

Today S.3

L16



Today S.3

L16

Friction



Today S.3  
Friday 5.4, 5.5

L16



Today S.3  
Friday 5.4, 5.5

Circular  
motion  
dynamics

Today S.3

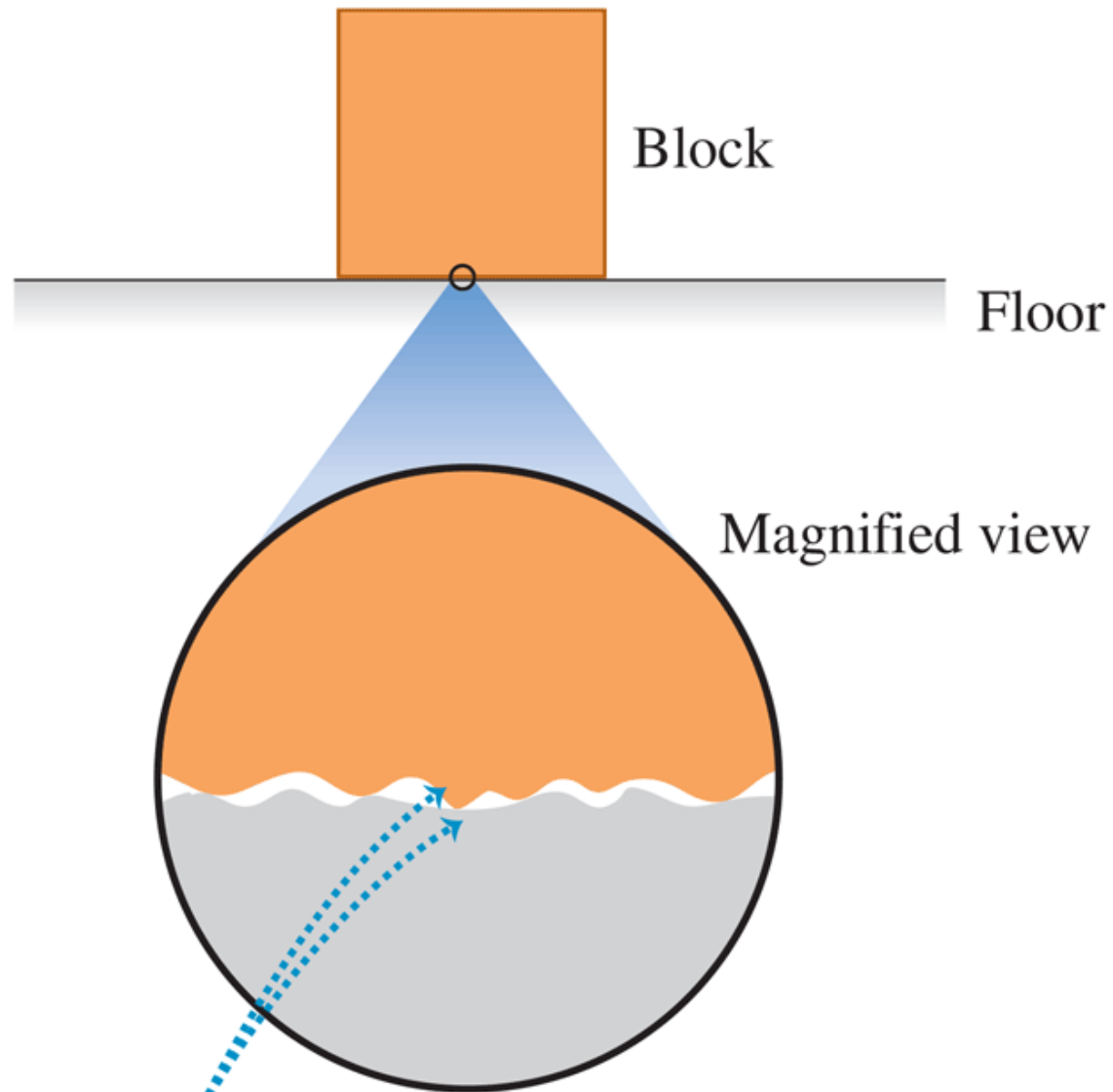
Friday 5.4, 5.5

L16

The Fundamental  
forces of  
nature

# Friction

# Friction



The friction and normal forces result from interactions between molecules in the block and in the floor where the two rough surfaces touch.

# Friction

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Materials	Coefficient of Static Friction, $\mu_s$	Coefficient of Kinetic Friction, $\mu_k$
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Brass on steel	0.51	0.44
Zinc on cast iron	0.85	0.21
Copper on cast iron	1.05	0.29
Glass on glass	0.94	0.40
Copper on glass	0.68	0.53
Teflon on Teflon	0.04	0.04
Teflon on steel	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25

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$$F_f = N\mu_k, \text{ kinetic}$$

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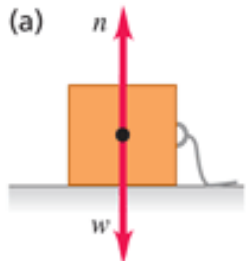
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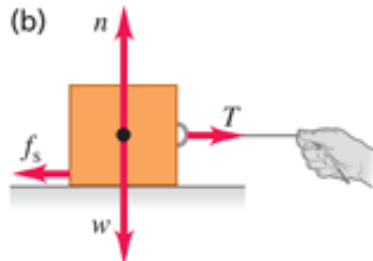
For static case,  $F_f$  only provides enough force to keep item static.

For both cases,  $N$  is the normal force between the two items that are in frictional contact.

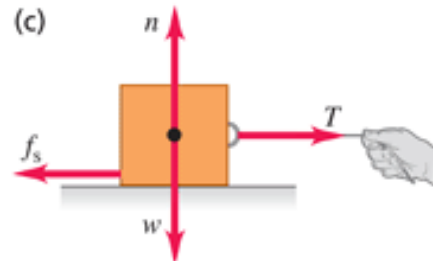
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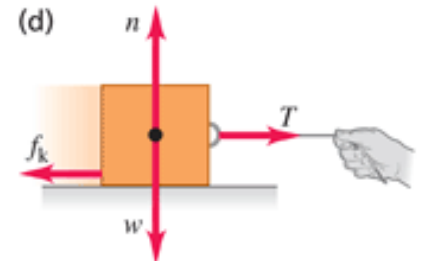
- ① No applied force, box at rest. No friction:  $f_s = 0$



- ② Weak applied force, box remains at rest. Static friction:  $f_s < \mu_s n$

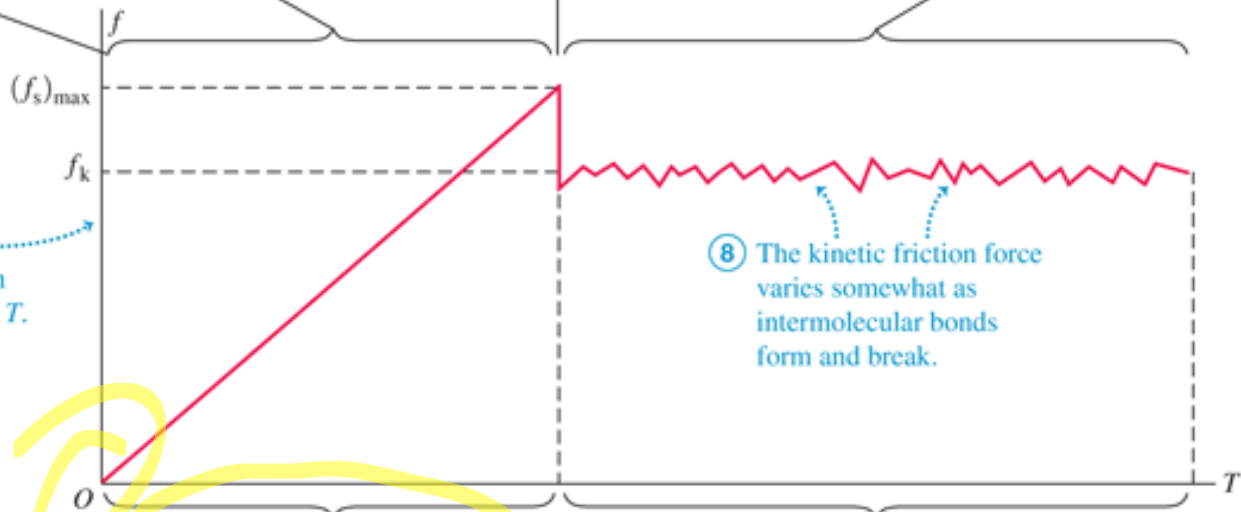


- ③ Stronger applied force, box just about to slide. Static friction:  $f_s = \mu_s n$



- ④ Box sliding at constant speed. Kinetic friction:  $f_k = \mu_k n$

(e)



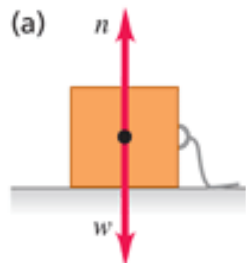
- ⑤ This graph shows the friction force magnitude  $f$  as a function of the pulling force magnitude  $T$ .

- ⑧ The kinetic friction force varies somewhat as intermolecular bonds form and break.

