

Today 6.1 & 6.2

L18



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Wednesday 6.1 & 6.2

} Work &  
kinetic energy

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\*  $\vec{F} = m\vec{a}$  + math  $\Rightarrow$  Work =  $\Delta$  kinetic energy

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\* As book does

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We will look at this subject  
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\*  $\vec{F} = m\vec{a}$  + Math  $\Rightarrow$  Work =  $\Delta$  Kinetic energy

\* As book does with  $W$  defined (in 6.1)  
& then connected to Kinetic energy  
(in 6.2)

I want to show you that

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$$\sum \vec{F} = m\vec{a} \oplus$$

I want to show you that  
[ $\Sigma \vec{F} = m\vec{a} \oplus$  Calculus]



I want to show you that  
[ $\Sigma \vec{F} = m\vec{a} \oplus$  Calculus]



Work = { Change in  
Kinetic energy }

1-2 case

$$\Sigma F = ma$$



1-2 case

$$\Sigma F = ma \quad \text{But} \quad a = v \frac{dv}{dx}$$

1-2 case

$$\Sigma F = ma$$

But  $a = v \frac{dv}{dx}$

so  $\Sigma F = m v \frac{dv}{dx}$



1-2 case

$$\Sigma F = ma$$

$$\text{But } a = v \frac{dv}{dx}$$

$$\text{so } \Sigma \int F dx = m \int v \frac{dv}{dx} dx$$

1-2 case

$$\Sigma F = ma$$

But  $a = v \frac{dv}{dx}$

so  $\Sigma \int F dx = m \int v \frac{dv}{dx} dx \Rightarrow \Sigma \int F dx = m \int v dv$

1-2 case

$$\Sigma F = ma$$

$$\text{But } a = v \frac{dv}{dx}$$

$$\text{so } \Sigma \int F dx = m \int v \frac{dv}{dx} dx \Rightarrow \Sigma \int_{x_1}^{x_2} F dx = m \int_{v_1}^{v_2} v dv$$

$$\Rightarrow \Sigma \int_{x_1}^{x_2} F dx = \frac{1}{2} m v^2 \Big|_{v_1}^{v_2}$$

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Define:  $W_{1 \rightarrow 2} \equiv$  Work from point 1 to point 2

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Define :  $W_{1 \rightarrow 2} \equiv$  Work from point 1 to point 2  
 $\nabla W_{1 \rightarrow 2} \equiv \Sigma \int_{x_1}^{x_2} F dx$

1-2 case

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Define :  $W_{1 \rightarrow 2} \equiv$  Work from point 1 to point 2

$$\& W_{1 \rightarrow 2} \equiv \Sigma \int_{x_1}^{x_2} F dx$$

$$\& K \equiv \text{Kinetic energy}$$

1-2 case

$$\Sigma F = ma \quad \text{But } a = v \frac{dv}{dx}$$

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Define :  $W_{1 \rightarrow 2} \equiv$  Work from point 1 to point 2

$$\& W_{1 \rightarrow 2} \equiv \Sigma \int_{x_1}^{x_2} F dx$$

&  $k \equiv$  Kinetic energy

$$\& k \equiv \frac{1}{2} m v^2$$

Now

$$W_{1 \rightarrow 2} = k_2 - k_1$$



1-2 case

$$\Sigma F = ma \quad \text{But } a = v \frac{dv}{dx}$$

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Now

$$W_{1 \rightarrow 2} = K_2 - K_1 \quad \text{or} \quad W_{1 \rightarrow 2} = \Delta K$$



1-2 case

$$\Sigma F = ma \quad \text{But } a = v \frac{dv}{dx}$$

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Define :  $W_{1 \rightarrow 2} \equiv$  Work from point 1 to point 2

$$\& W_{1 \rightarrow 2} \equiv \Sigma \int_{x_1}^{x_2} F dx$$

&  $K \equiv$  Kinetic energy

$$\& K \equiv \frac{1}{2} m v^2$$

Now

$$W_{1 \rightarrow 2} = K_2 - K_1 \quad \text{or} \quad W_{1 \rightarrow 2} = \Delta K \quad \text{or} \quad K_1 + W_{1 \rightarrow 2} = K_2$$



For 3-D case  $U_{1 \rightarrow 2}$  generalizes as

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\* Proof for 3-D case is slightly more difficult & put at end of these notes.

For 3-D case  $U_{1 \rightarrow 2}$  generalizes as

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We will now follow  
the treatment provided  
in the book

# Work

In physics we sometimes have very particular definitions of words used in everyday language. A good example is "work"

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In physics we sometimes have very particular definitions of words used in everyday language. A good example is "work"

$$\text{Work} = [\text{force in direction of motion}] \\ * \\ [\text{distance travelled}]$$



These people are doing work as they push on the car because they exert a force on the car as it moves.

---

# Work

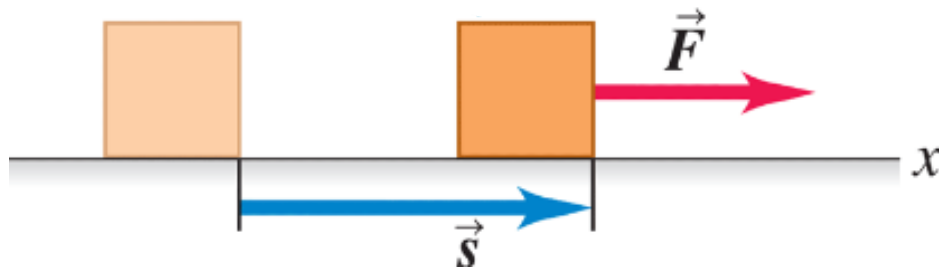
$$W = Fs$$

# Work

$W = Fs$ , where  $F$  is in direction of straight-line displacement  $s$

# Work

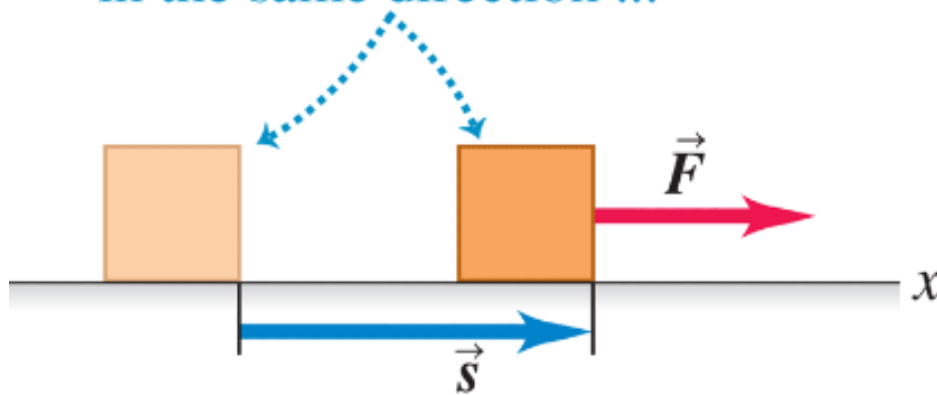
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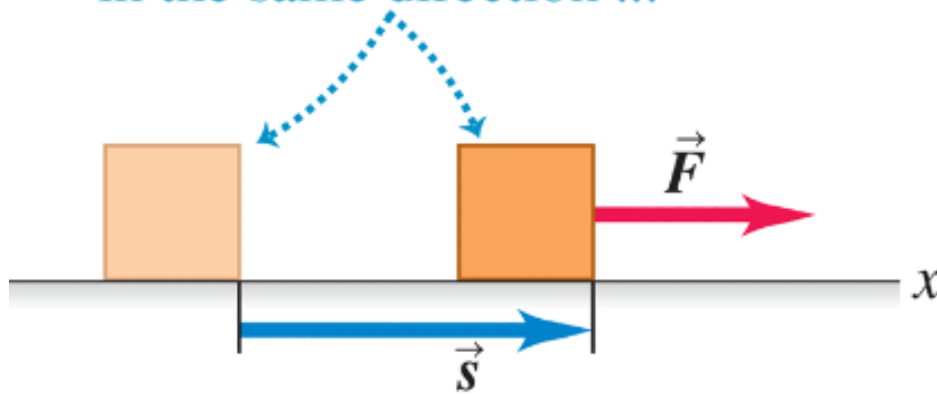
If a particle moves through a displacement  $\vec{s}$  while a constant force  $\vec{F}$  acts on it in the same direction ...



# Work

$W = Fs$ , where  $F$  is in direction of straight-line displacement  $s$

If a particle moves through a displacement  $\vec{s}$  while a constant force  $\vec{F}$  acts on it in the same direction ...



... the work done by the force on the particle is  $W = Fs$ .

# Work

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\* Don't confuse  $W$  [work] with  $w$  [weight]

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\* Units

$$\text{Joule} = (\text{Newton})(\text{meter})$$

# Work

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\* Units

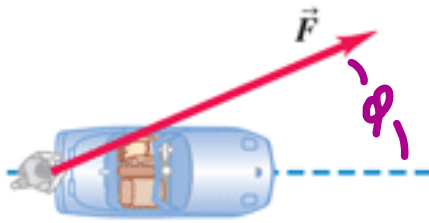
$$\text{Joule} = (\text{Newton})(\text{meter}) \quad \text{or} \quad J = N \cdot m$$

# Work

\* Don't confuse  $W$  [work] with  $w$  [weight]

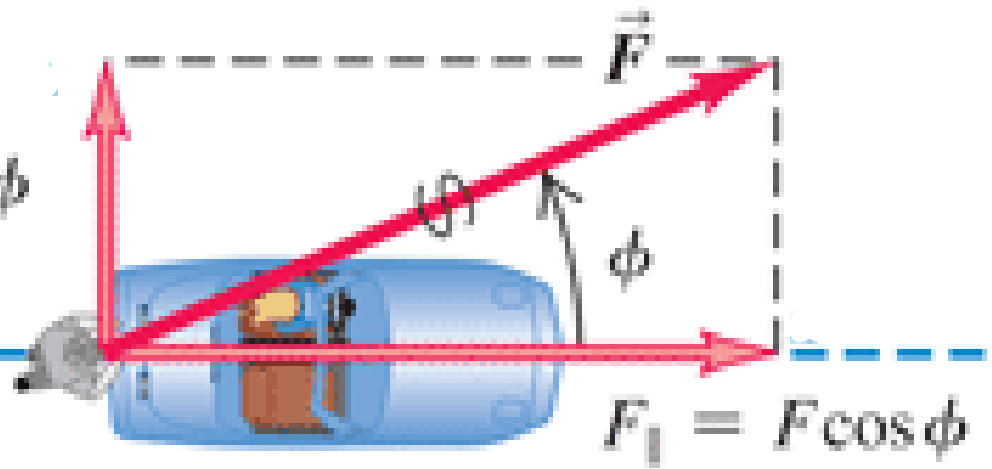
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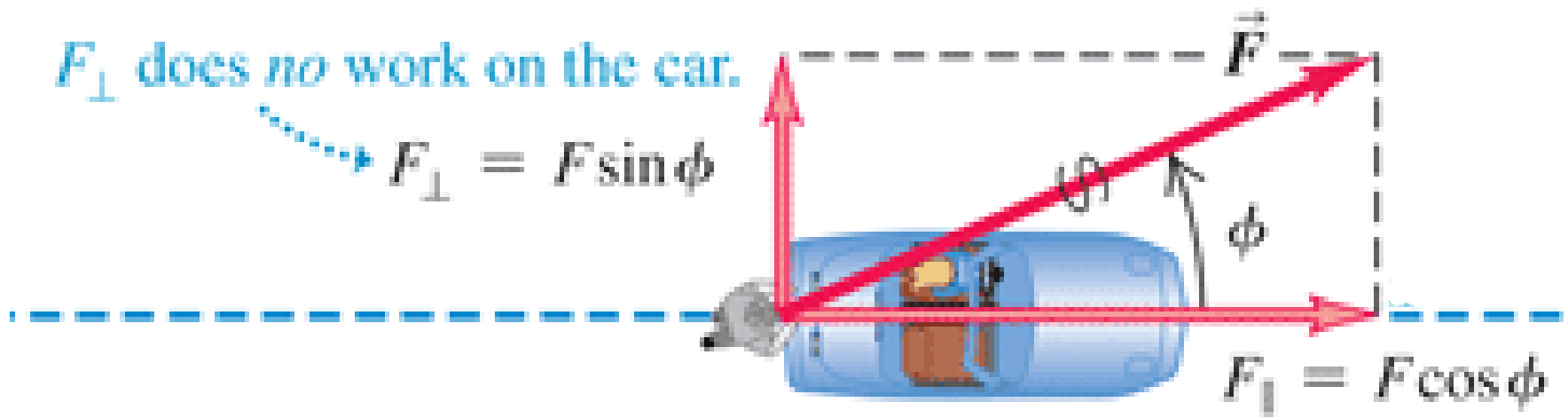
The car moves through displacement  $\vec{s}$  while a constant force  $\vec{F}$  acts on it at an angle  $\phi$  to the displacement.

$$F_{\perp} = F \sin \phi$$



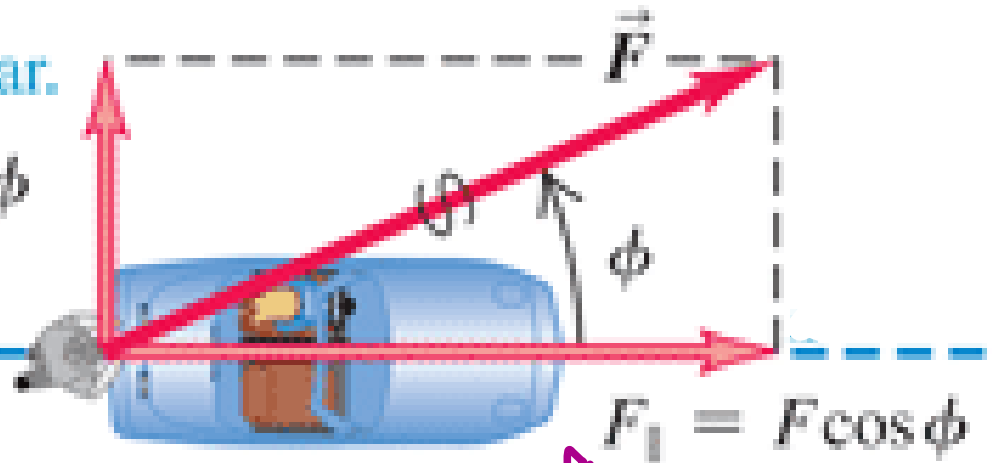
$F_{\perp}$  does *no* work on the car.

