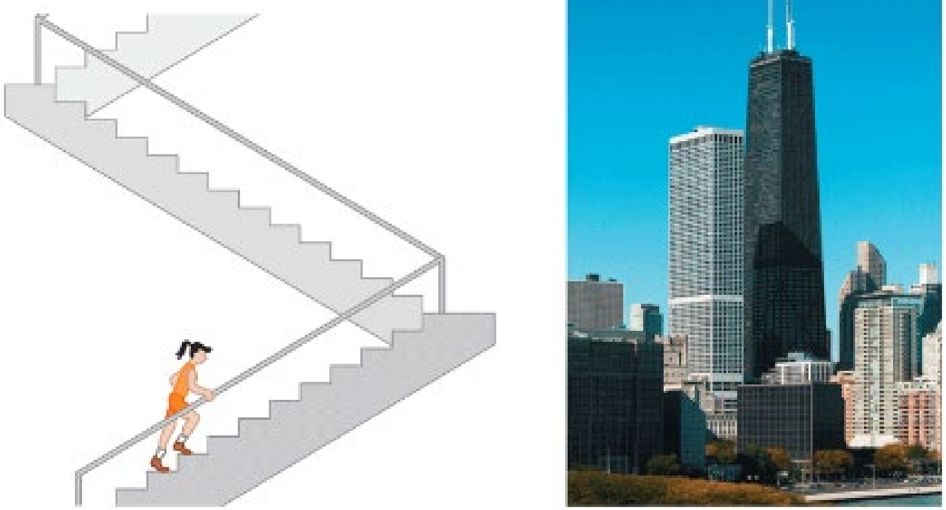
$m = 50kg$, $\Delta y = 443 m$,

$t = 15 min \left(\frac{60 s}{min} \right)$



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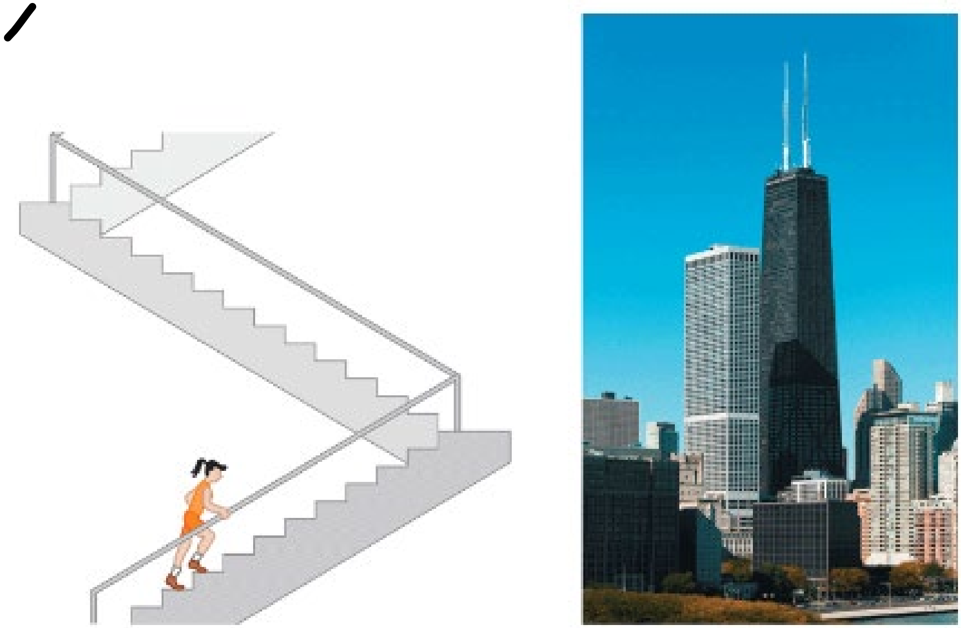
$$P_{AVE} = \frac{F_{\parallel}\Delta y}{\Delta t}$$