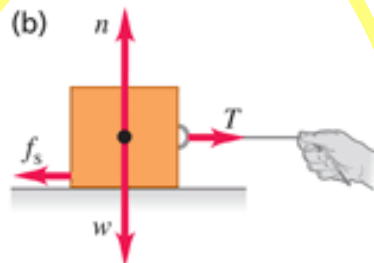
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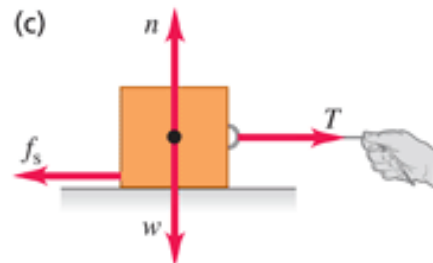
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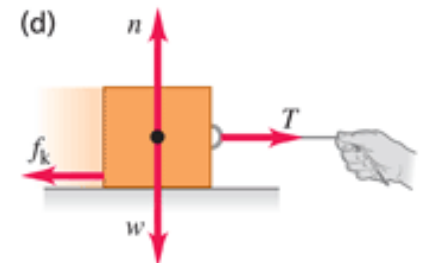
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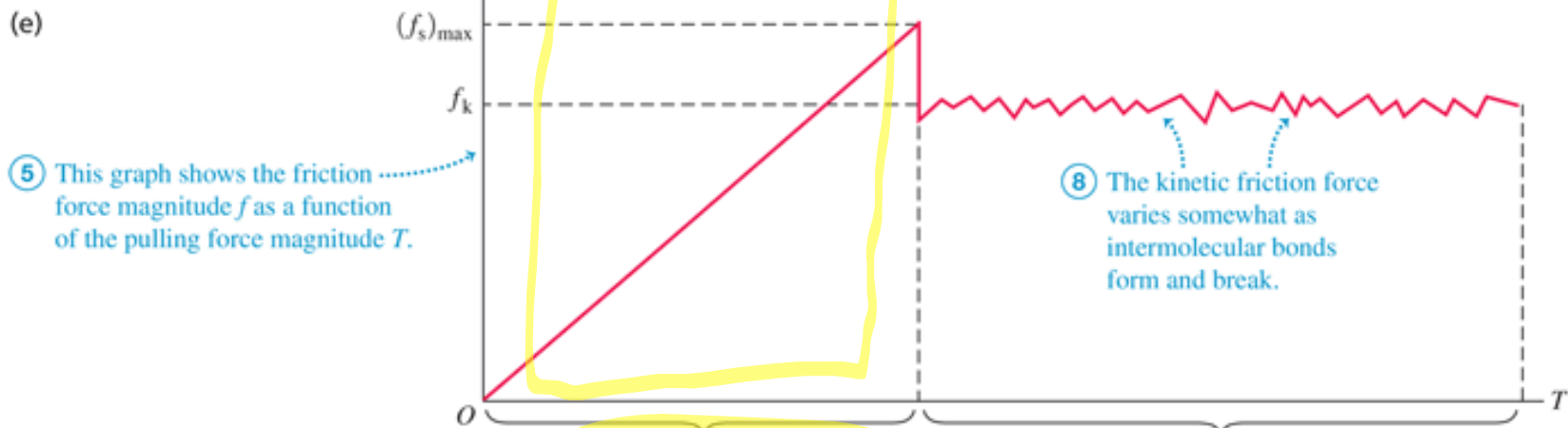
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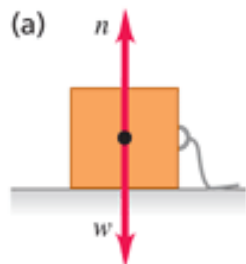
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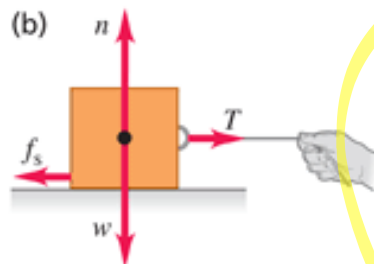
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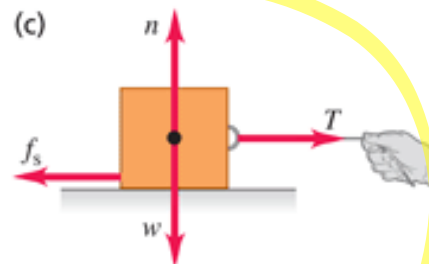
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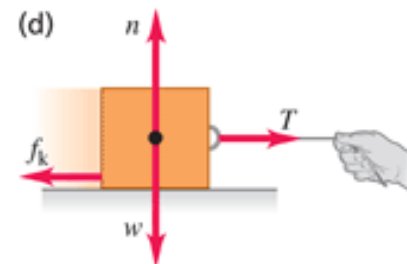
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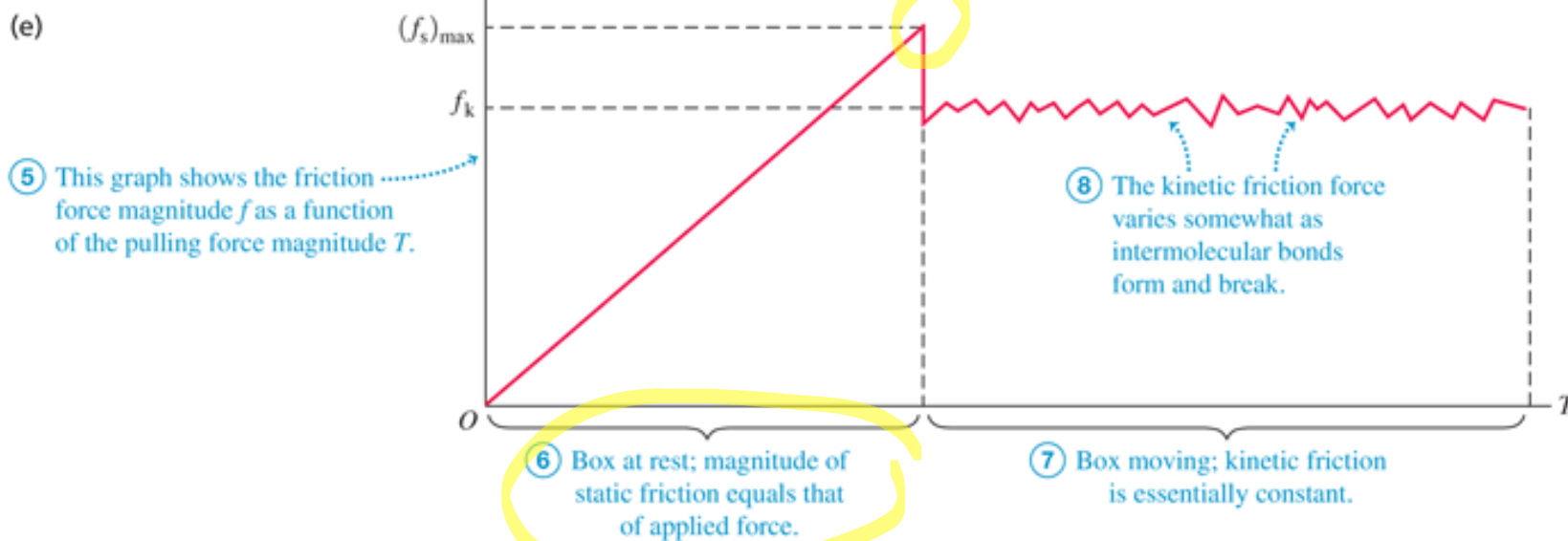
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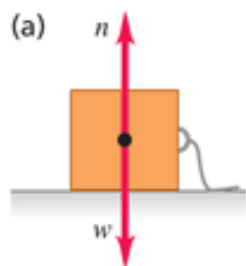
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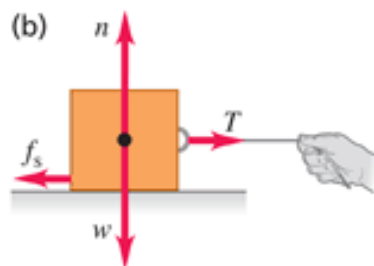
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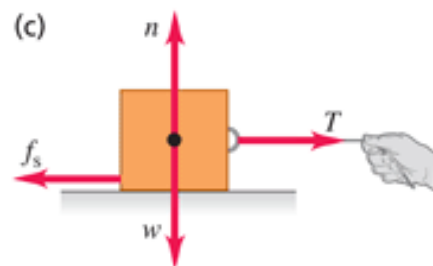
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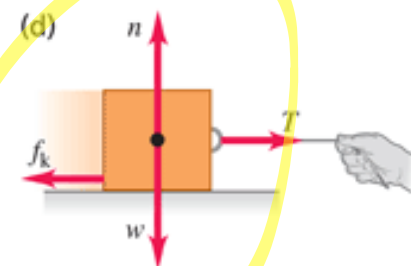
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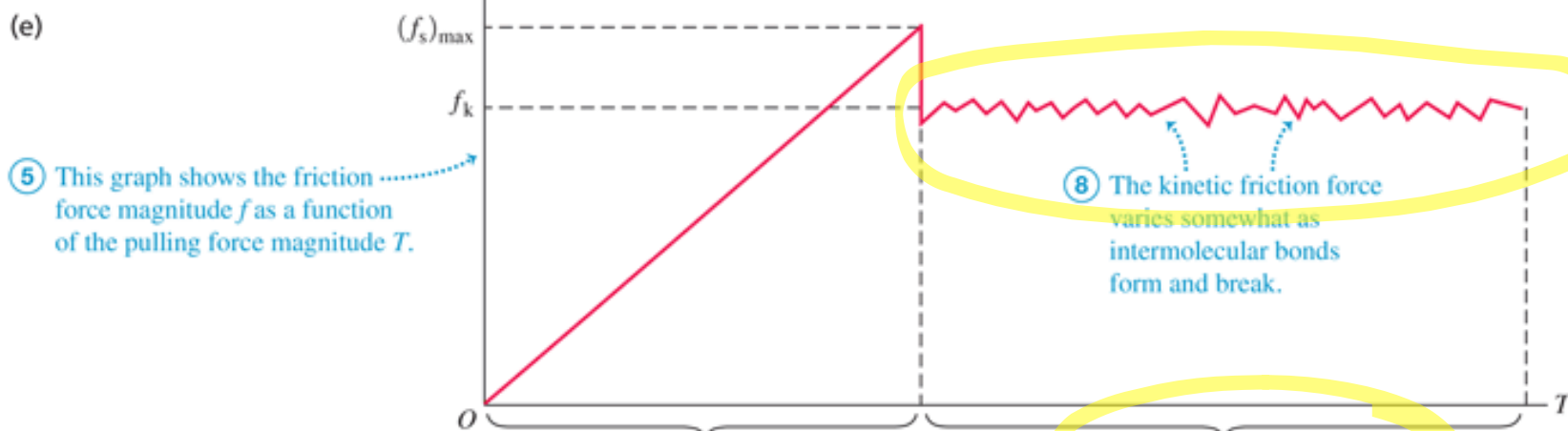
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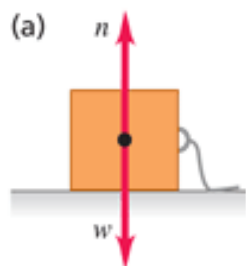
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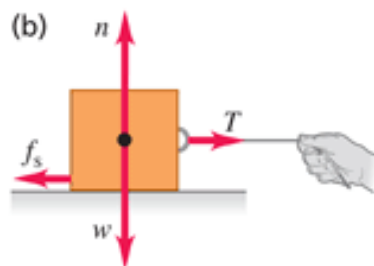
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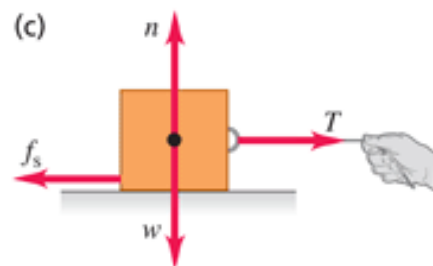
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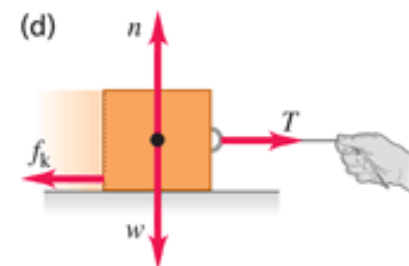
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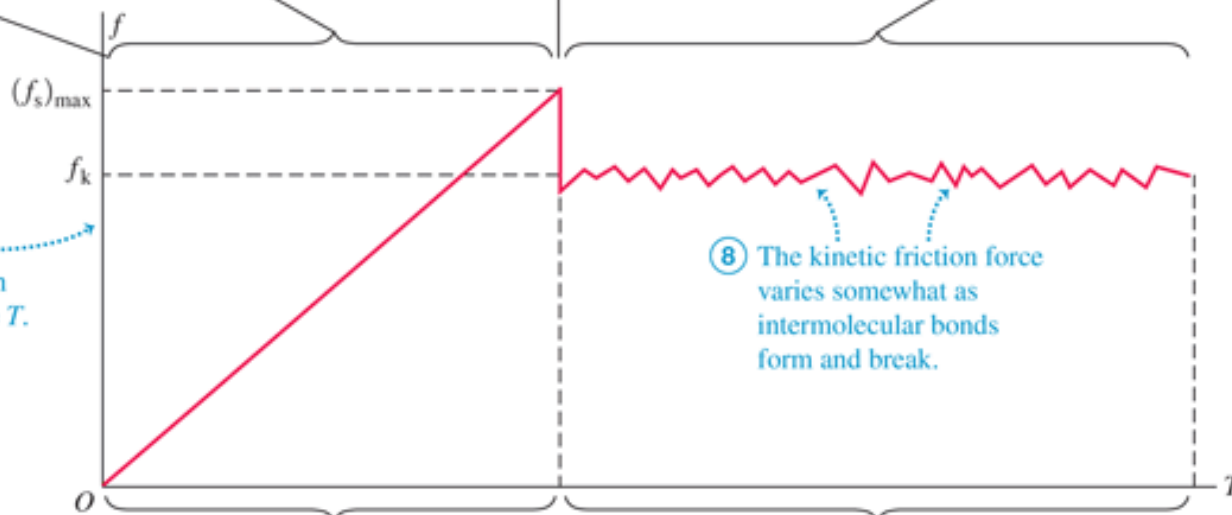


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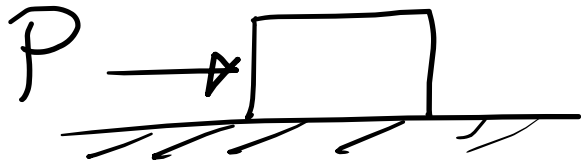
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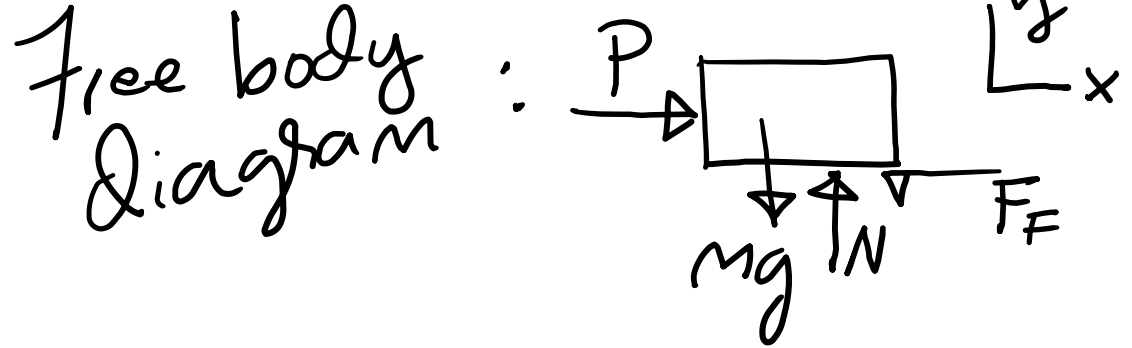
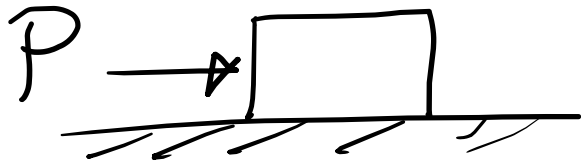
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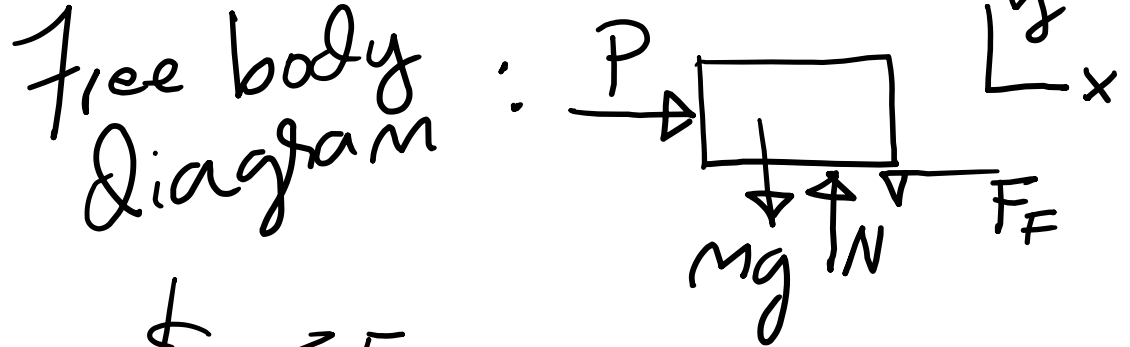
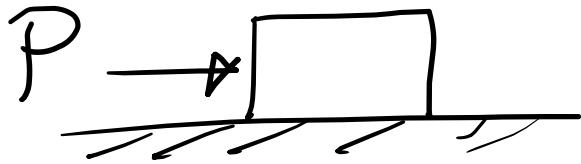
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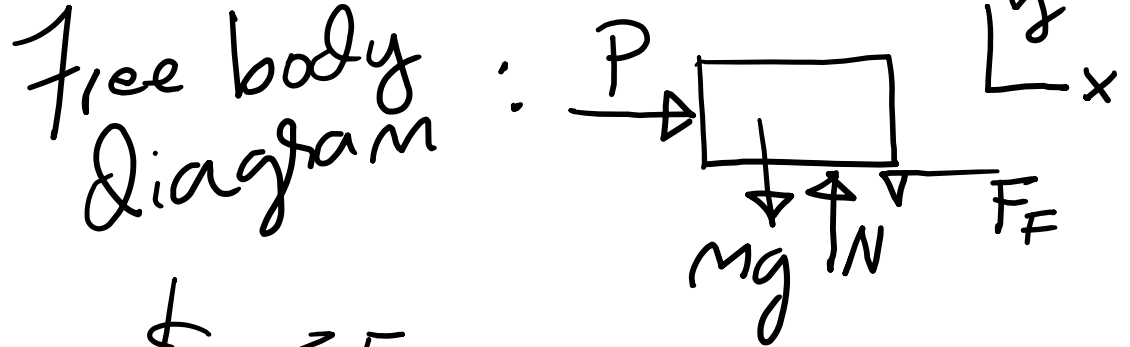
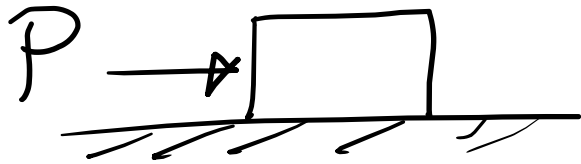