$F_{\perp} = F \sin \phi$



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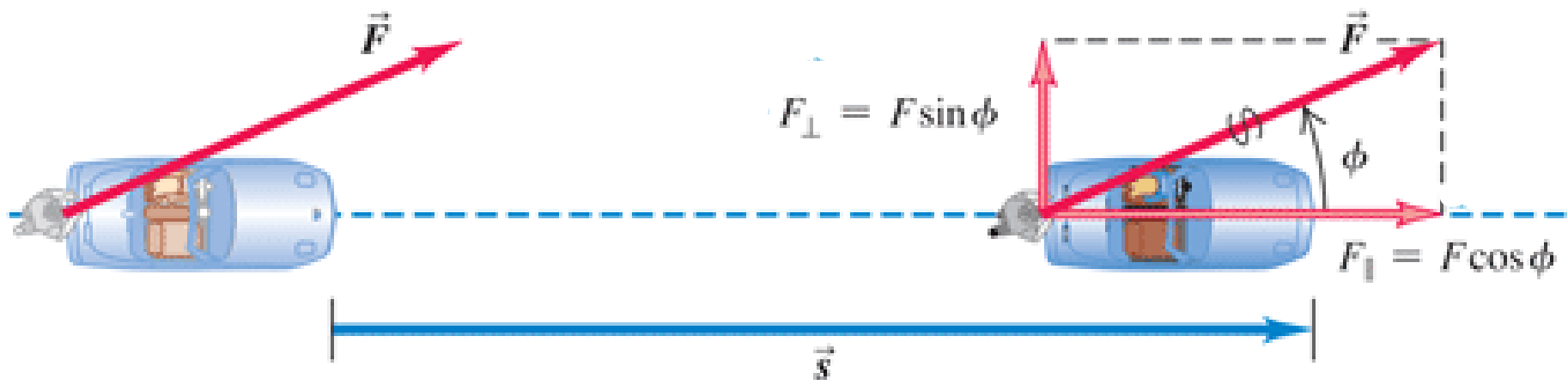
$F_{\perp} = F \sin \phi$



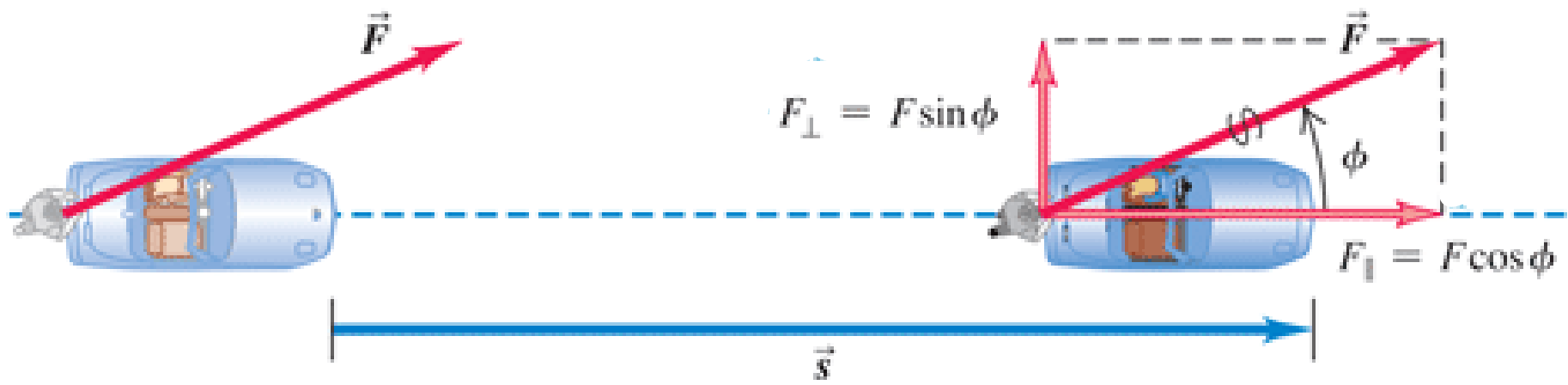
$F_{\parallel} = F \cos \phi$

Only  $F_{\parallel}$  does work on the car:

$$W = F_{\parallel} s = (F \cos \phi) s$$
$$= F s \cos \phi$$

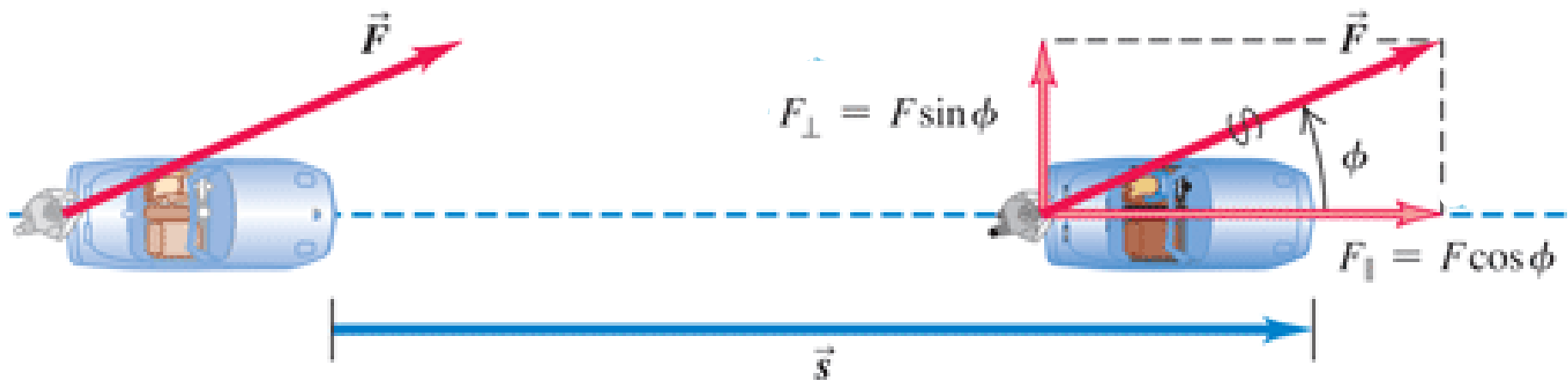


$$W = F s \cos \phi$$



Work done on a particle  
by constant force  $\vec{F}$  during  
straight-line displacement  $\vec{s}$

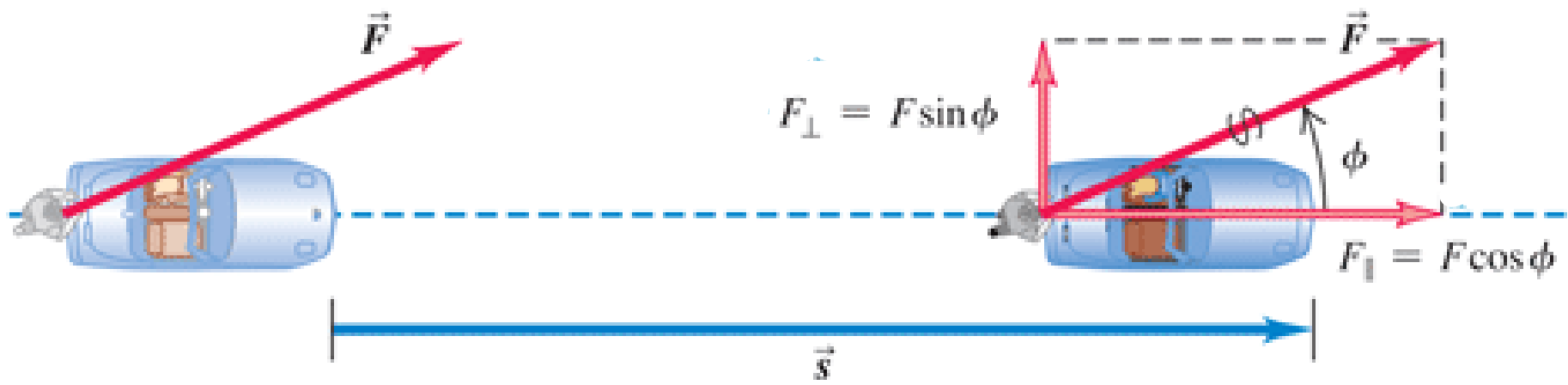
$$W = Fs \cos \phi$$



Work done on a particle by constant force  $\vec{F}$  during straight-line displacement  $\vec{s}$

Magnitude of  $\vec{F}$

$$W = Fs \cos \phi$$

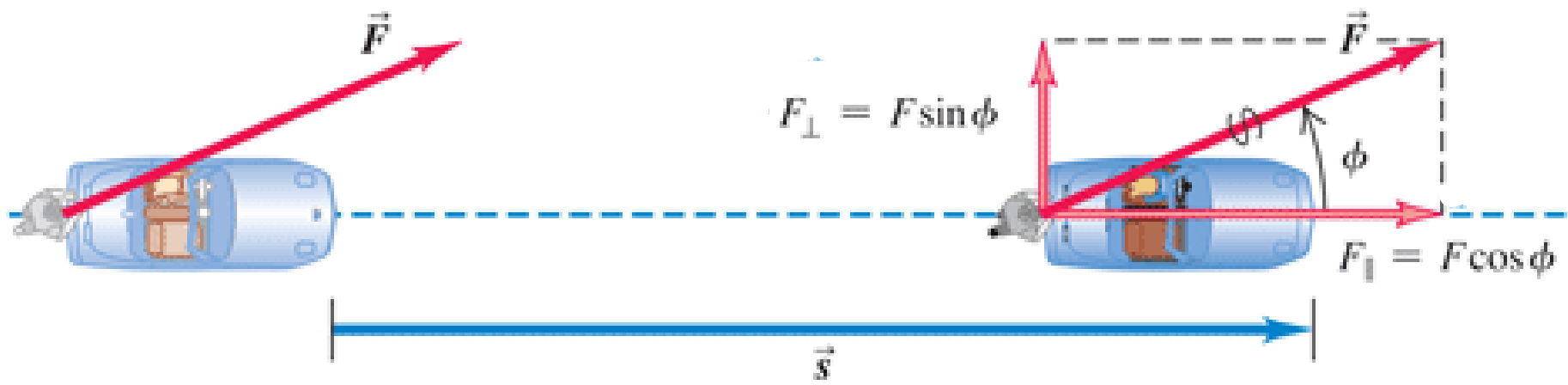


Work done on a particle by constant force  $\vec{F}$  during straight-line displacement  $\vec{s}$

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Magnitude of  $\vec{F}$

Magnitude of  $\vec{s}$



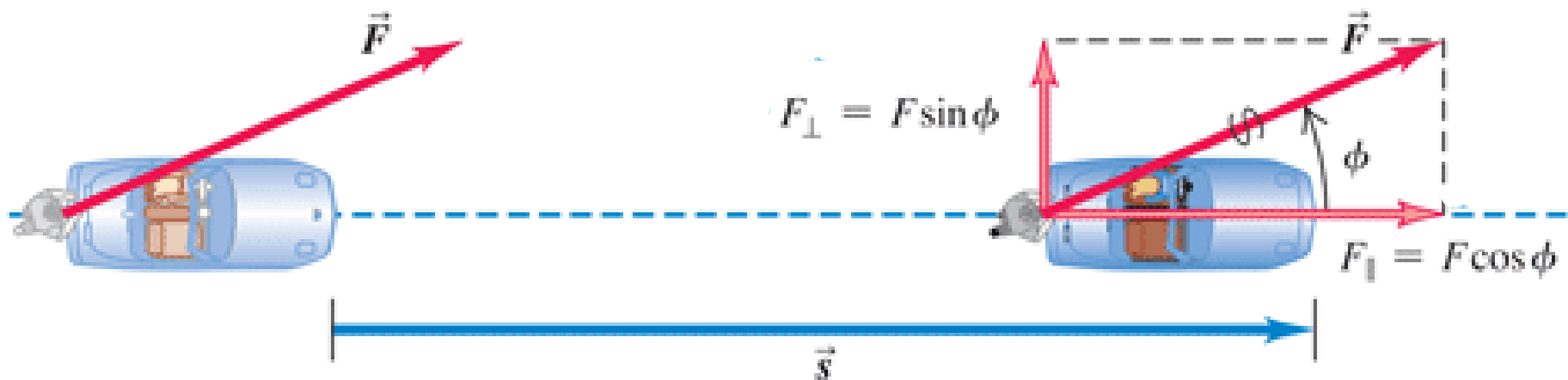
Work done on a particle by constant force  $\vec{F}$  during straight-line displacement  $\vec{s}$

$$W = F s \cos \phi$$

Magnitude of  $\vec{F}$   
 Angle between  $\vec{F}$  and  $\vec{s}$   
 Magnitude of  $\vec{s}$