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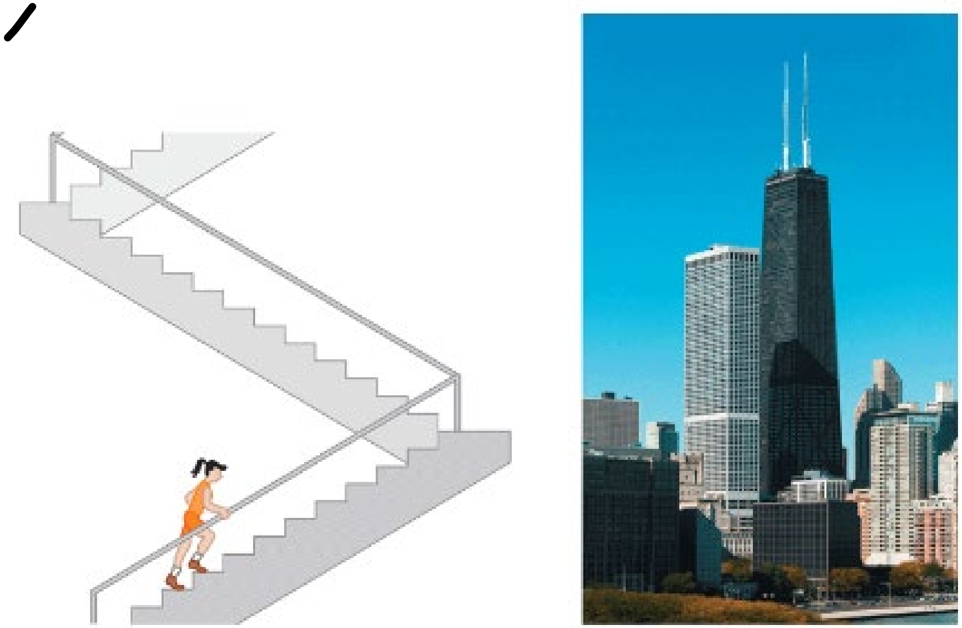
$$P_{AVE} = \frac{F_{\parallel}\Delta y}{\Delta t}$$

$$= \frac{9.8*50*443}{15*60}\,W$$



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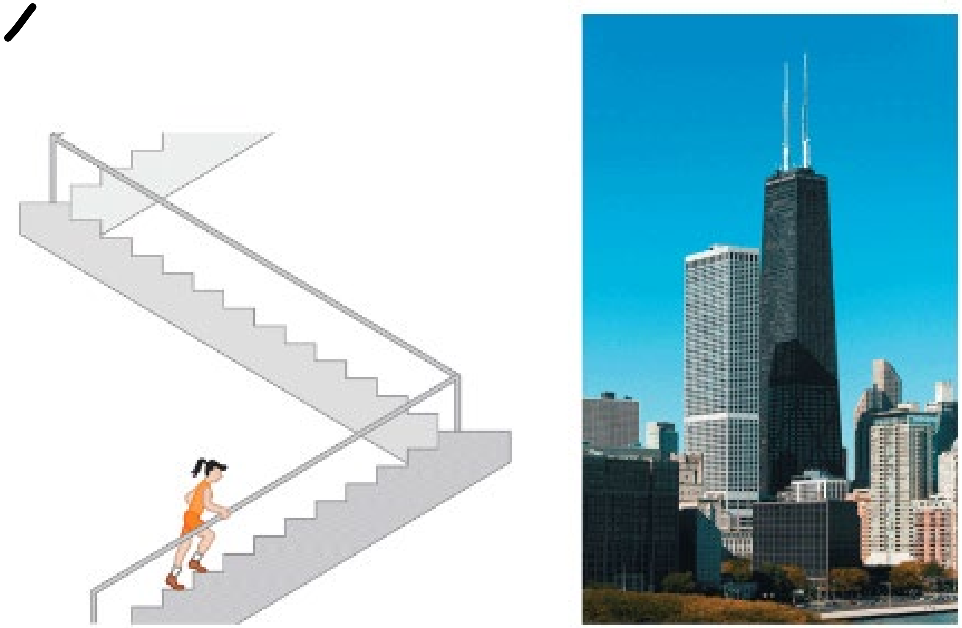

$$P_{AVE} = \frac{F_{\parallel}\Delta y}{\Delta t}$$

$$= \frac{9.8*50*443}{15*60}\,W$$

$$\Rightarrow \boxed{P_{AVE} = 241\,W}$$