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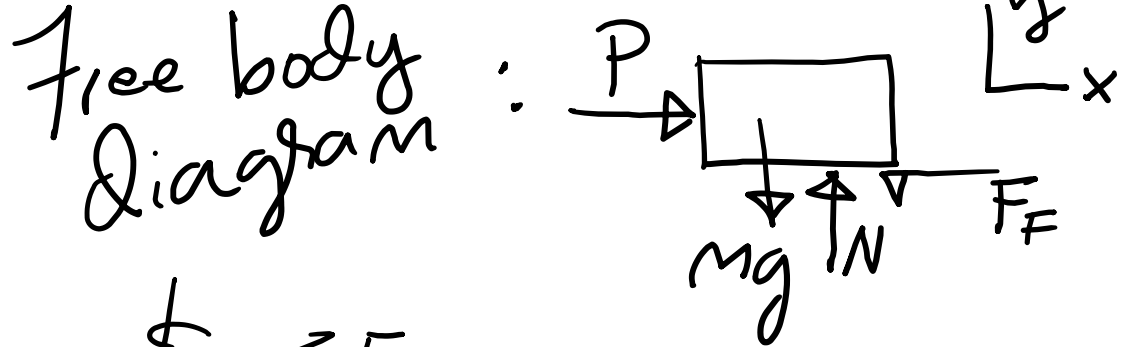
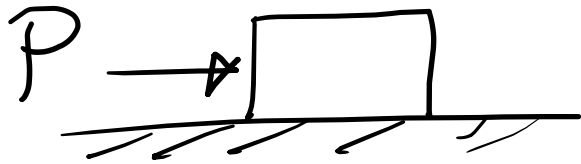
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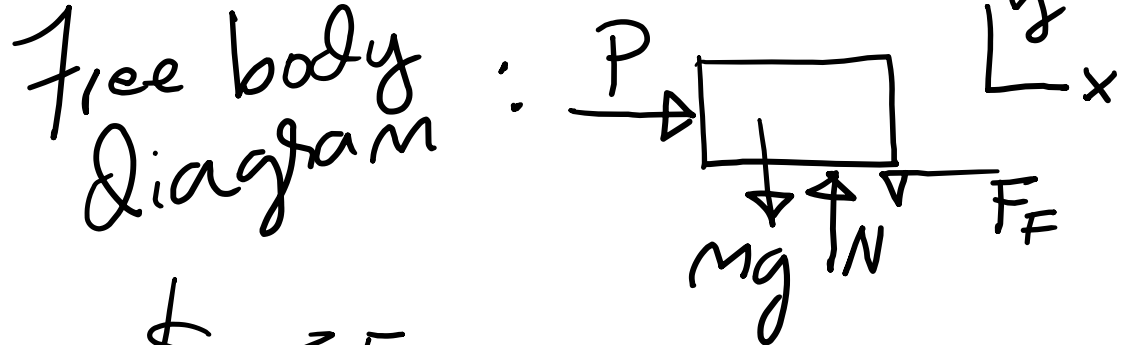
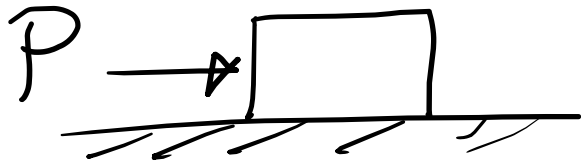
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Kinetic case:  $\sum F_x = ma_x \Rightarrow P - F_f = ma_x$   
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 $P = 2N$  &  $m = 1\text{kg}$  &  $\mu_k = 0.1$

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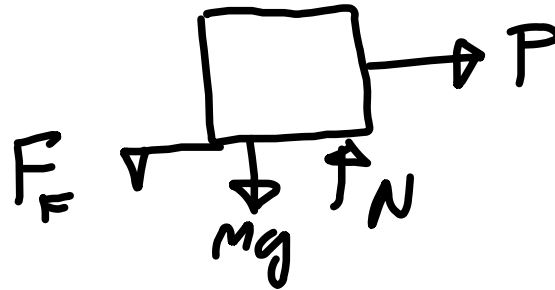
$P = 2N$  &  $m = 1\text{ kg}$  &  $\mu_k = 0.1$ , then

  $a_x = (2 - 0.981)^{1/2} = 1.019 \text{ m/s}^2$

You want to move a 500 N crate across a level floor. To start the crate moving, you have to pull with a 230 N horizontal force. Once the crate starts to move, you can keep it moving at constant velocity with only 200 N. What are the coefficients of static and kinetic friction?

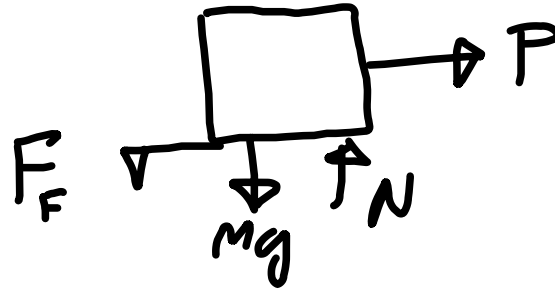


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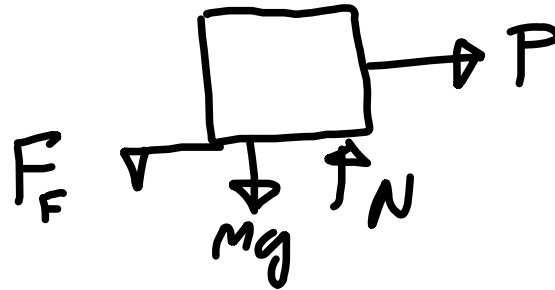
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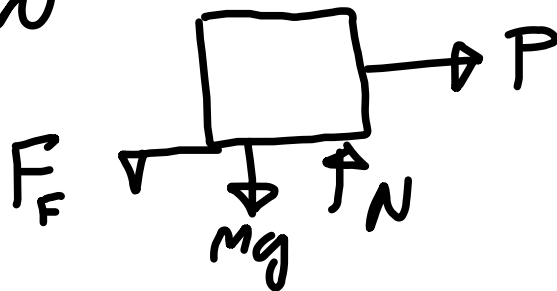
KINETIC



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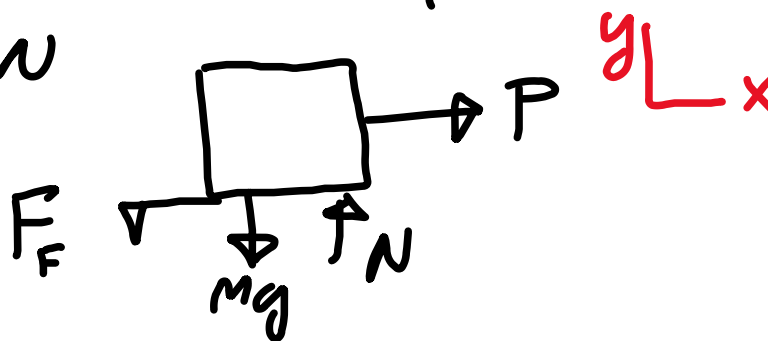
KINETIC:  $P_2 = 200\text{ N}$   
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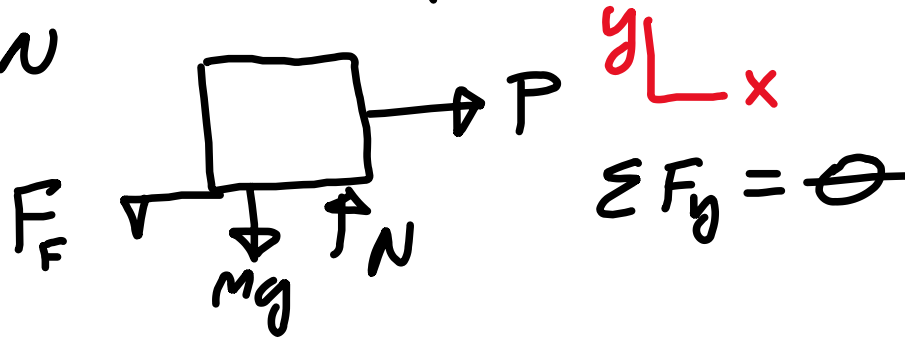
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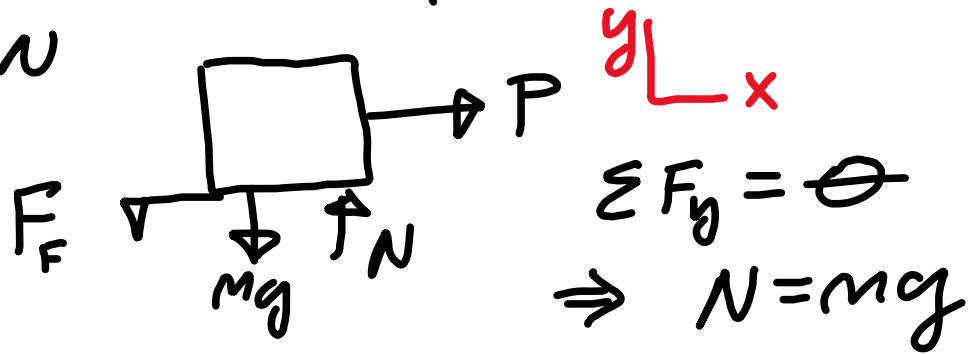
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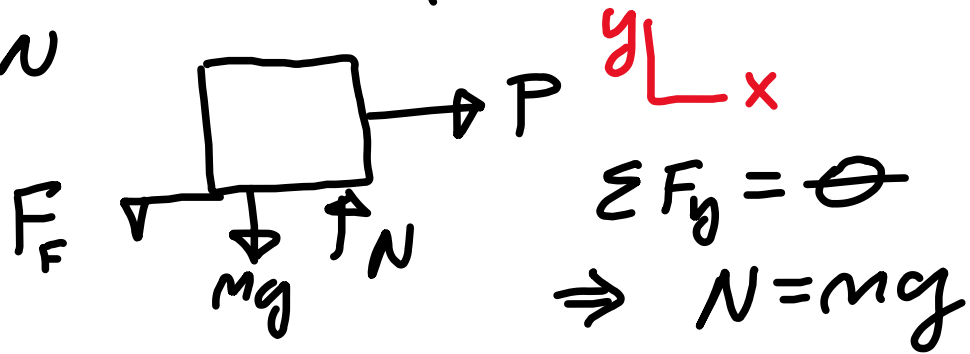
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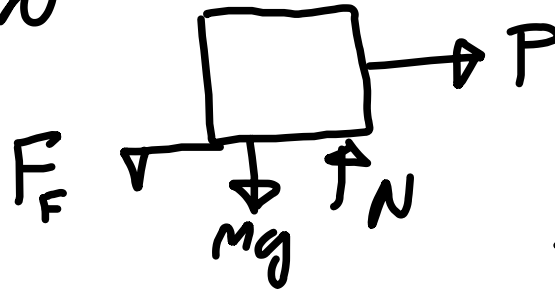


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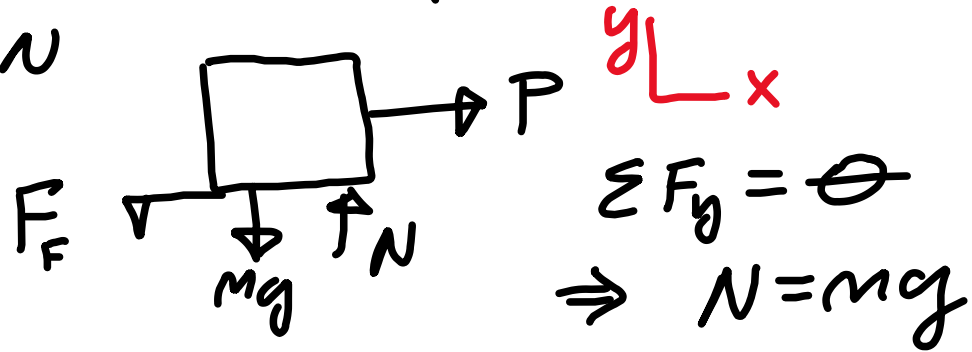
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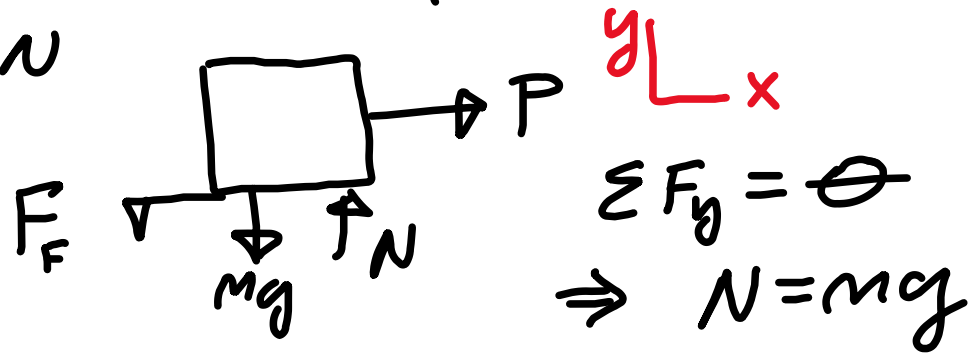