Same as

$$W = \vec{F} \cdot \vec{s}$$



Work done on a particle by constant force  $\vec{F}$  during straight-line displacement  $\vec{s}$

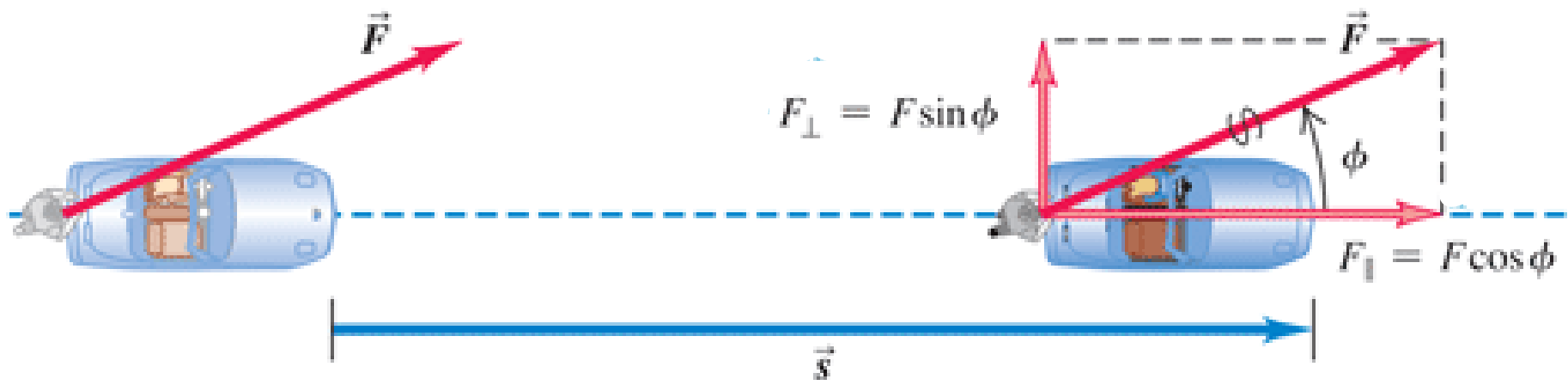
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*Same as*

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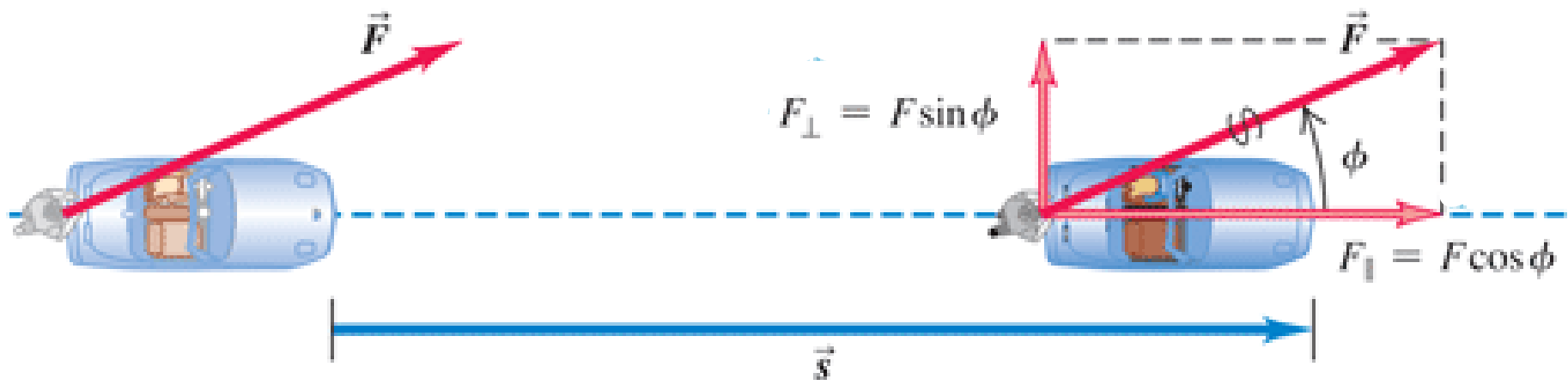
Magnitude of  $\vec{F}$   
 Angle between  $\vec{F}$  and  $\vec{s}$   
 Magnitude of  $\vec{s}$

*Same as*

Work done on a particle by constant force  $\vec{F}$  during straight-line displacement  $\vec{s}$

$$W = \vec{F} \cdot \vec{s}$$

Scalar product (dot product) of vectors  $\vec{F}$  and  $\vec{s}$



Work done on a particle by constant force  $\vec{F}$  during straight-line displacement  $\vec{s}$  is given by the equation:

$$W = F s \cos \phi$$

where  $F$  is the magnitude of  $\vec{F}$ ,  $s$  is the magnitude of  $\vec{s}$ , and  $\phi$  is the angle between  $\vec{F}$  and  $\vec{s}$ .

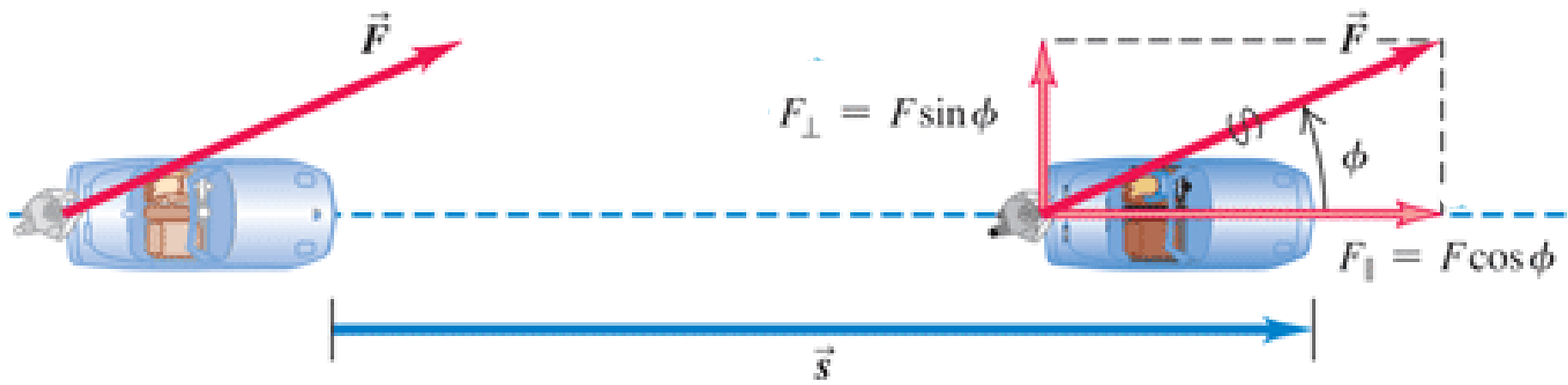
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Scalar product (dot product) of vectors  $\vec{F}$  and  $\vec{s}$

NOTE: Work is a scalar !!



Work done on a particle by constant force  $\vec{F}$  during straight-line displacement  $\vec{s}$  is given by:

$$W = F s \cos \phi$$

Labels for the equation above:

- Magnitude of  $\vec{F}$  (points to  $F$ )
- Magnitude of  $\vec{s}$  (points to  $s$ )
- Angle between  $\vec{F}$  and  $\vec{s}$  (points to  $\phi$ )


Same as

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Scalar product (dot product) of vectors  $\vec{F}$  and  $\vec{s}$

NOTE: Work is a scalar !!! NOT vector !!!

(a) Steve exerts a steady force of magnitude 210 N (about 47 lb) on the stalled car in **Fig. 6.3**  as he pushes it a distance of 18 m. The car also has a flat tire, so to make the car track straight Steve must push at an angle of  $30^\circ$  to the direction of motion. How much work does Steve do?

(a) Steve exerts a steady force of magnitude 210 N (about 47 lb) on the stalled car in Fig. 6.3 as he pushes it a distance of 18 m. The car also has a flat tire, so to make the car track straight Steve must push at an angle of  $30^\circ$  to the direction of motion. How much work does Steve do?

$$F = 210\text{ N}$$

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$F = 210\text{ N}$  at  $30^\circ$  to displacement



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$$F = 210\text{ N at } 30^\circ \text{ to displacement}$$

$$s = 18\text{ m}$$

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$$\Rightarrow W = 3300\text{ J}$$

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$$\vec{F} = [160\hat{i} - 40\hat{j}] \text{ N} \quad \&$$
$$\vec{s} = [14\hat{i} + 11\hat{j}] \text{ m}$$

(a) Steve exerts a steady force of magnitude 210 N (about 47 lb) on the stalled car in Fig. 6.3 as he pushes it a distance of 18 m. The car also has a flat tire, so to make the car track straight Steve must push at an angle of  $30^\circ$  to the direction of motion. How much work does Steve do? (b) In a helpful mood, Steve pushes a second stalled car with a steady force  $\vec{F} = (160 \text{ N})\hat{i} - (40 \text{ N})\hat{j}$ . The displacement of the car is  $\vec{s} = (14 \text{ m})\hat{i} + (11 \text{ m})\hat{j}$ . How much work does Steve do in this case?

$$\vec{F} = [160\hat{i} - 40\hat{j}] \text{ N} \quad \&$$
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$$W = \vec{F} \cdot \vec{s}$$

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$$\Rightarrow W = [160 * 14 - 40 * 11] \text{ J}$$

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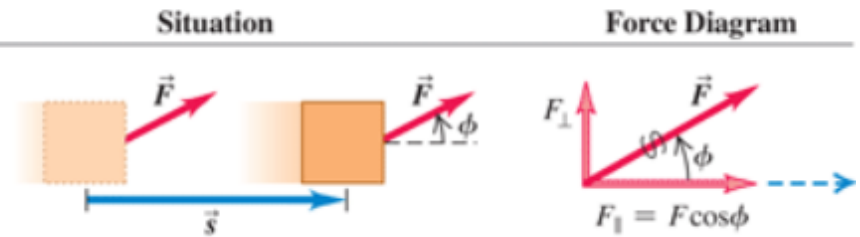
$$\Rightarrow W = [160 * 14 - 40 * 11] \text{ J} = 1800 \text{ J}$$



# Work: Positive, Negative, or Zero

## Direction of Force (or Force Component)

- (a) Force  $\vec{F}$  has a component in direction of displacement:  
 $W = F_{\parallel}s = (F\cos\phi)s$   
Work is *positive*.



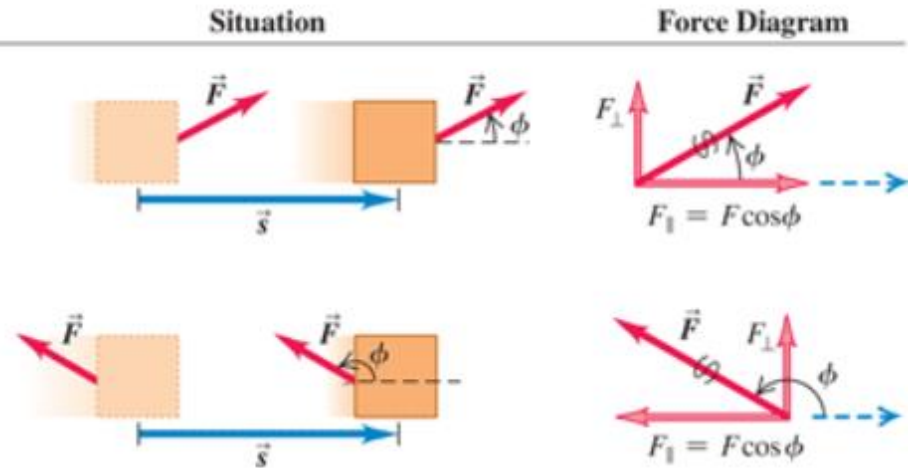
# Work: Positive, Negative, or Zero

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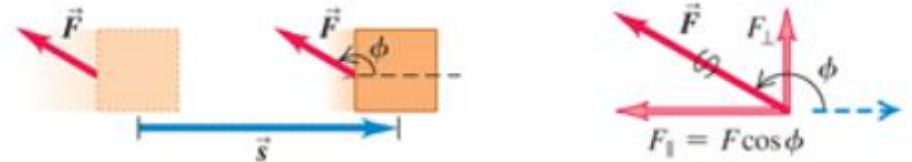
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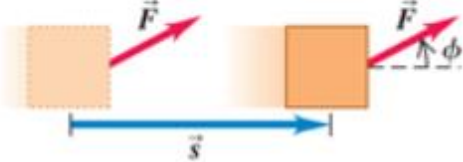
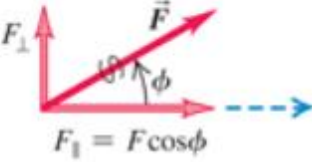
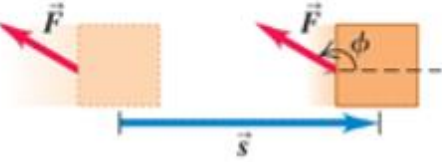
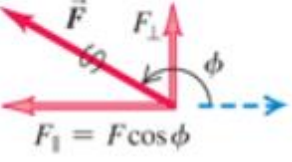
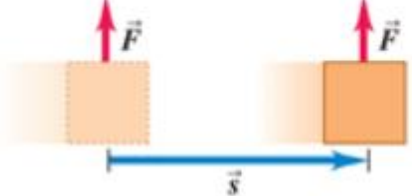
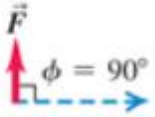


$$W = F_{\parallel}s = (F \cos \phi)s$$

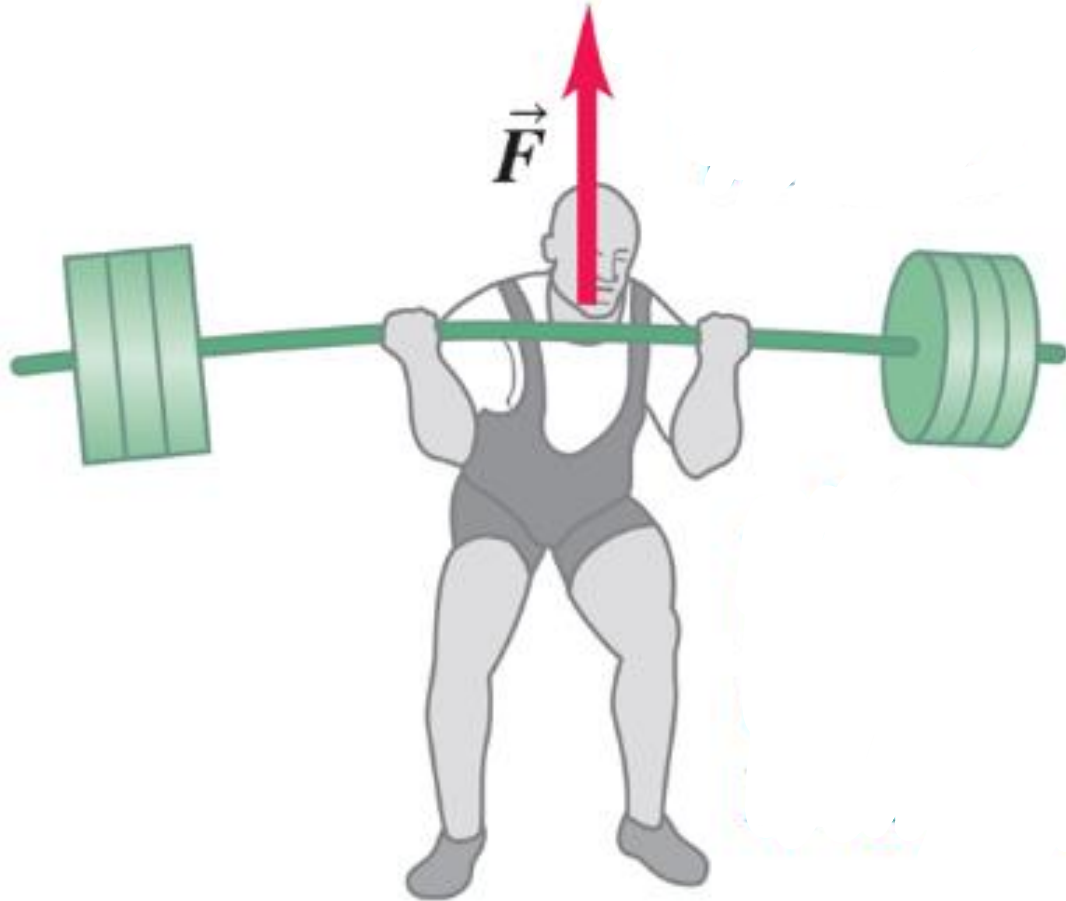
Work is *negative* (because  $F \cos \phi$  is negative for  $90^\circ < \phi < 180^\circ$ ).



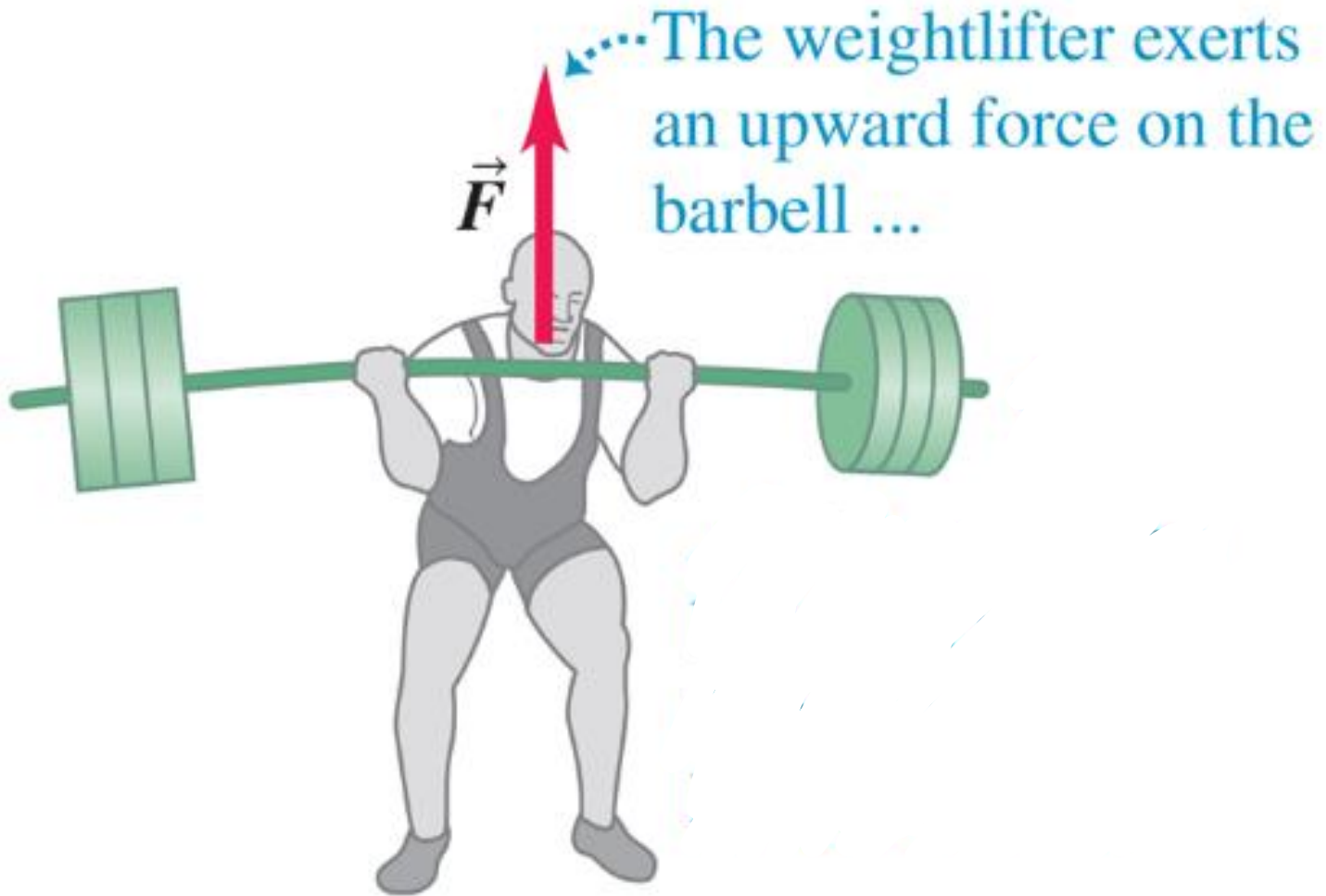
# Work: Positive, Negative, or Zero

Direction of Force (or Force Component)	Situation	Force Diagram
<p>(a) Force <math>\vec{F}</math> has a component in direction of displacement:  <math>W = F_{\parallel}s = (F \cos \phi)s</math>                      Work is <i>positive</i>.</p>		
<p>(b) Force <math>\vec{F}</math> has a component opposite to direction of displacement:  <math>W = F_{\parallel}s = (F \cos \phi)s</math>                      Work is <i>negative</i> (because <math>F \cos \phi</math> is negative for <math>90^\circ &lt; \phi &lt; 180^\circ</math>).</p>		
<p>(c) Force <math>\vec{F}</math> (or force component <math>F_{\perp}</math>) is <b>perpendicular to direction of displacement</b>: The force (or force component) does <i>no</i> work on the object.</p>		

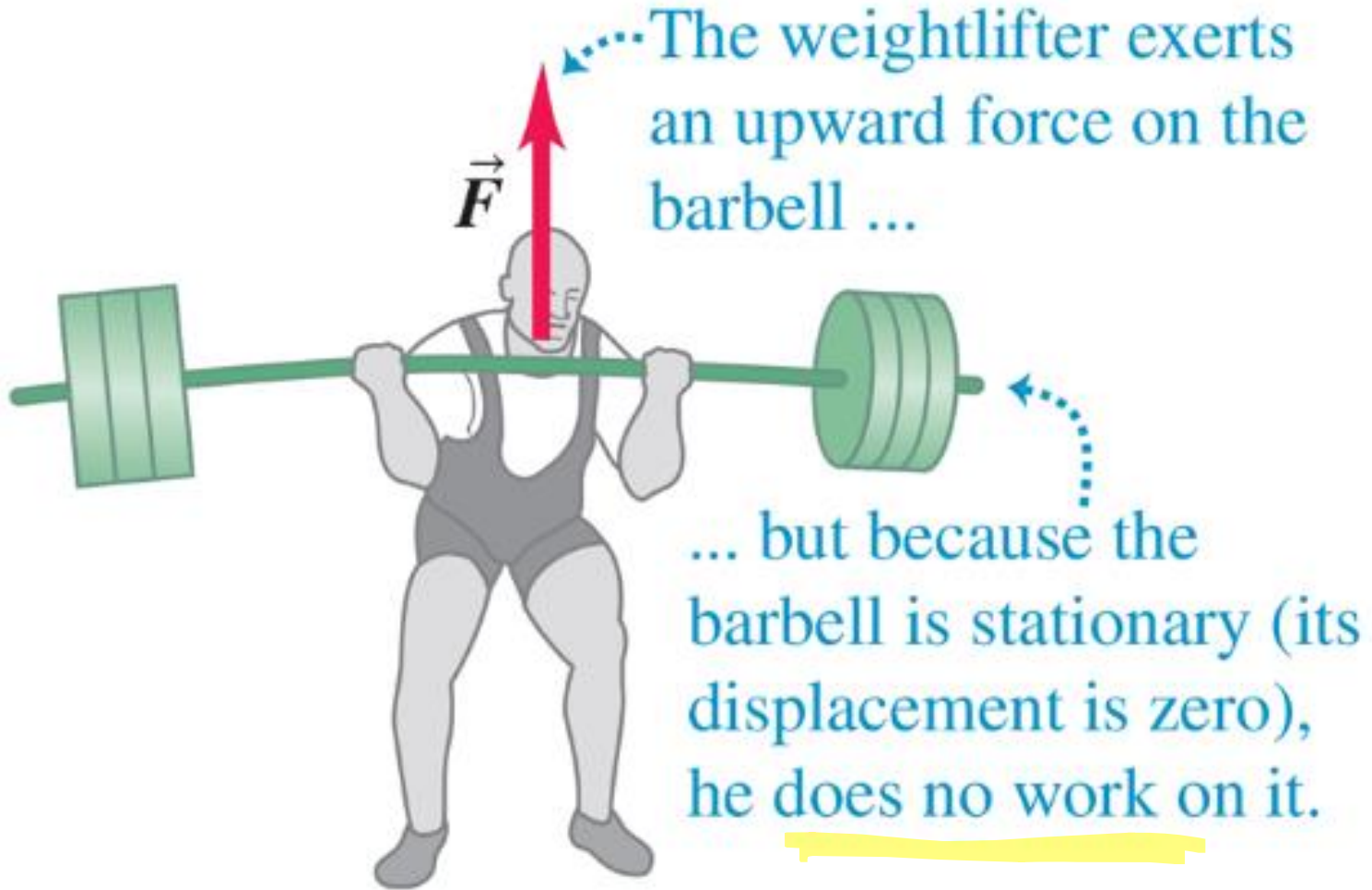
# Weights held stationary



# Weights held stationary



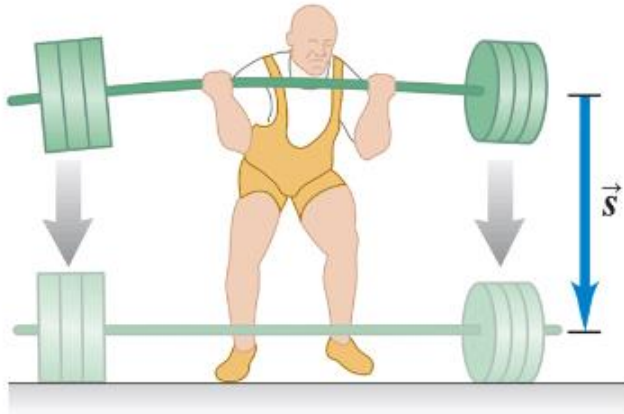
# Weights held stationary



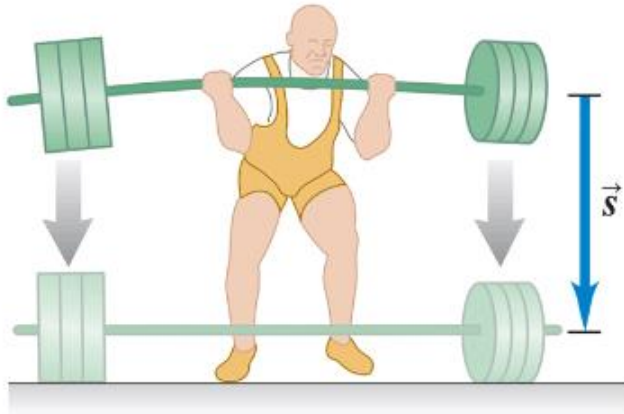
The weightlifter exerts an upward force on the barbell ...