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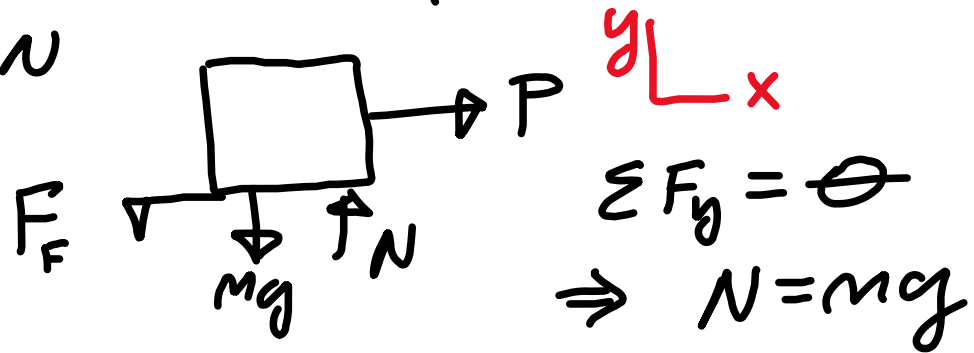
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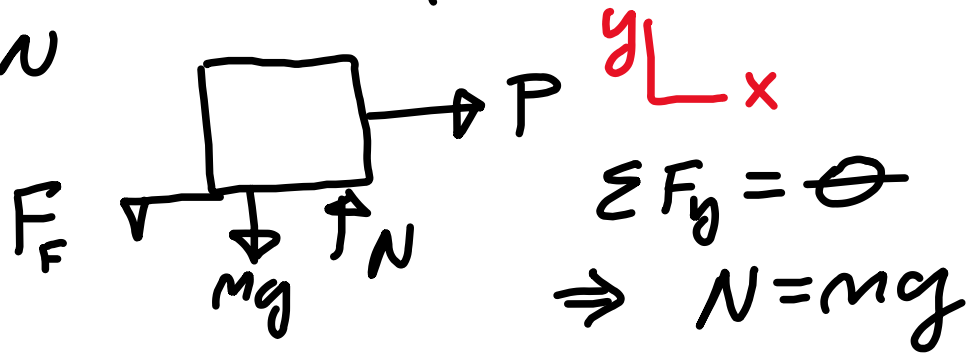
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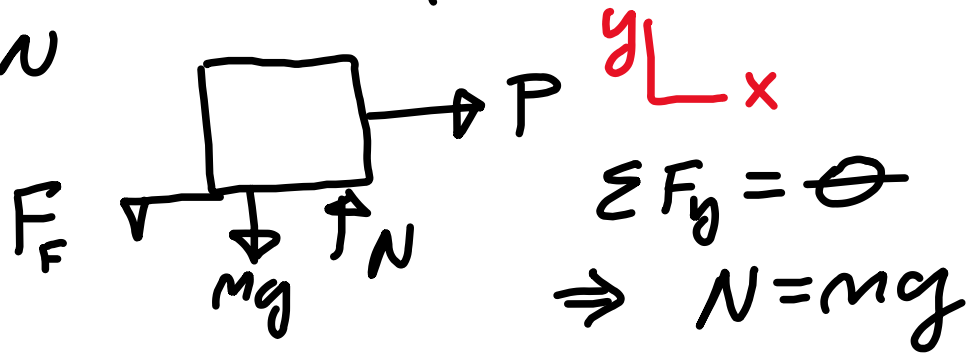
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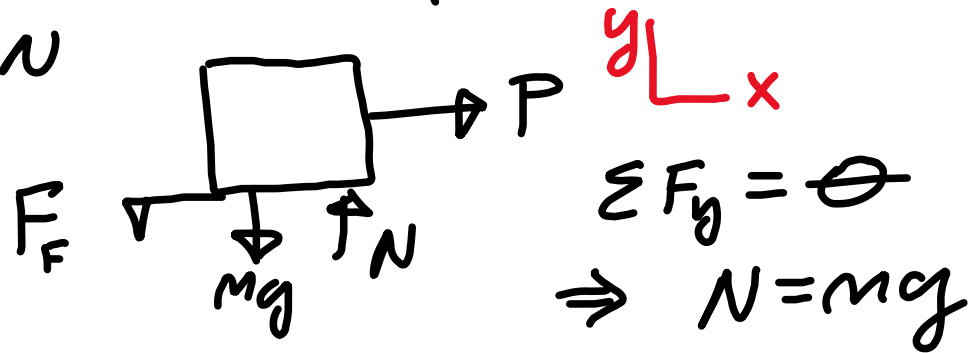
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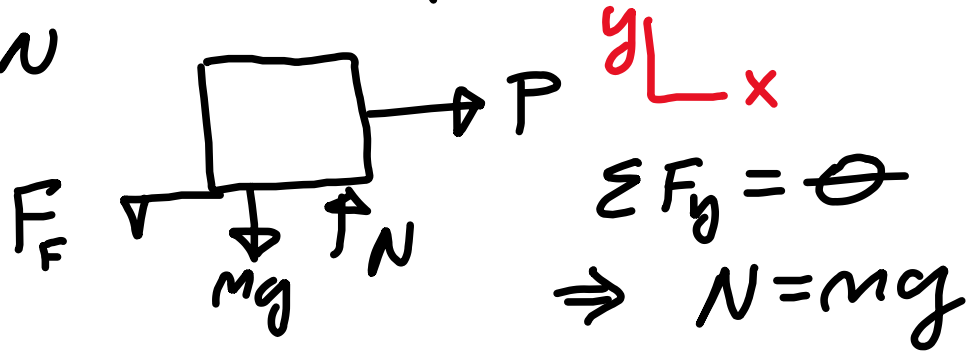
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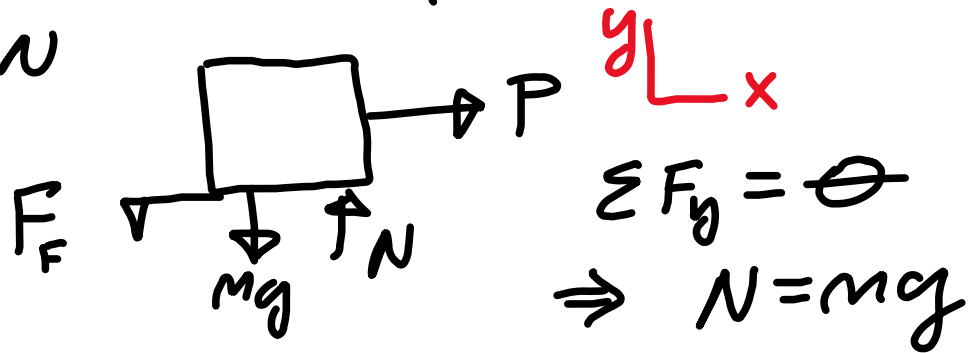
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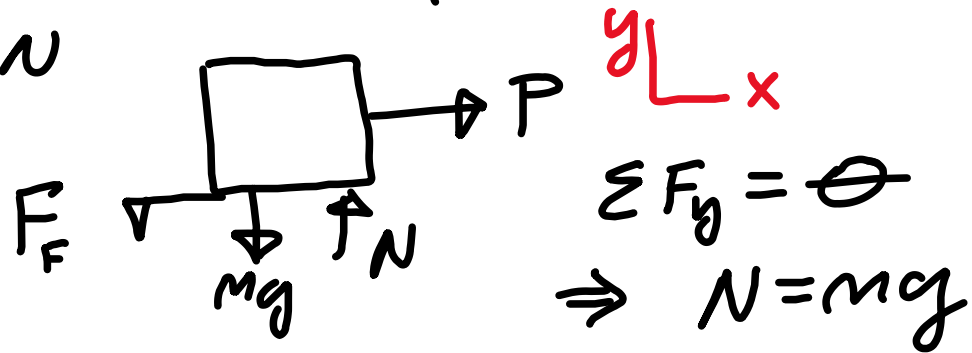
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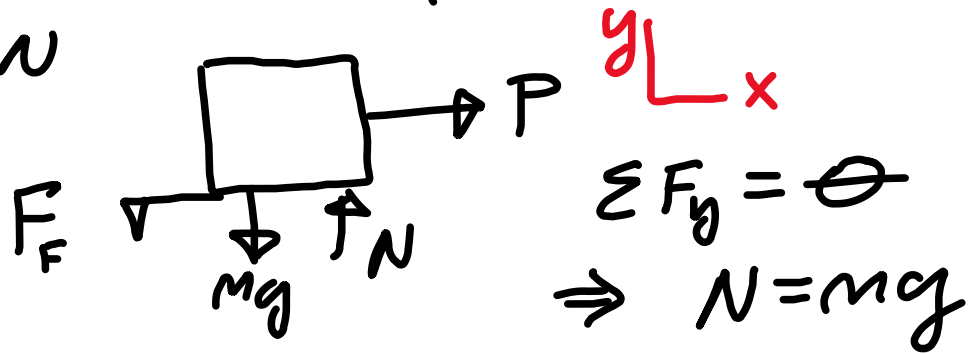
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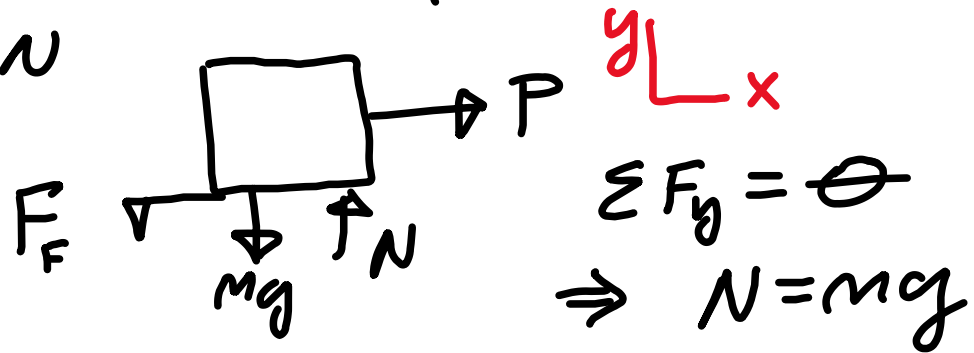
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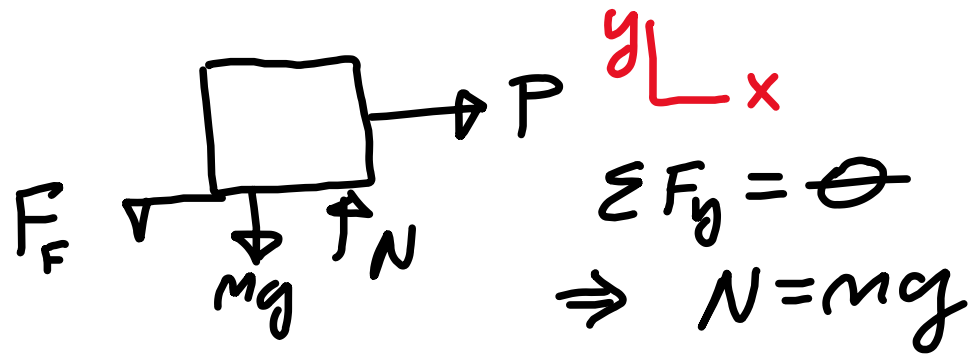
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In **Example 5.13** , what is the friction force if the crate is at rest on the surface and a horizontal force of 50 N is applied to it?

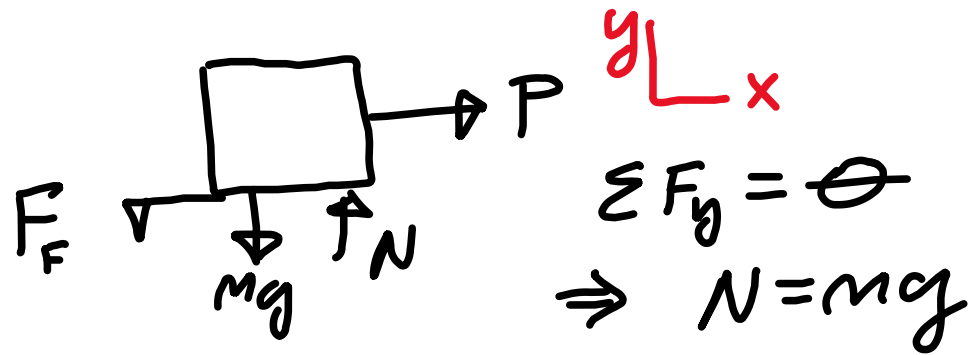
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$$\Sigma F_x = 0$$

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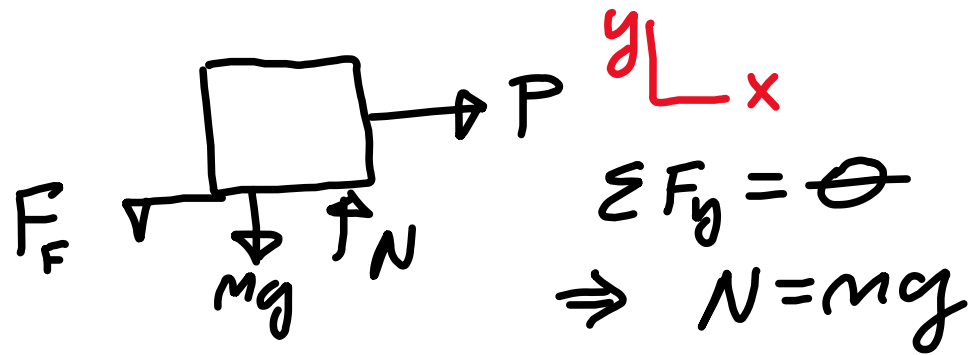
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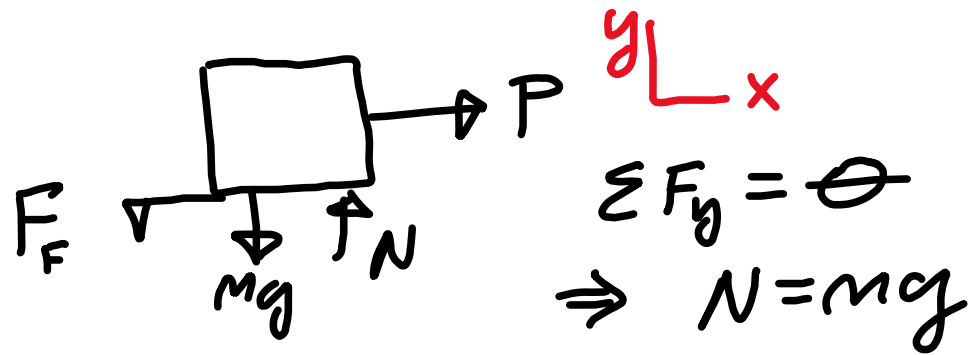
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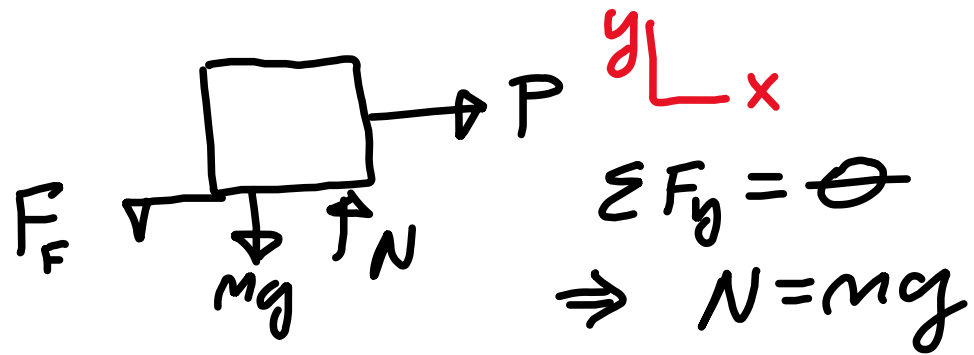
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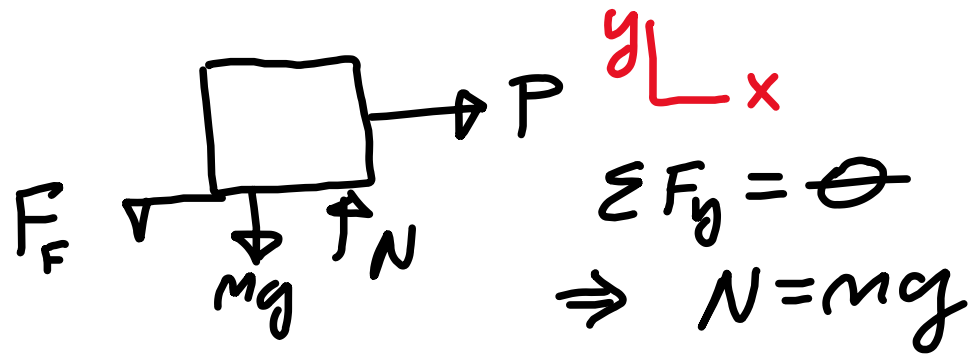


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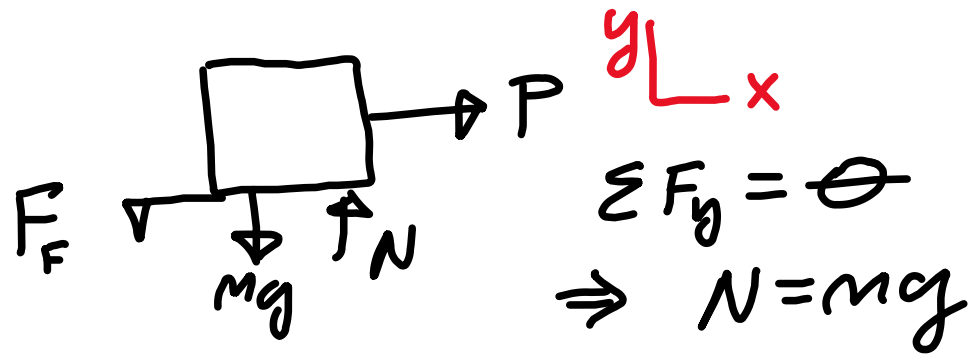
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where  $N\mu_s = (500 \text{ N})(0.46)$

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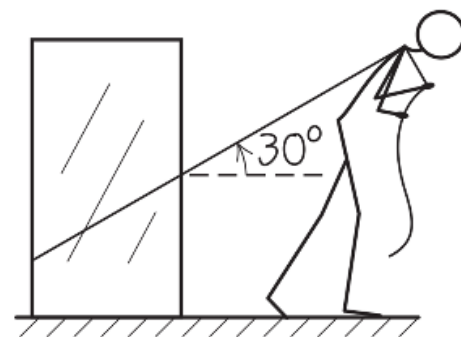
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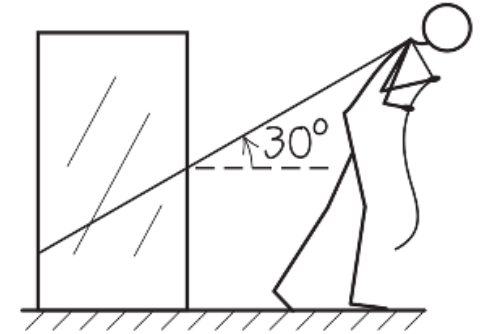
In **Example 5.13**, suppose you move the crate by pulling upward on the rope at an angle of  $30^\circ$  above the horizontal. How hard must you pull to keep it moving with constant velocity? Assume that  $\mu_k = 0.40$ .