... but because the barbell is stationary (its displacement is zero), he does no work on it.

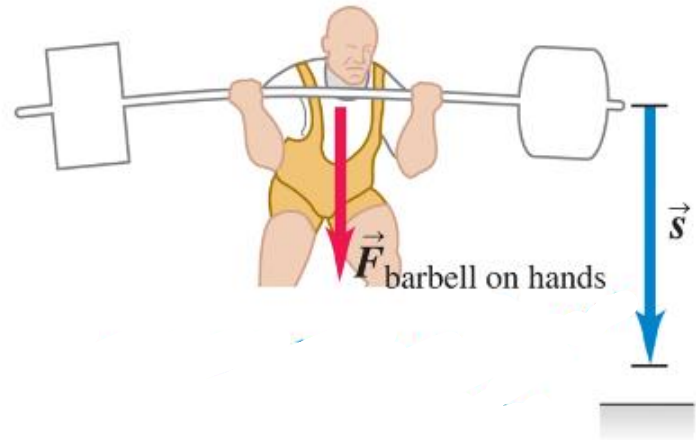
(a) A weightlifter lowers a barbell to the floor.



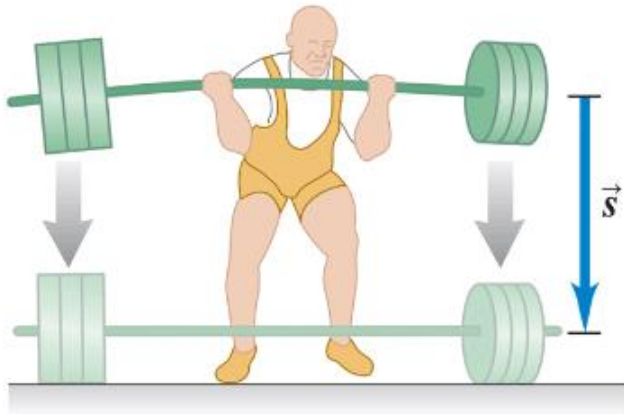
(a) A weightlifter lowers a barbell to the floor.



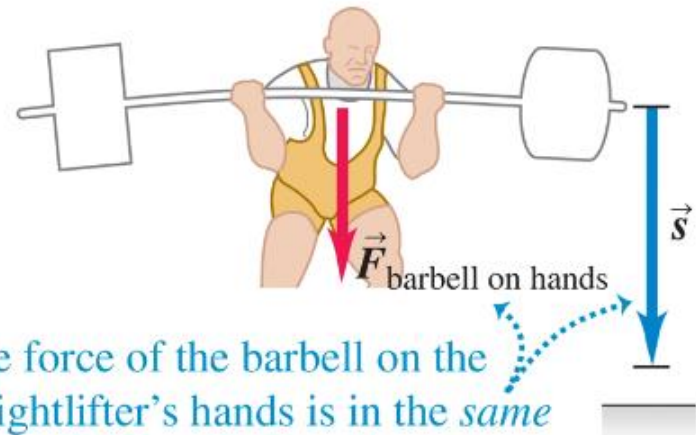
(b) The barbell does *positive* work on the weightlifter's hands.



(a) A weightlifter lowers a barbell to the floor.

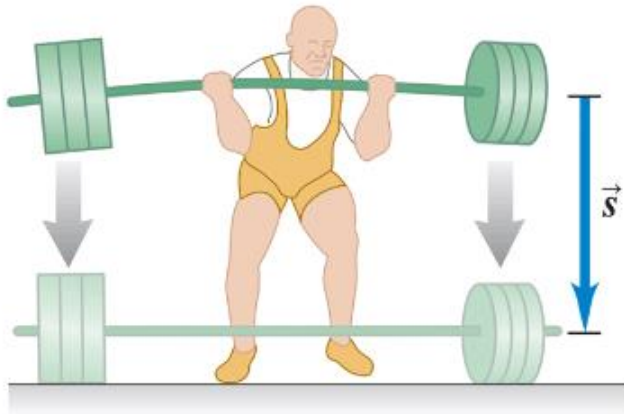


(b) The barbell does *positive* work on the weightlifter's hands.

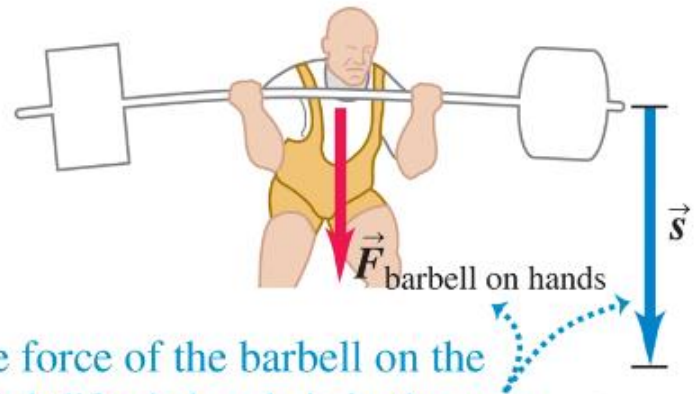


The force of the barbell on the weightlifter's hands is in the *same* direction as the hands' displacement.

(a) A weightlifter lowers a barbell to the floor.

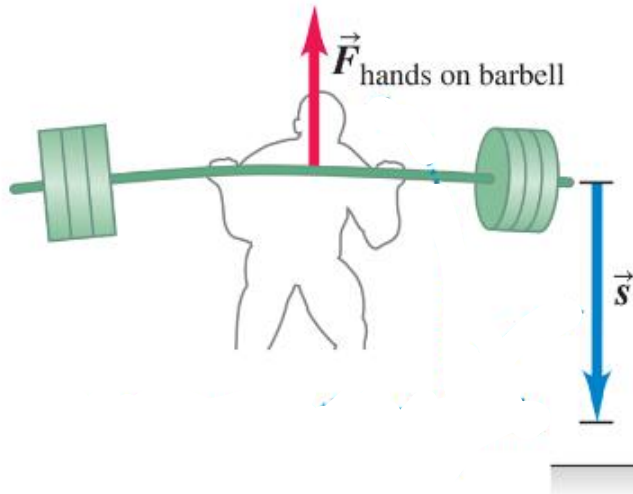


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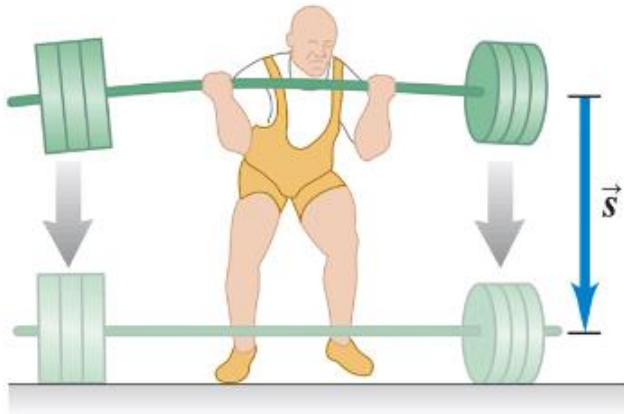


The force of the barbell on the weightlifter's hands is in the *same* direction as the hands' displacement.

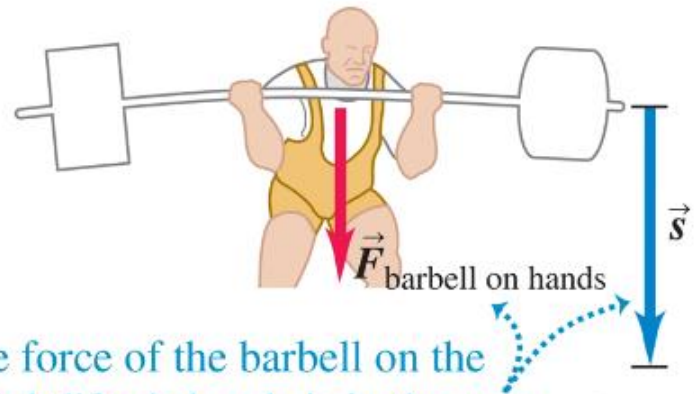
(c) The weightlifter's hands do *negative* work on the barbell.



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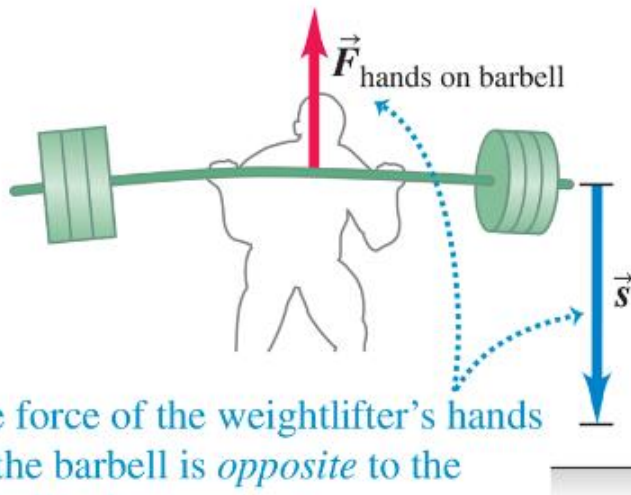


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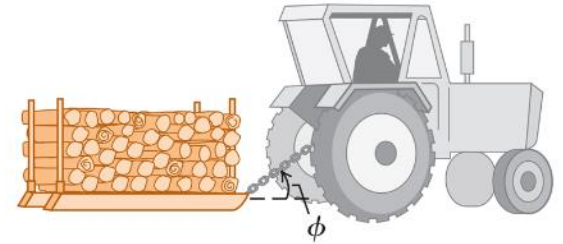
The force of the barbell on the weightlifter's hands is in the *same* direction as the hands' displacement.

(c) The weightlifter's hands do *negative* work on the barbell.

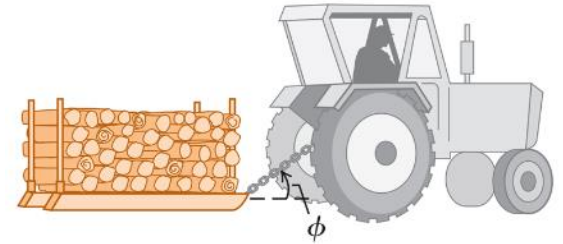


The force of the weightlifter's hands on the barbell is *opposite* to the barbell's displacement.

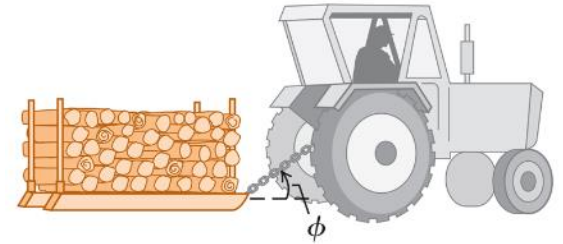
A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground (Fig. 6.7a). The total weight of sled and load is 14,700 N. The tractor exerts a constant 5000 N force at an angle of  $36.9^\circ$  above the horizontal. A 3500 N friction force opposes the sled's motion. Find the work done by each force acting on the sled and the total work done by all the forces.



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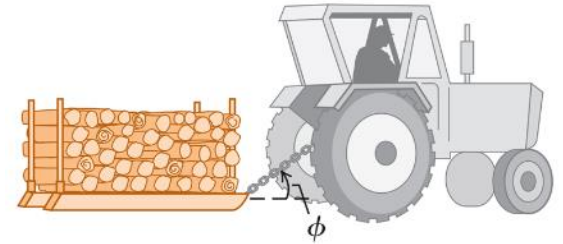
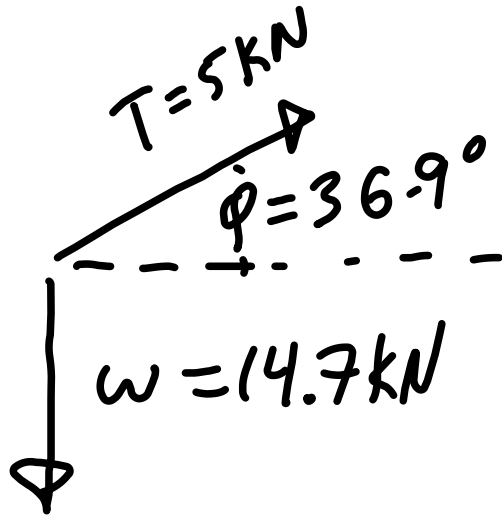


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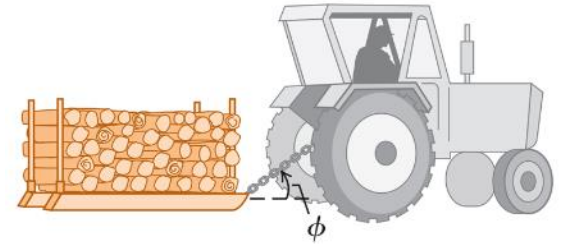
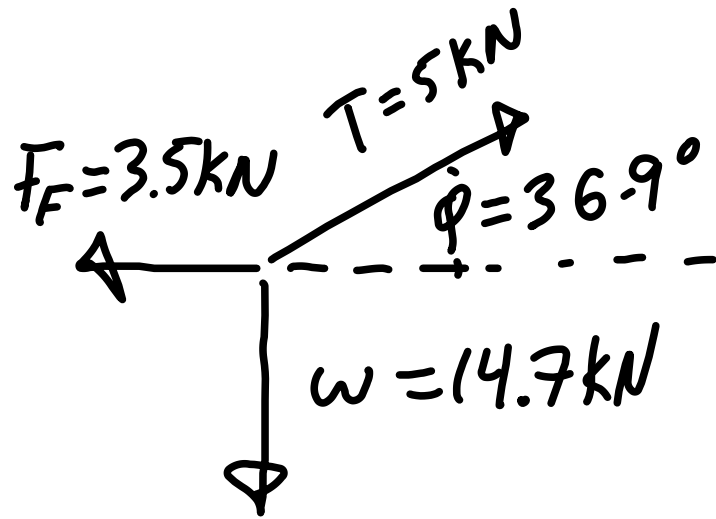


$$\downarrow w = 14.7\text{ kN}$$

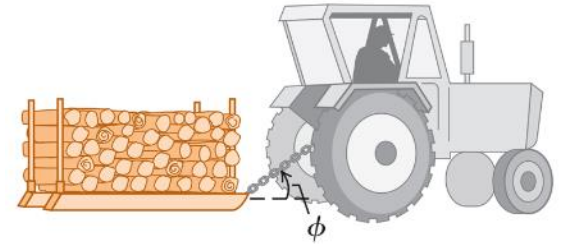
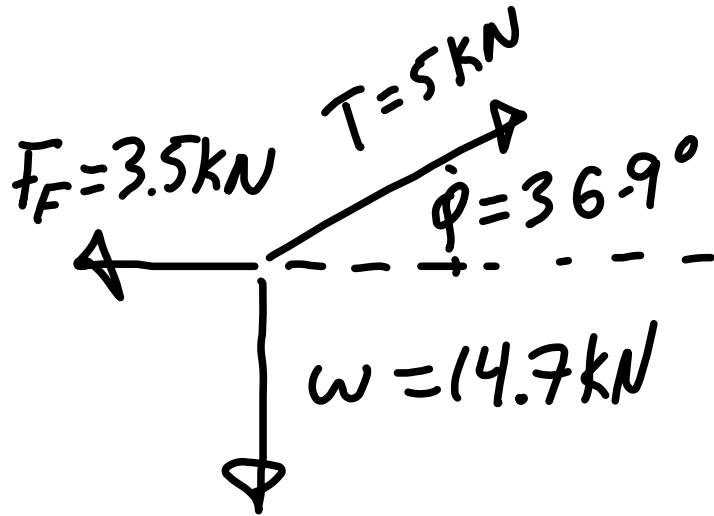
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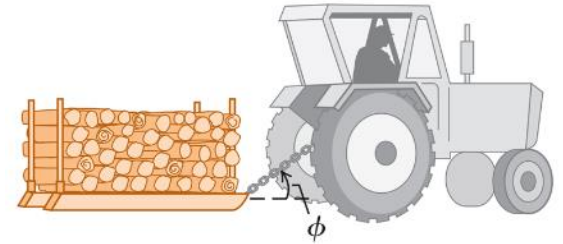
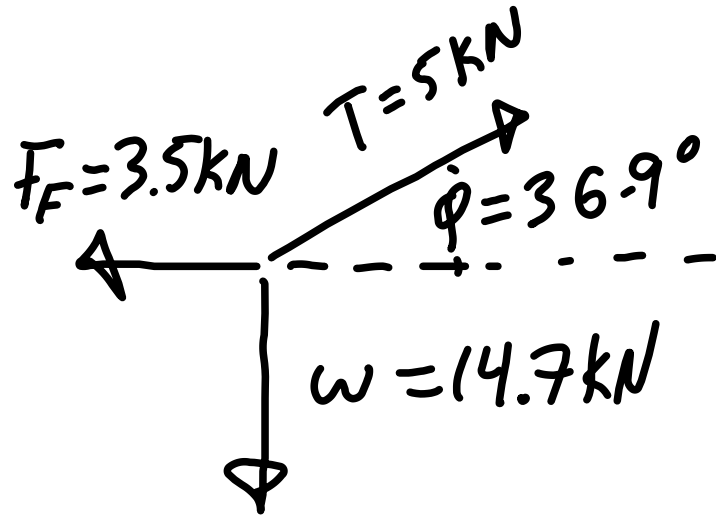


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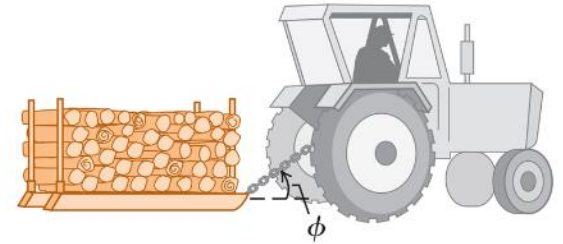
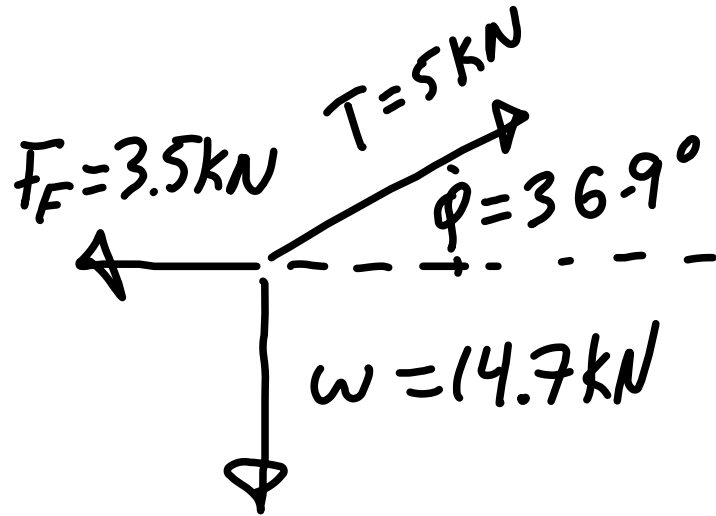
$$W_T = Td \cos \phi$$

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$$W_T = Td \cos \phi$$
$$= (5 * 20 \cos 36.9^\circ) \text{ kJ}$$

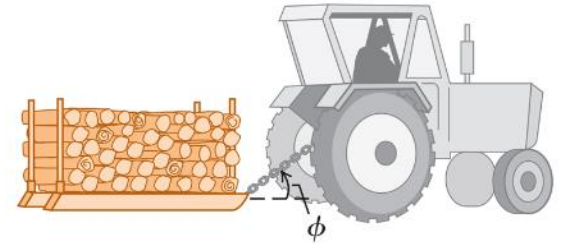
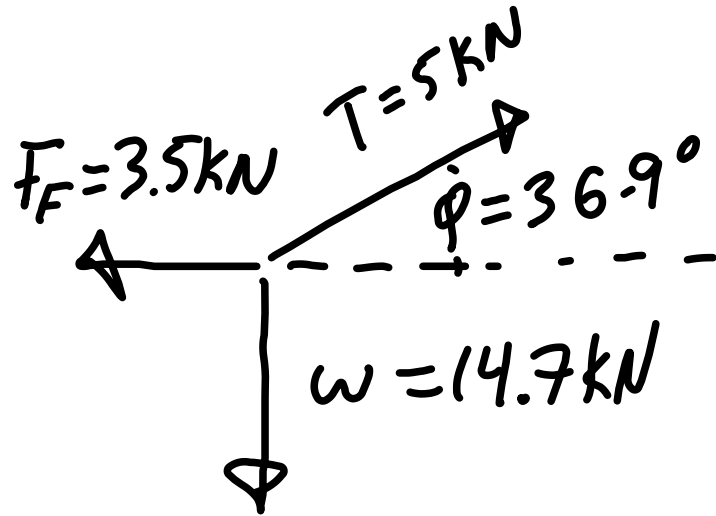
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$$W_T = Td \cos \phi$$

$$= 80\text{ kJ}$$

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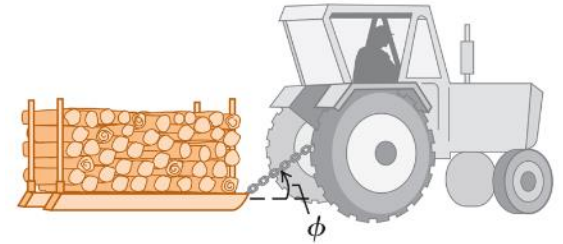
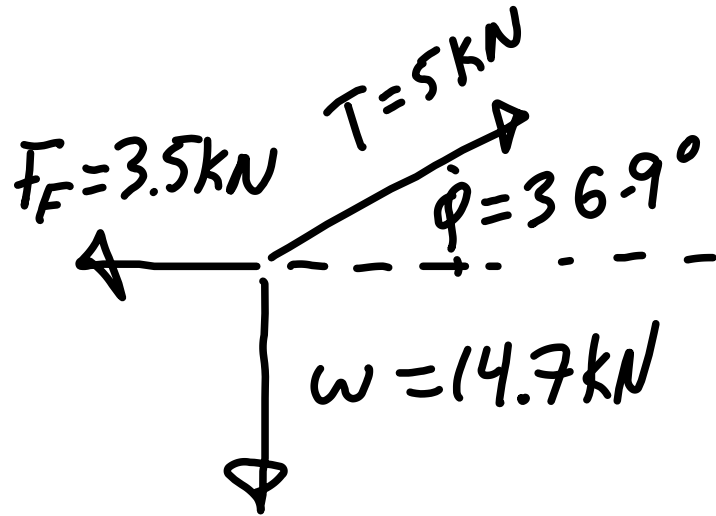


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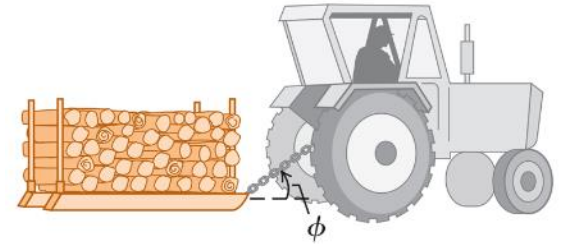
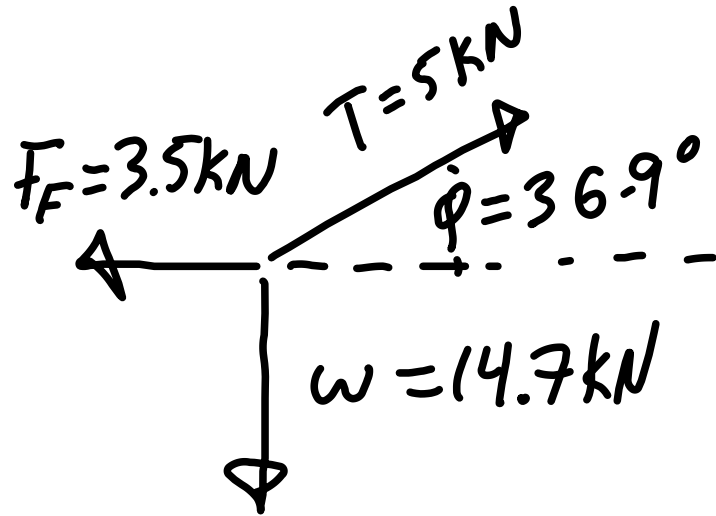
$$W_F = F_f d \cos(180^\circ)$$

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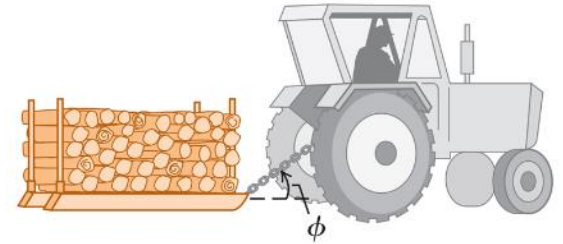
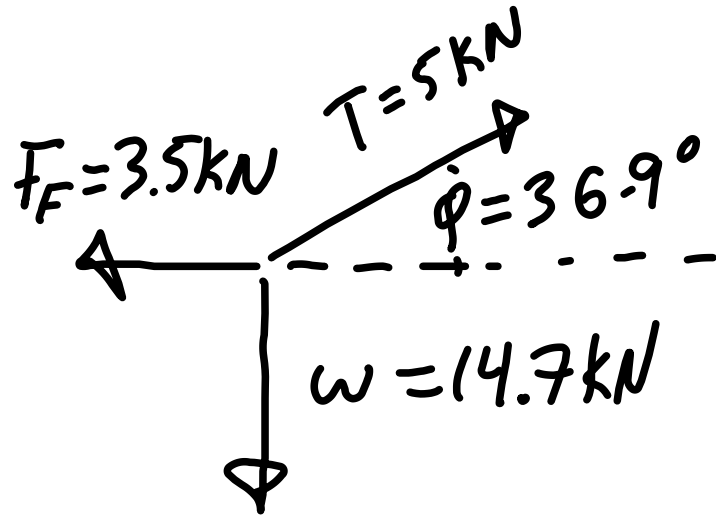
$$\begin{aligned}
 W_T &= T d \cos \phi \\
 &= 80 \text{ kJ} \\
 W_F &= F_f d \cos(180^\circ) \\
 &= -35 * 20 \text{ kJ}
 \end{aligned}$$