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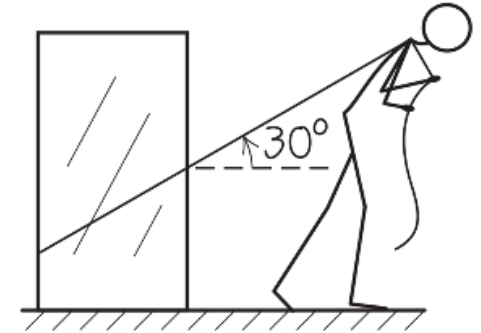
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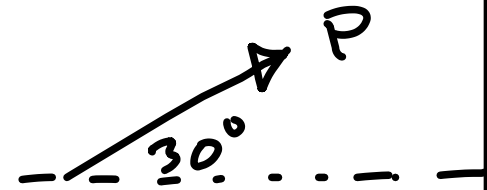
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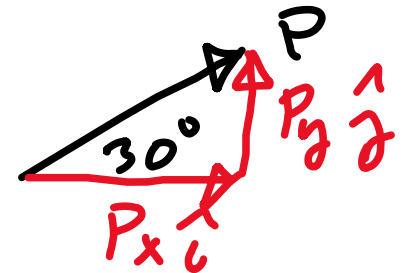
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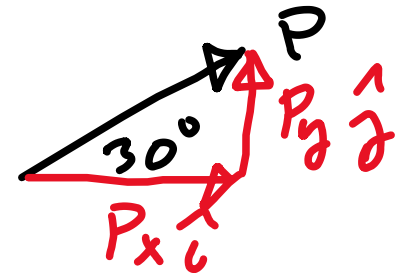


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$$\vec{P} = P_x \hat{i} + P_y \hat{j}$$

y  
L  
x

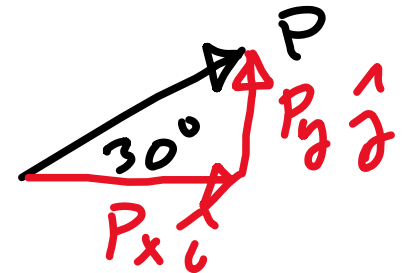


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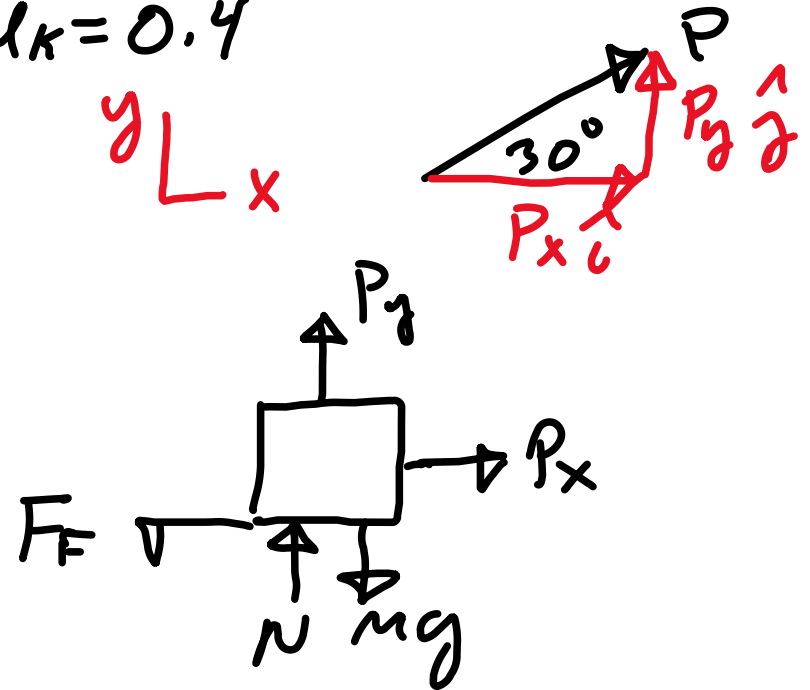
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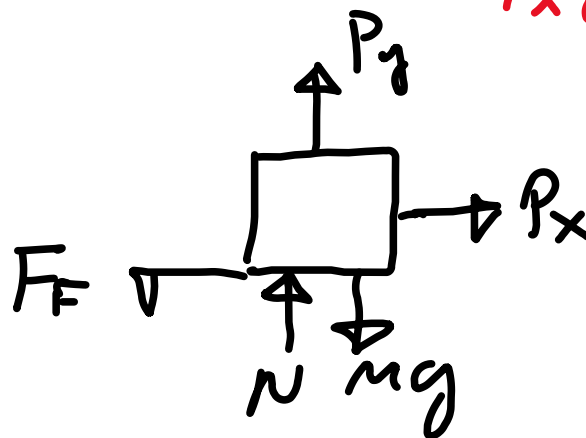
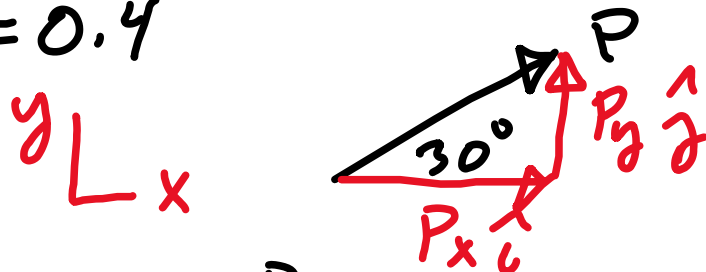


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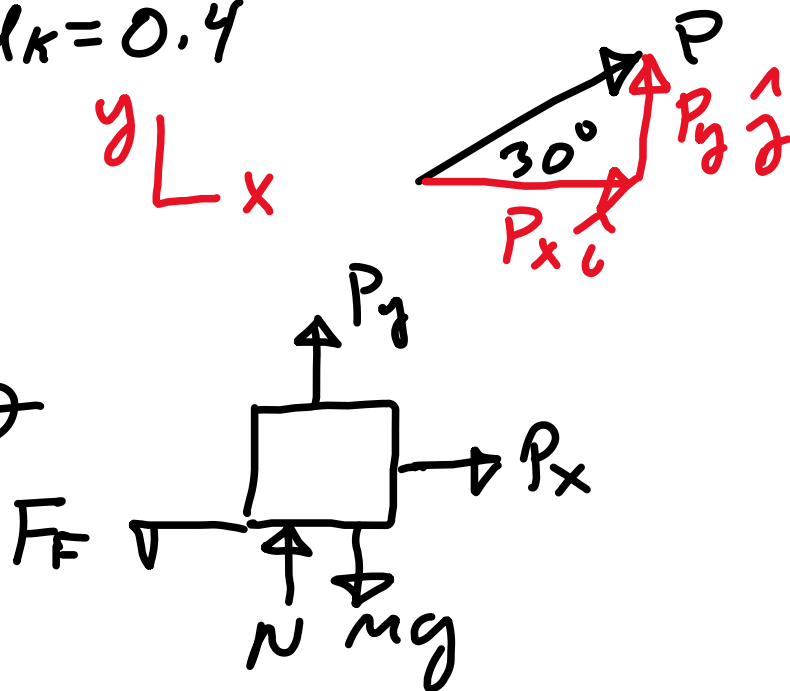
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$$\Sigma F_y = 0 \Rightarrow P_y + N - mg = 0$$



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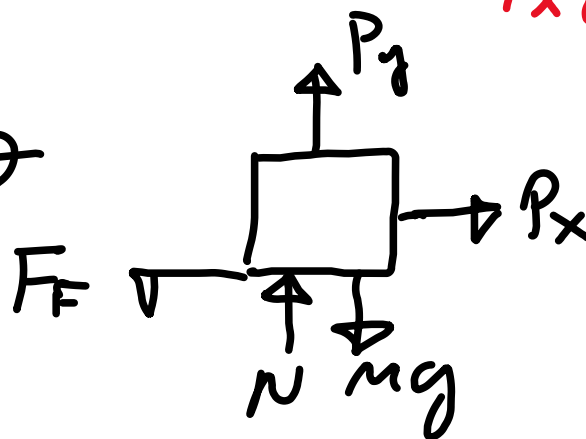
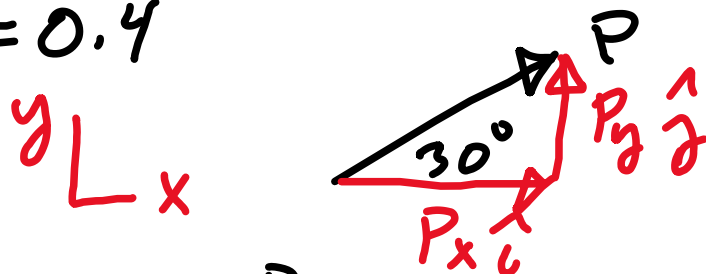
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$$N = mg - P_y$$



In **Example 5.13**, suppose you move the crate by pulling upward on the rope at an angle of  $30^\circ$  above the horizontal. How hard must you pull to keep it moving with constant velocity?

Assume that  $\mu_k = 0.40$ .  $W = 500\text{ N}$ ,  $\mu_k = 0.4$

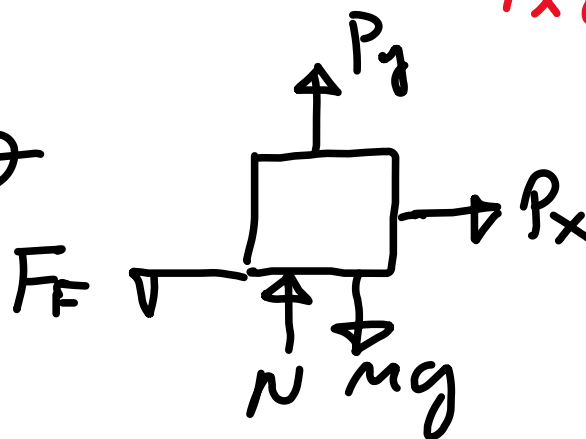
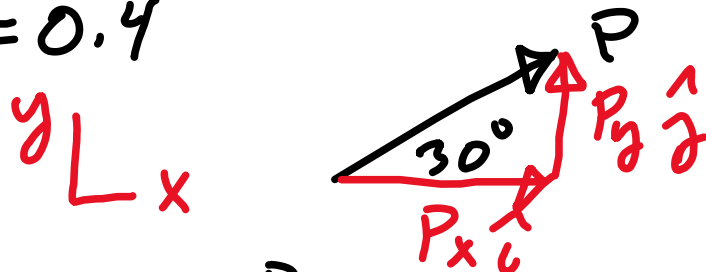
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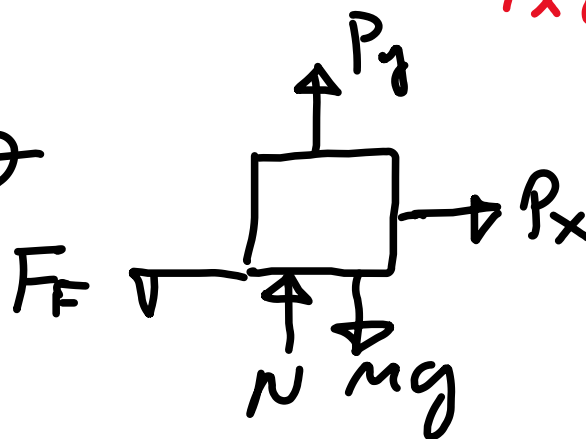
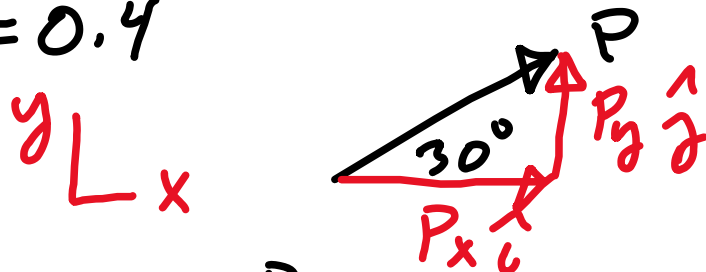
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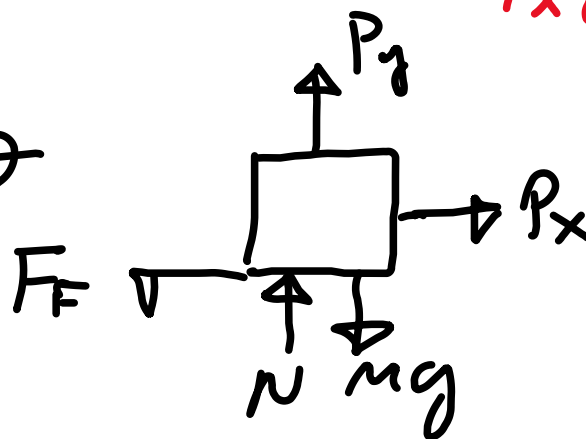
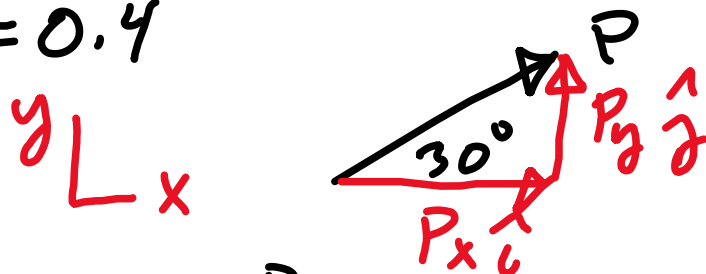
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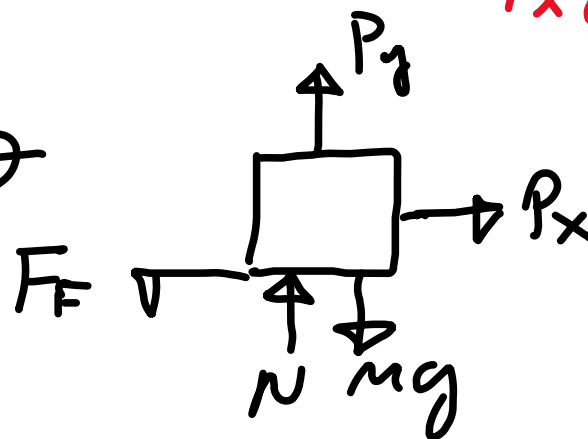
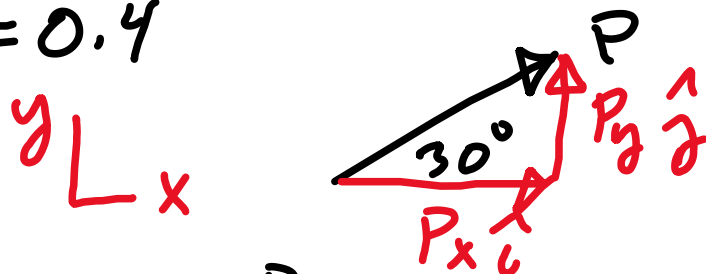
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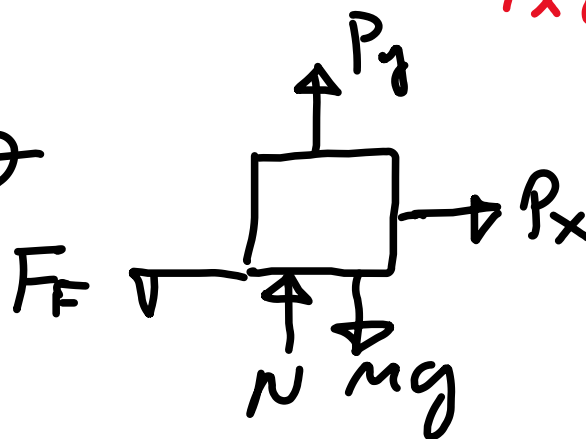
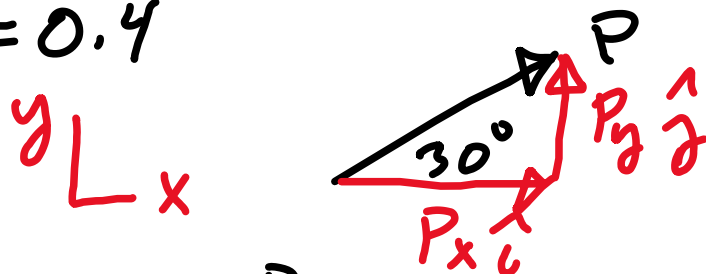
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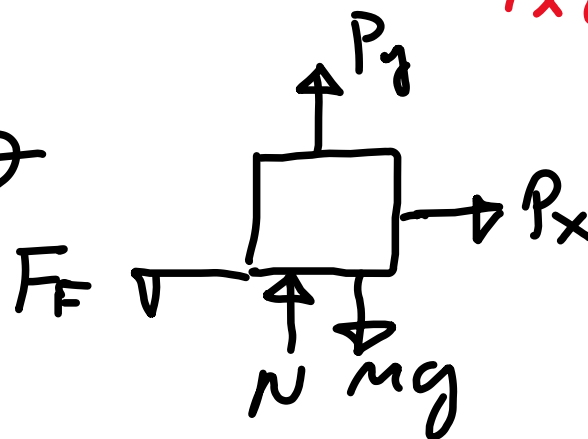
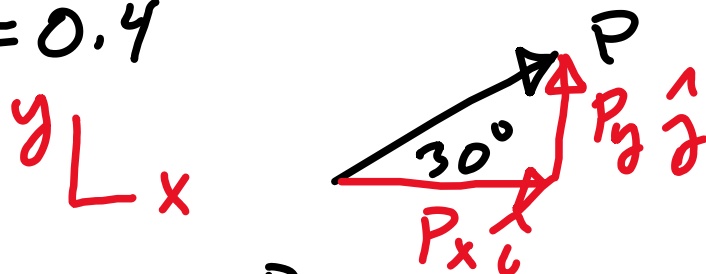
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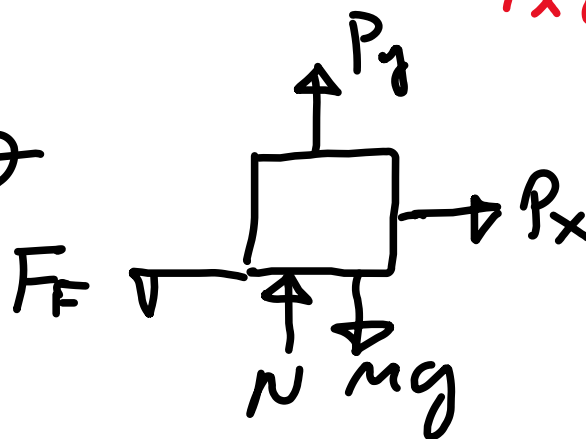
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$$\Rightarrow P = \frac{mg \mu_k}{\cos 30^\circ + \mu_k \sin 30^\circ}$$