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$$\begin{aligned}
 W_T &= T d \cos \phi \\
 &= 80 \text{ kJ} \\
 W_F &= F_f d \cos(180^\circ) \\
 &= -70 \text{ kJ}
 \end{aligned}$$

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$$W_T = Td \cos \phi$$

$$= 80 \text{ kJ}$$

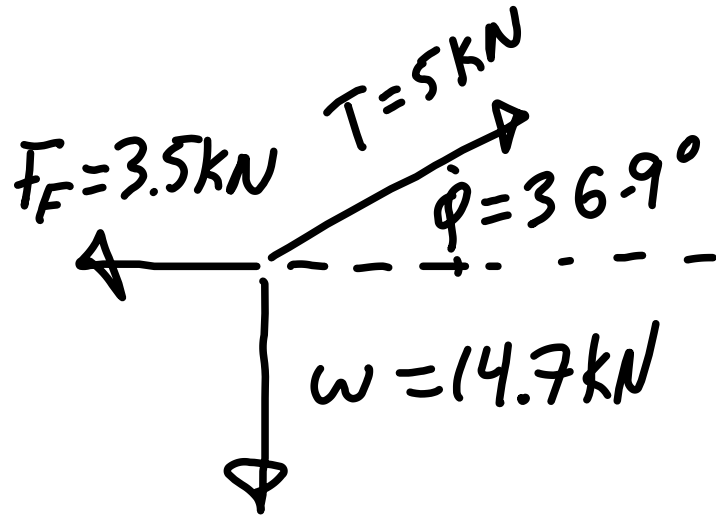
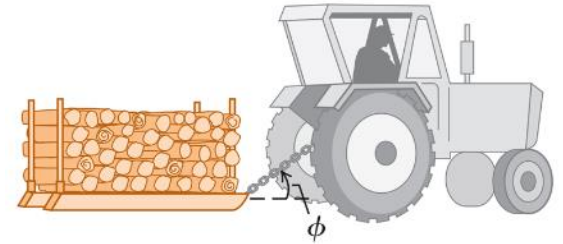
$$W_F = F_f d \cos(180^\circ)$$

$$= -70 \text{ kJ}$$

$$W_w = w d \cos(90^\circ)$$

$$= 0$$

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$$W_T = Td \cos \phi$$

$$= 80 \text{ kJ}$$

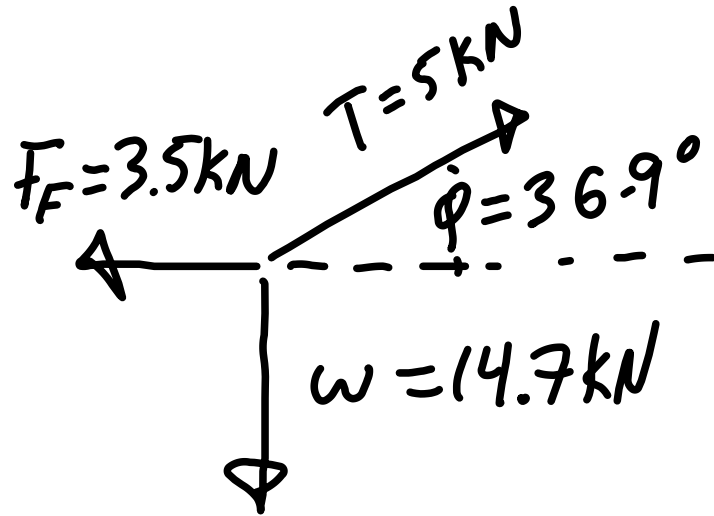
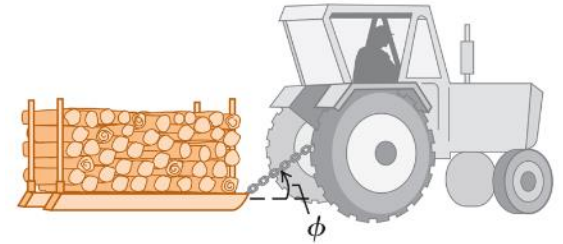
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$$= -70 \text{ kJ}$$

$$W_w = w d \cos(90^\circ)$$

$$= 0$$

A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground (Fig. 6.7a). The total weight of sled and load is 14,700 N. The tractor exerts a constant 5000 N force at an angle of  $36.9^\circ$  above the horizontal. A 3500 N friction force opposes the sled's motion. Find the work done by each force acting on the sled and the total work done by all the forces.  $d = 20\text{ m}$



$$W_T = Td \cos \phi$$

$$= 80 \text{ kJ}$$

$$W_F = F_f d \cos(180^\circ)$$

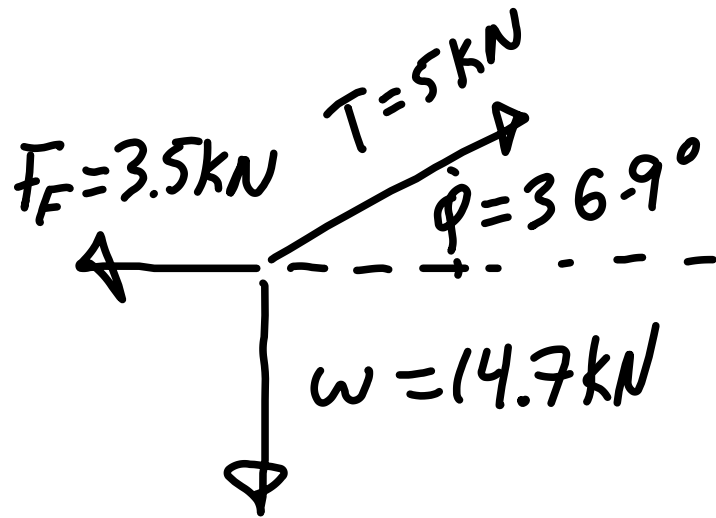
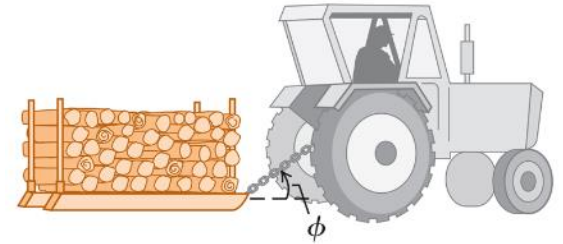
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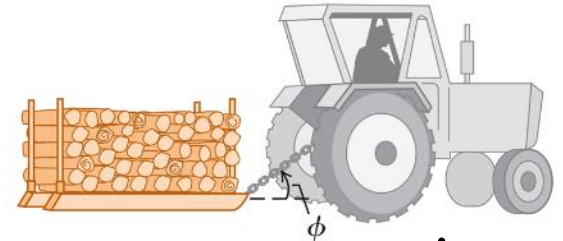
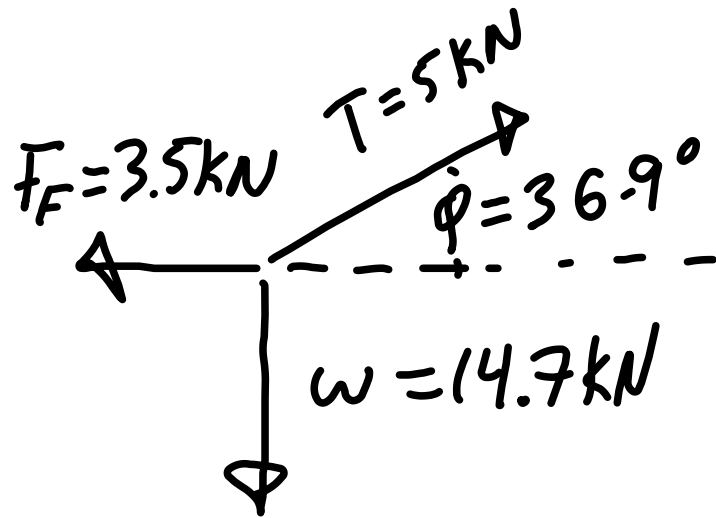
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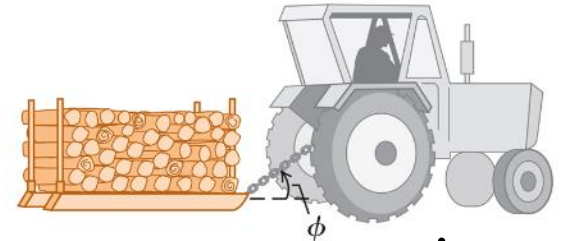
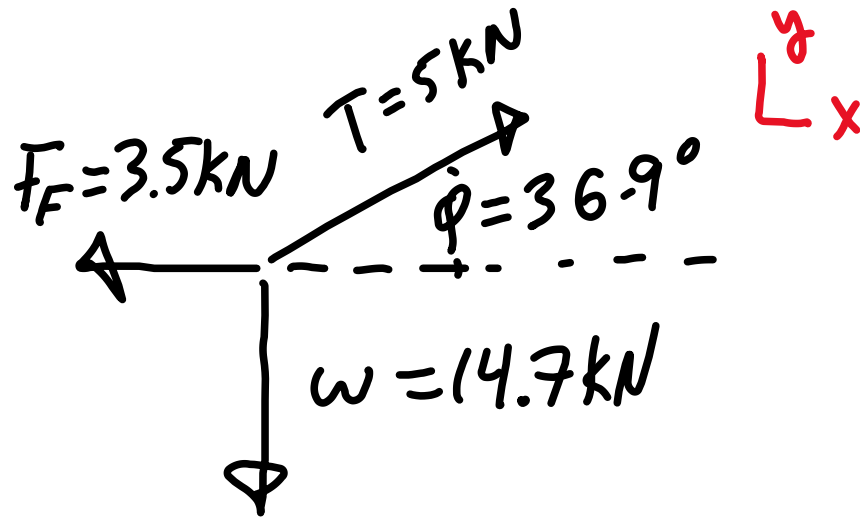
$$= (80 - 70) \text{ kJ} = 10 \text{ kJ}$$