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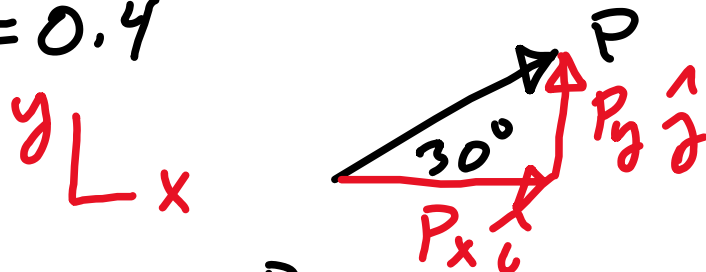
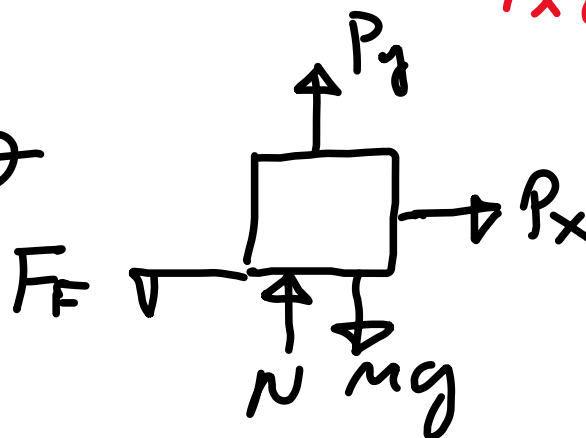
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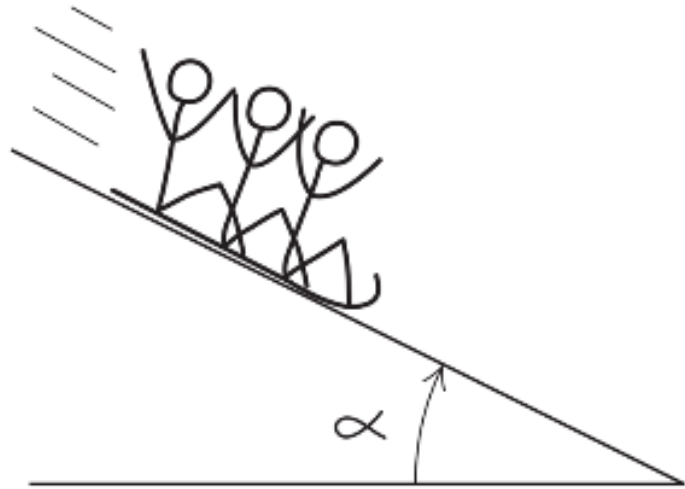
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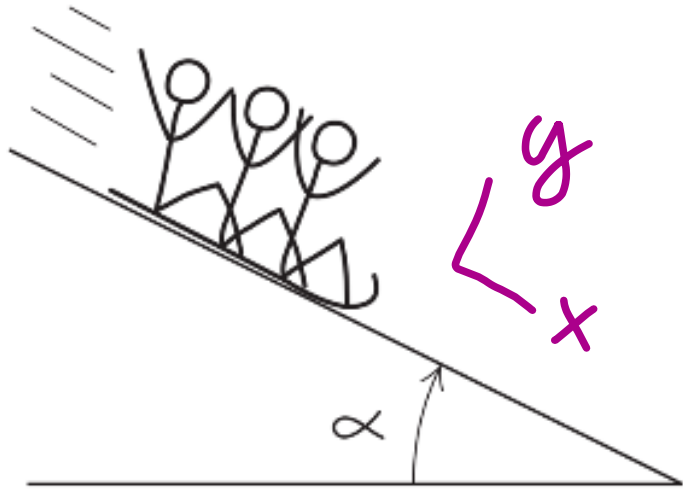
$$\Rightarrow P = \frac{(500\text{ N})(0.4)}{\cos 30^\circ + (0.4) \sin 30^\circ} = 188\text{ N}$$

Let's go back to the toboggan we studied in [Example 5.10](#). The wax has worn off, so there is now a nonzero coefficient of kinetic friction  $\mu_k$ . The slope has just the right angle to make the toboggan slide with constant velocity. Find this angle in terms of  $w$  and  $\mu_k$ .

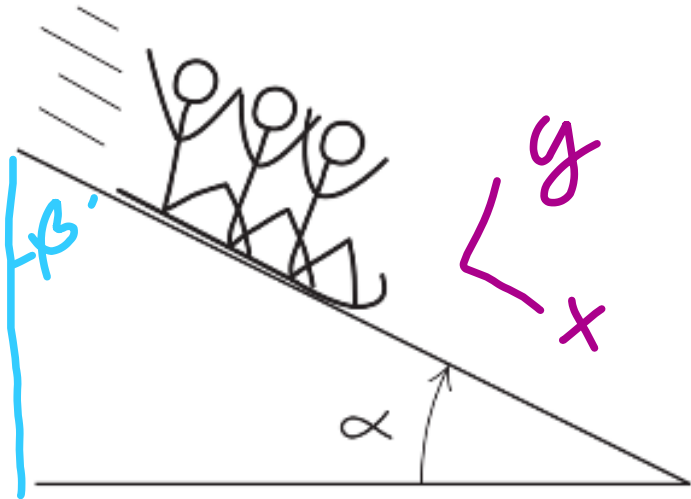
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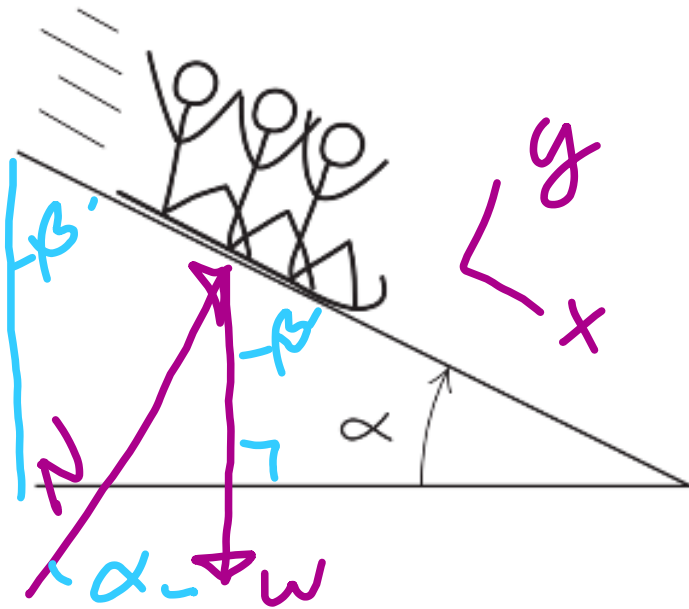
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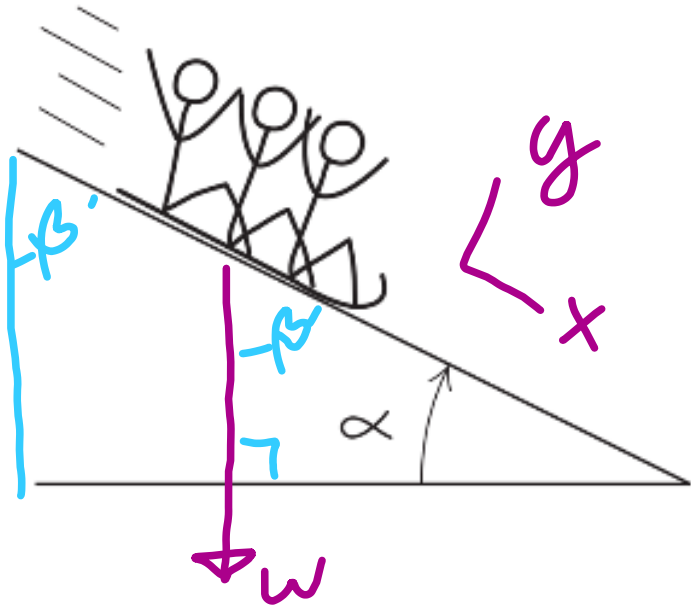
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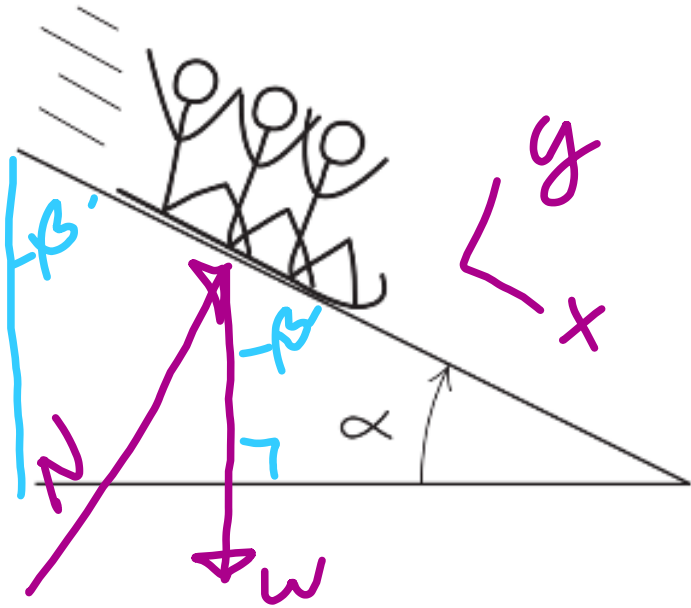
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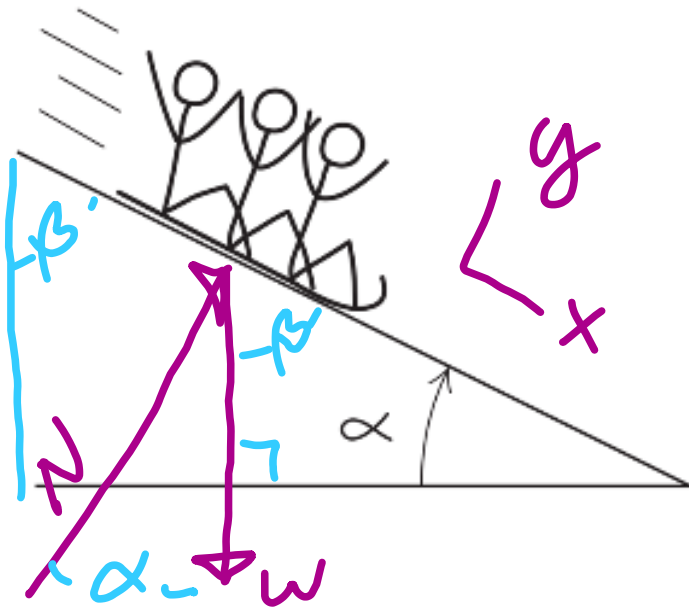
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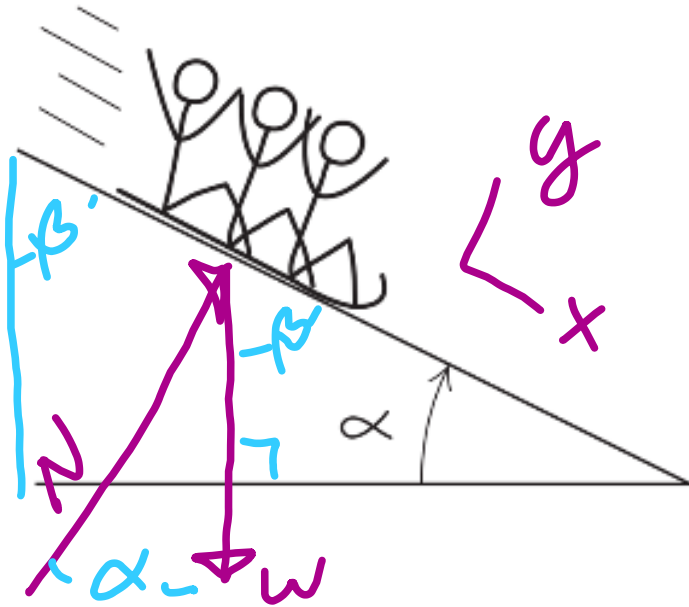


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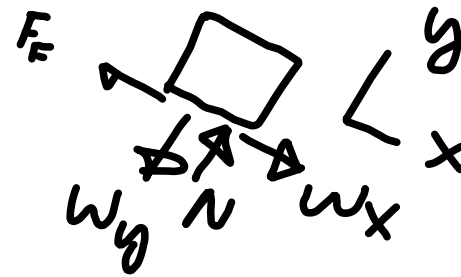
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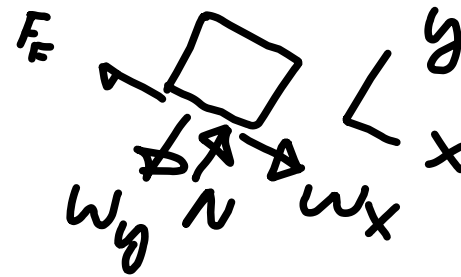
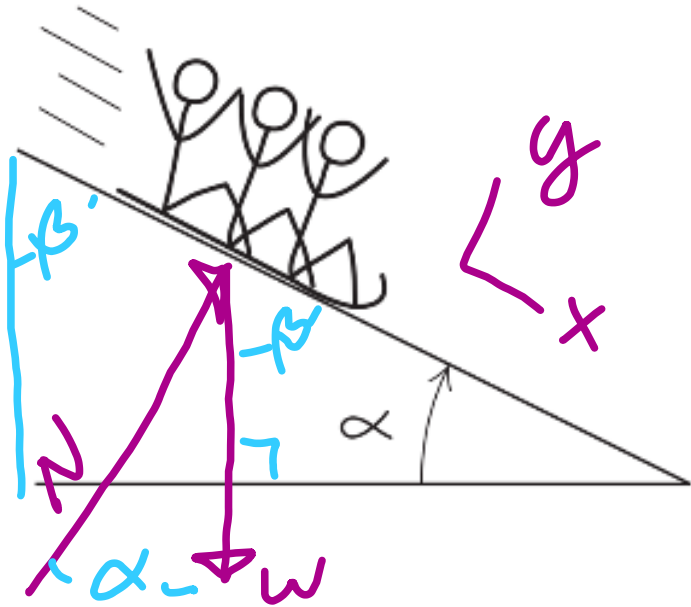
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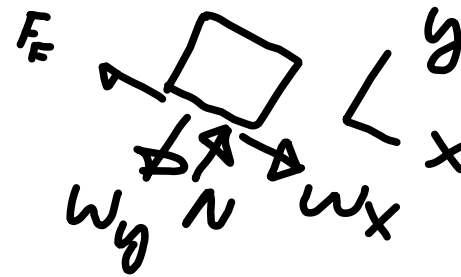
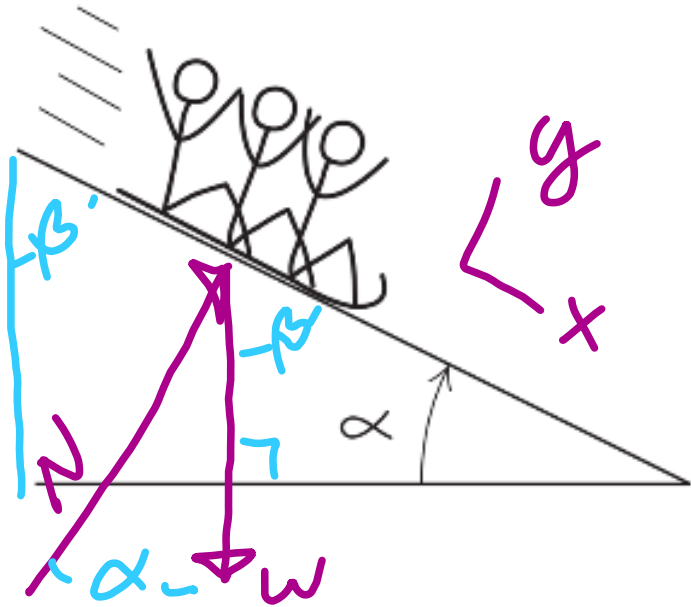
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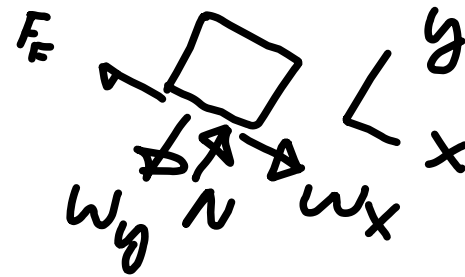
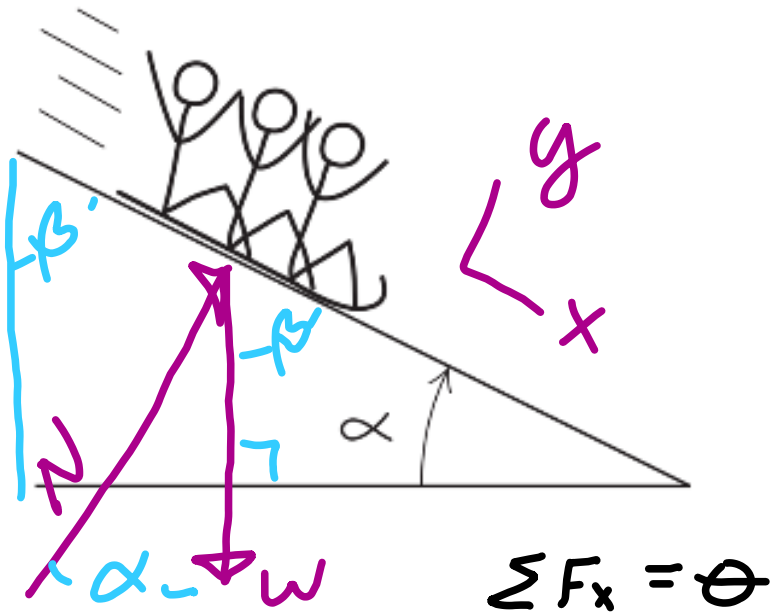
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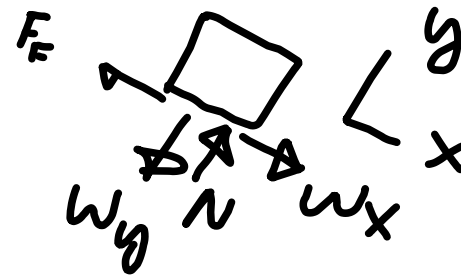
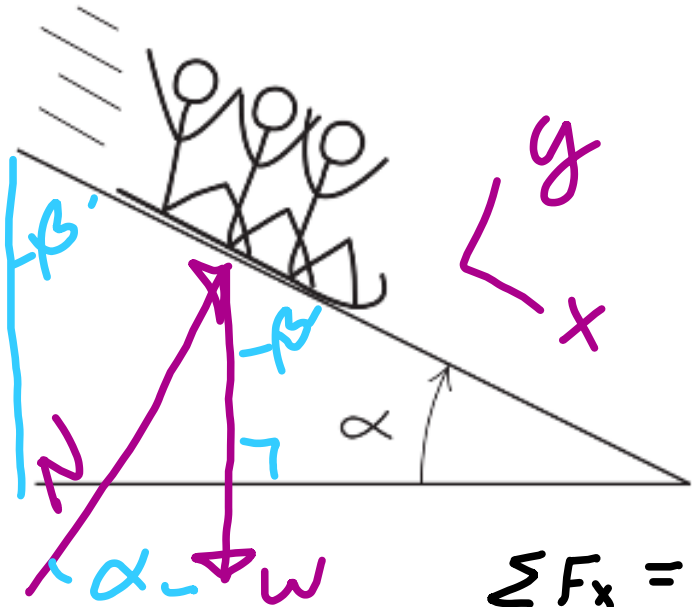
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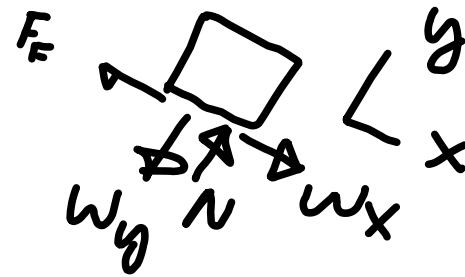
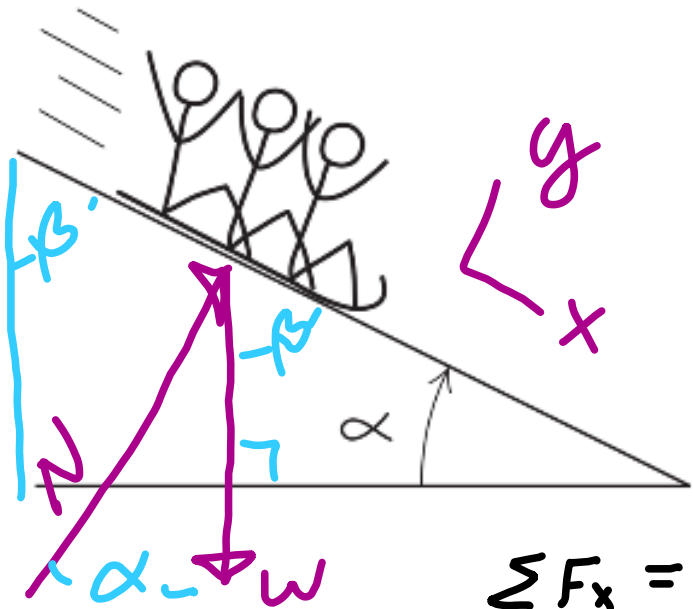


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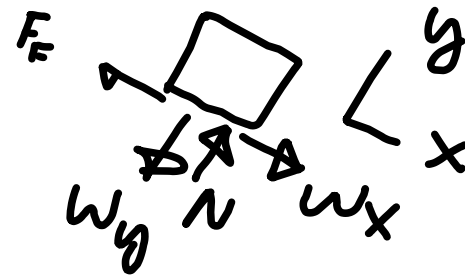
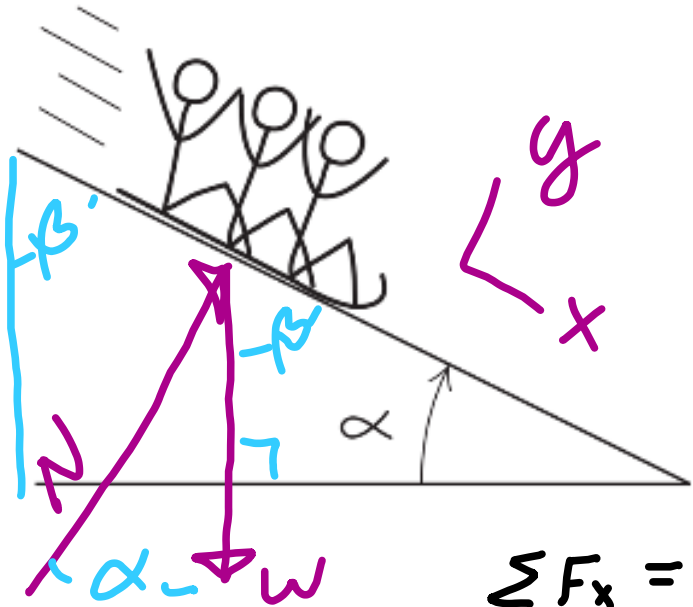
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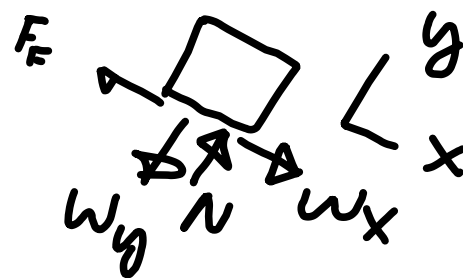
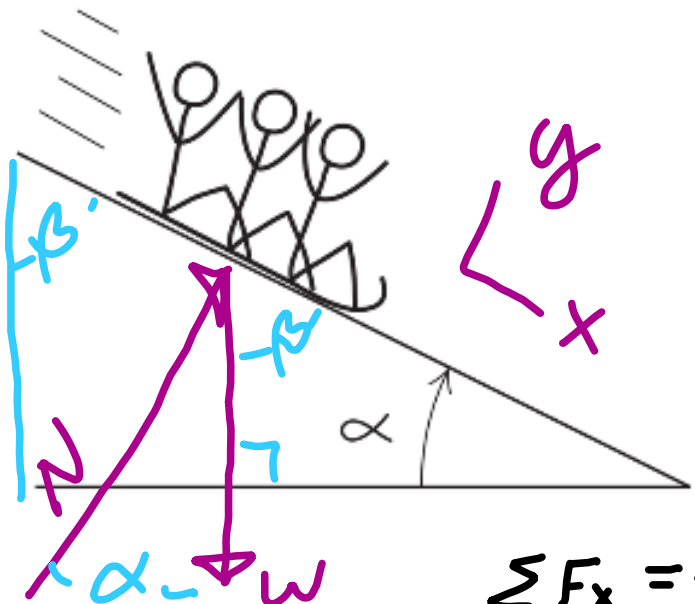
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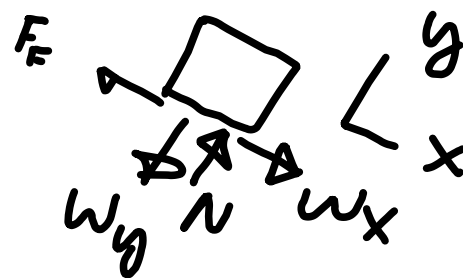
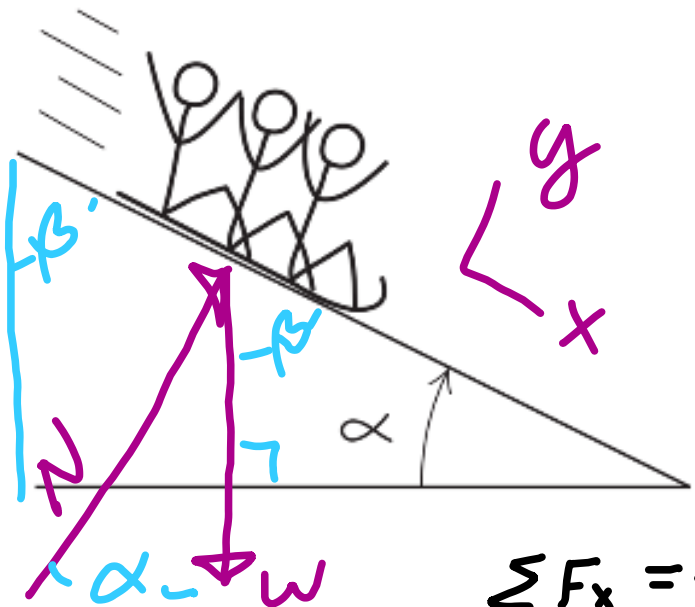
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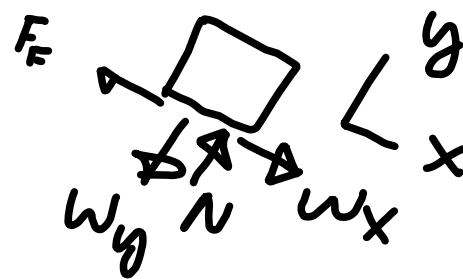
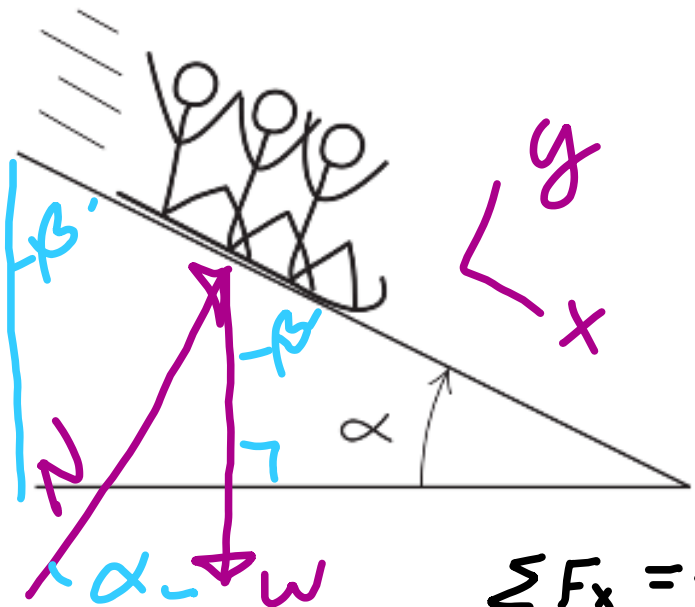
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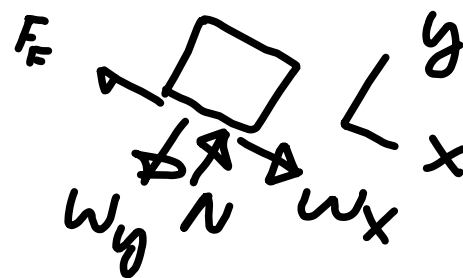
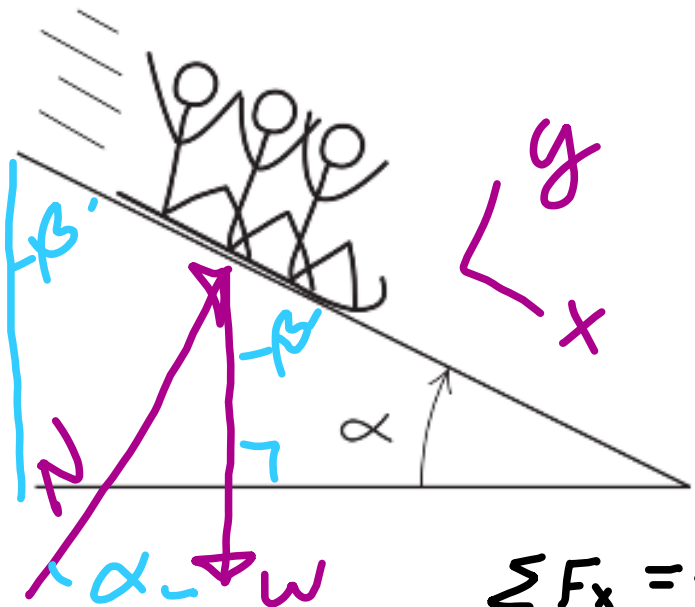
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Let's go back to the toboggan we studied in [Example 5.10](#). The wax has worn off, so there is now a nonzero coefficient of kinetic friction  $\mu_k$ . The slope has just the right angle to make the toboggan slide with constant velocity. Find this angle in terms of  $w$  and  $\mu_k$ .