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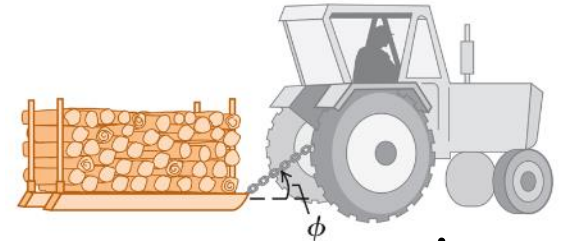
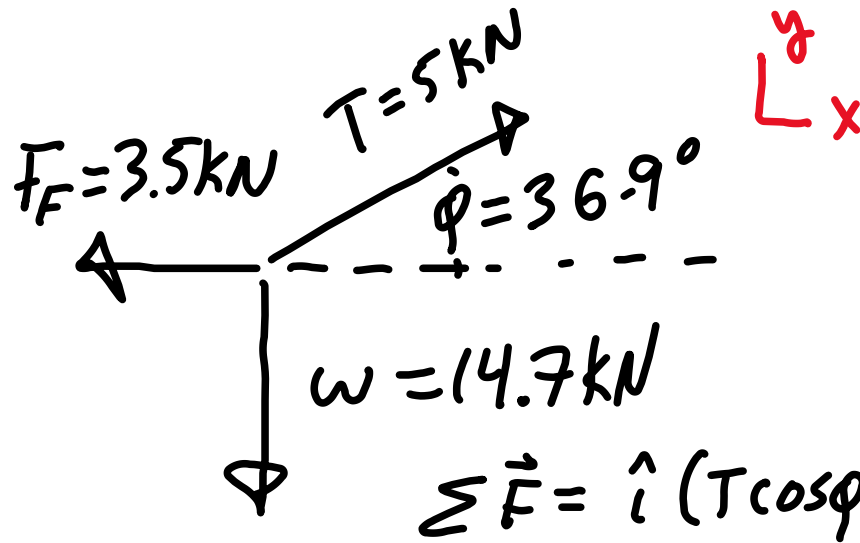
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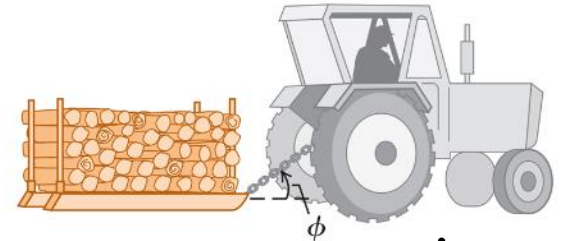
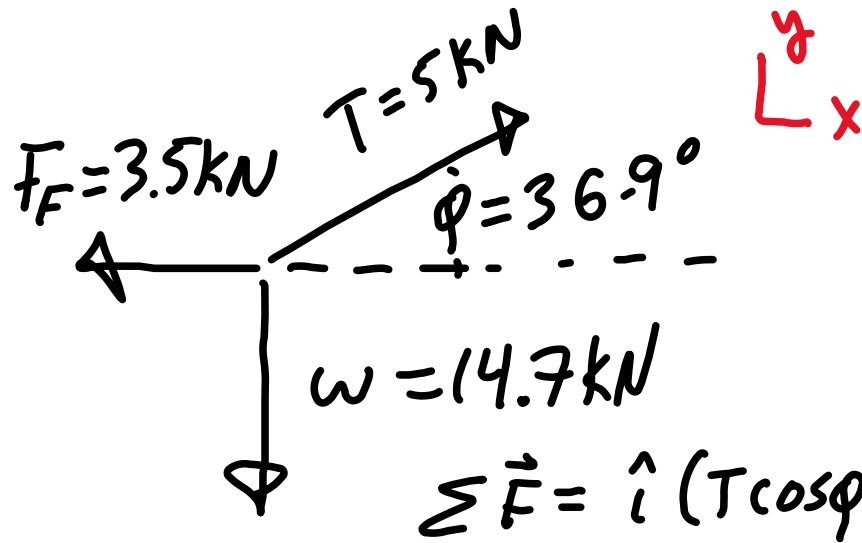
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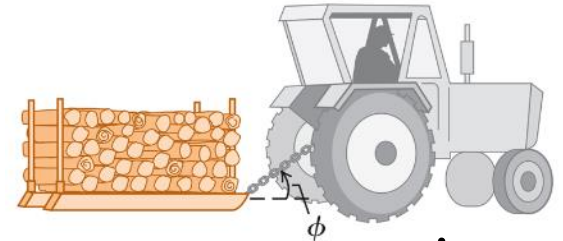
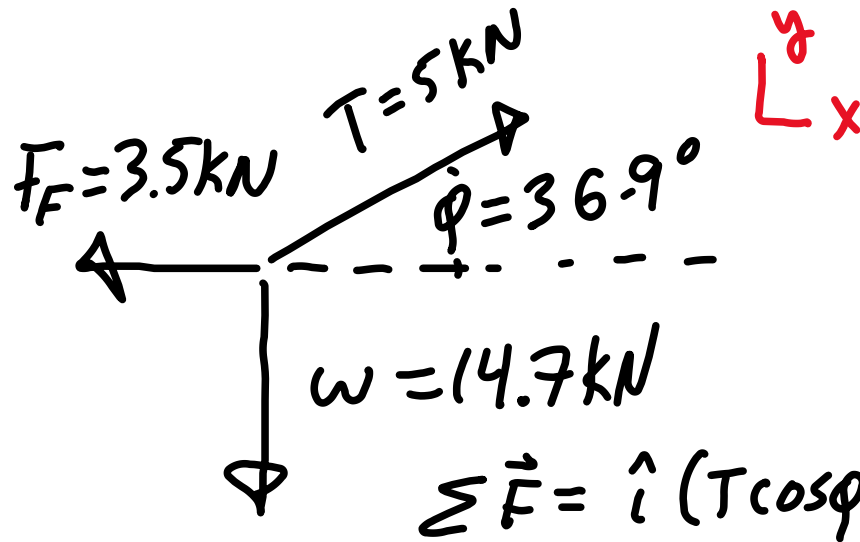


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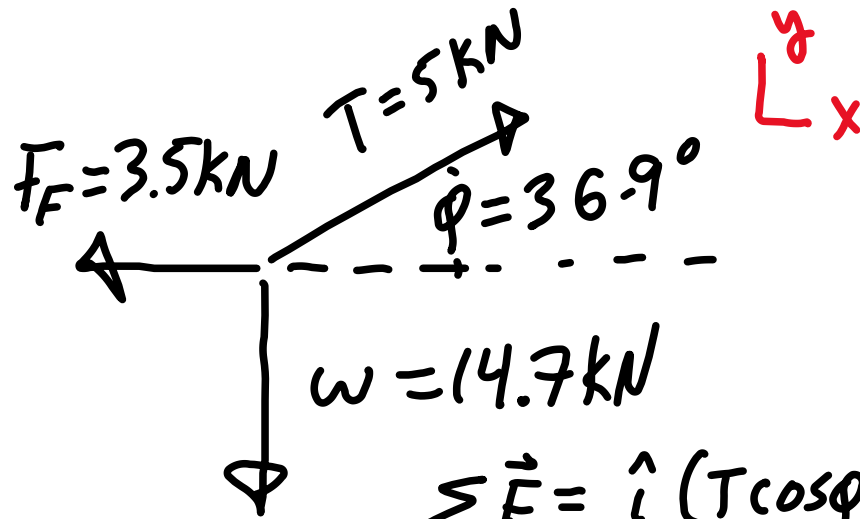
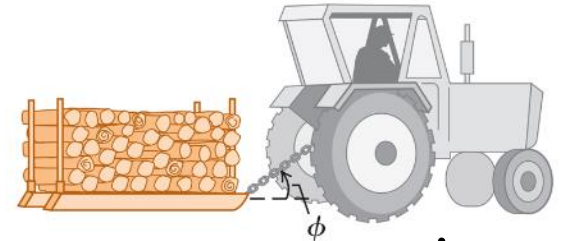


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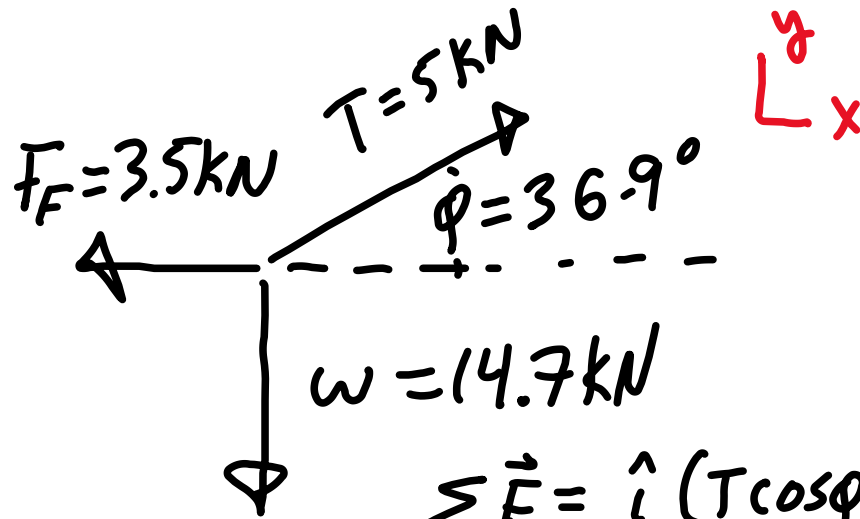
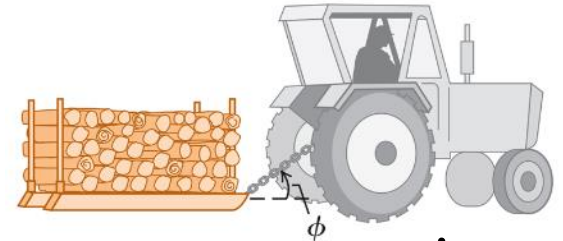
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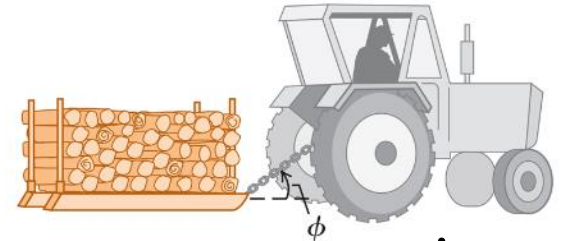
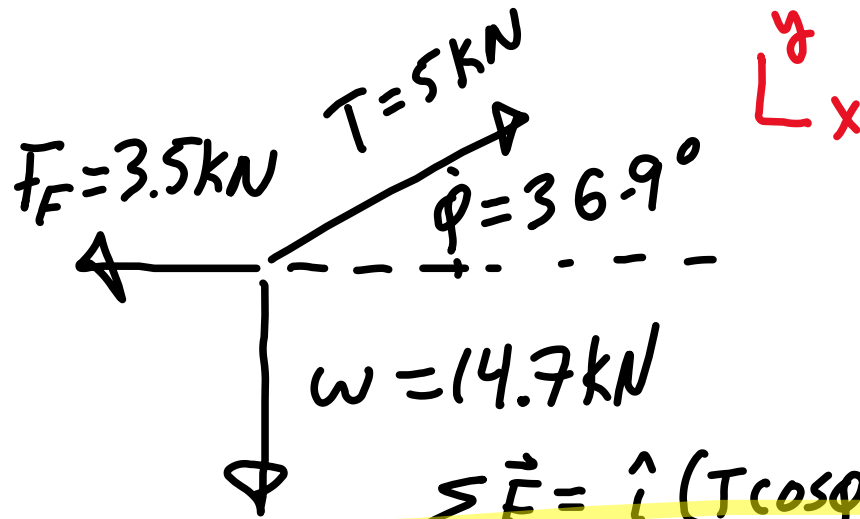
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$$\begin{aligned}\sum \vec{F} &= \hat{i}(T \cos \phi - F_f) - w \hat{j} \\ W &= (\sum \vec{F}) \cdot \vec{d} = [\hat{i}(T \cos \phi - F_f) - w \hat{j}] \cdot 20\text{m} \hat{i} \\ &= (T \cos \phi - F_f) 20\text{m} \\ &= [5 \cos(36.9^\circ) - 3.5] \times 20 \text{ kJ}\end{aligned}$$

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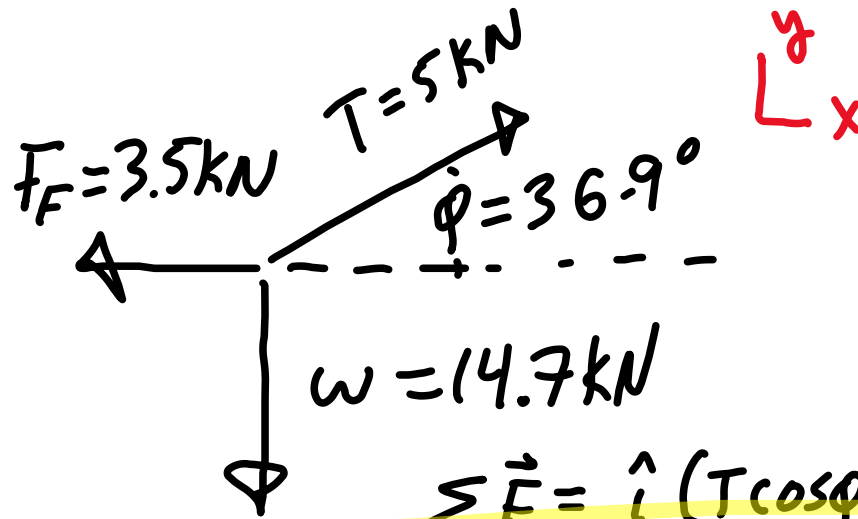
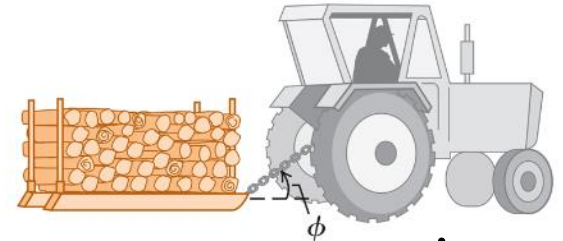


Another way to find  $W_{TOT}$ :

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$$\begin{aligned} W &= (\Sigma \vec{F}) \cdot \vec{d} = [\hat{i}(T \cos \phi - F_f) - w \hat{j}] \cdot 20\text{m} \hat{i} \\ &= (T \cos \phi - F_f) 20\text{m} \\ &= [5 \cos(36.9^\circ) - 3.5] \times 20 \text{ kJ} = 10 \text{ kJ} \end{aligned}$$

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$$= (T \cos \phi - F_f) 20\text{m}$$

$$= [5 \cos(36.9^\circ) - 3.5] \times 20 \text{ kJ} = 10 \text{ kJ}$$

*Same as before*

## 3d case

$$\Sigma \vec{F} = m\vec{a} \Rightarrow \Sigma \vec{F} \cdot d\vec{r} = m\vec{a} \cdot d\vec{r}, \text{ But}$$

$$d\vec{r} = \frac{d\vec{r}}{dt} dt = \vec{v} dt \quad \& \quad \vec{a} = \frac{d\vec{v}}{dt}$$

$$\text{so } \Sigma \vec{F} \cdot d\vec{r} = m\vec{v} \cdot \left(\frac{d\vec{v}}{dt}\right) dt \quad \& \quad \text{since}$$

$$d\vec{v} = \left(\frac{d\vec{v}}{dt}\right) dt, \text{ then } \int \Sigma \vec{F} \cdot d\vec{r} = \int m\vec{v} \cdot d\vec{v}$$

$$= M \left\{ \int v_x dv_x + \int v_y dv_y + \int v_z dv_z \right\} \Rightarrow \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \frac{M}{2} (v_x^2 + v_y^2 + v_z^2) \Big|_{v_I}^{v_F}$$

$$= \left(\frac{M}{2}\right) \{ v_F^2 - v_I^2 \} \Rightarrow \Sigma \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = T_2 - T_1 \quad \underline{\text{or}}$$

$$U_{1 \rightarrow 2} = \Delta T \quad \text{with} \quad U_{1 \rightarrow 2} \equiv \Sigma \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