$$\vec{w} = w\hat{i}\sin\alpha - w\hat{j}\cos\alpha$$



$$\sum F_y = 0 \Rightarrow N = mg \cos\alpha$$

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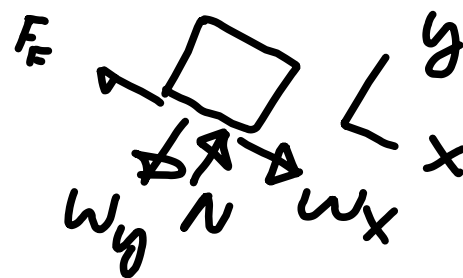
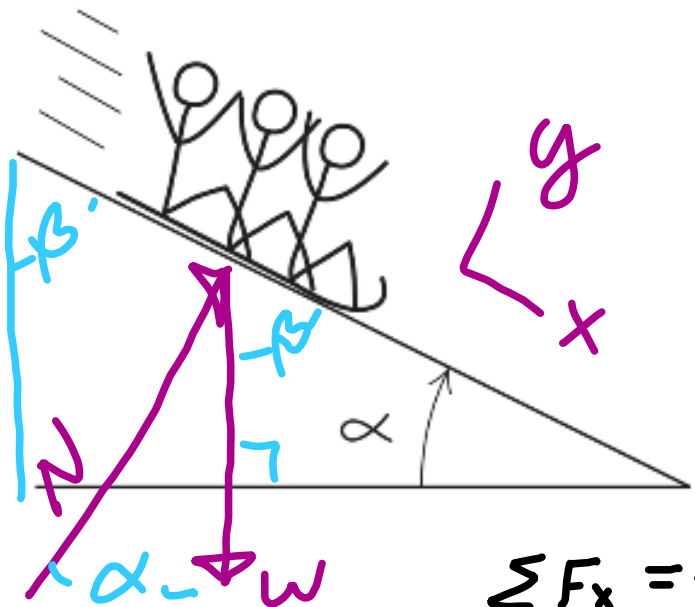
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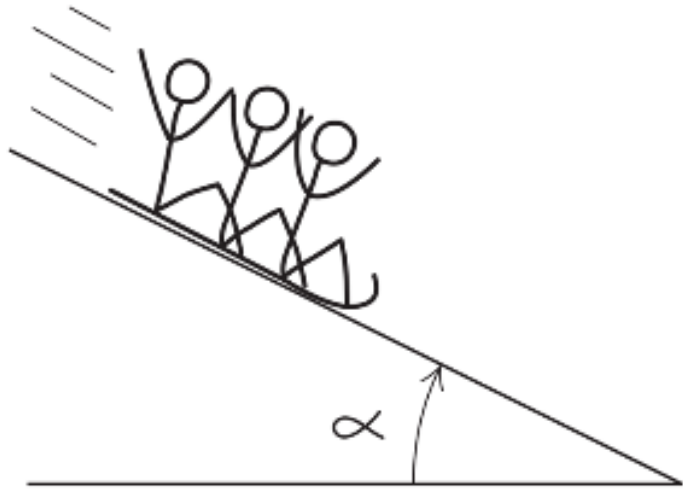
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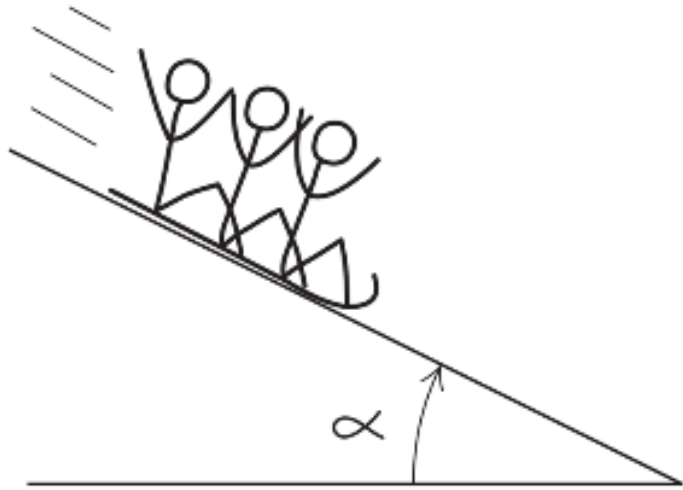
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The same toboggan with the same coefficient of friction as in **Example 5.16**  $\square$  accelerates down a steeper hill. Derive an expression for the acceleration in terms of  $g$ ,  $\alpha$ ,  $\mu_k$ , and  $w$ .

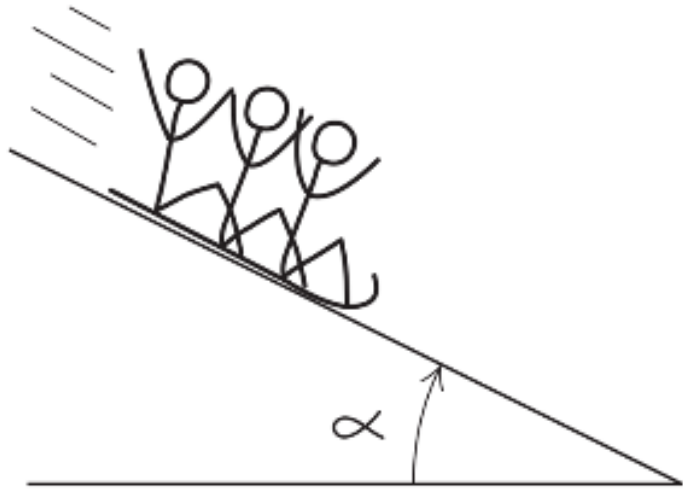


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Solution starts out the same & becomes different when summing  $F_x$

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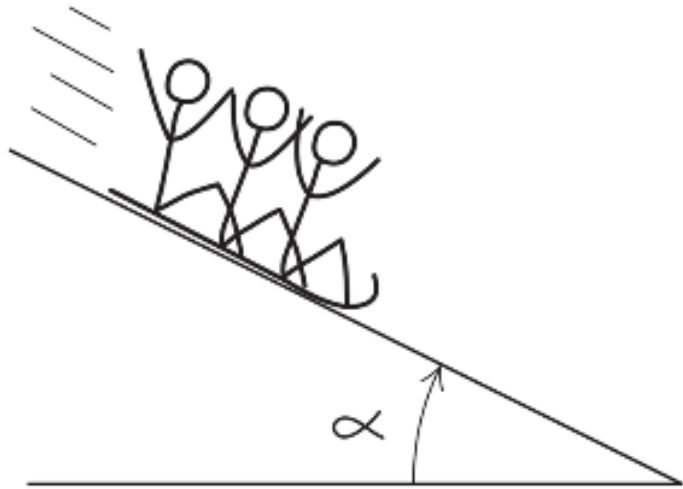


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previous problem

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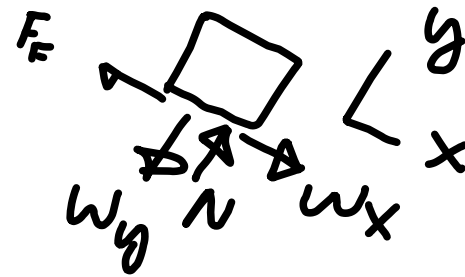
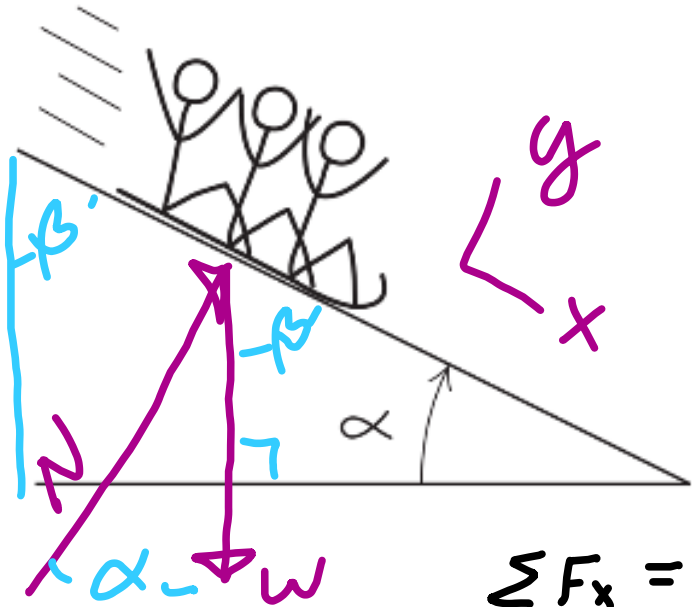
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→ now

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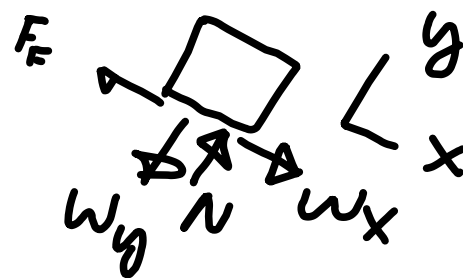
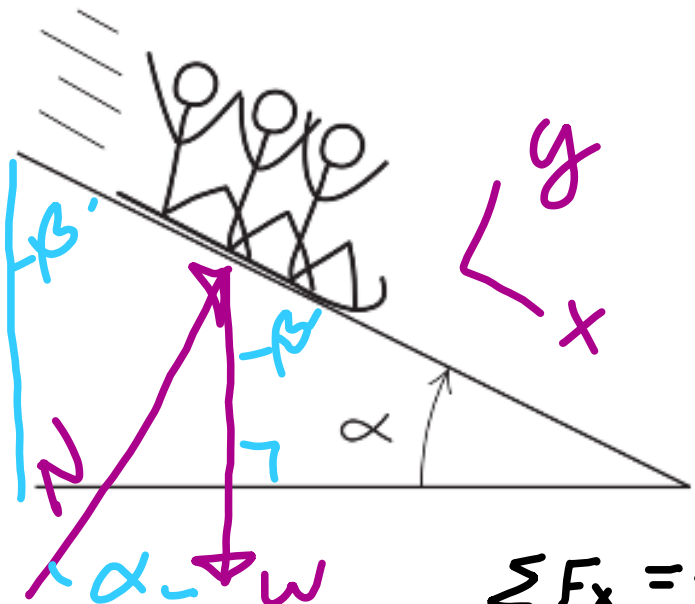


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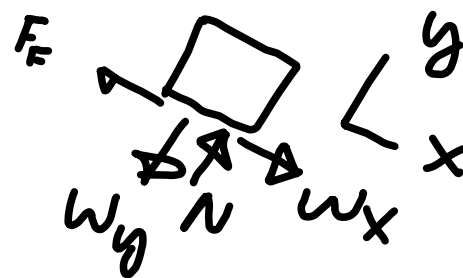
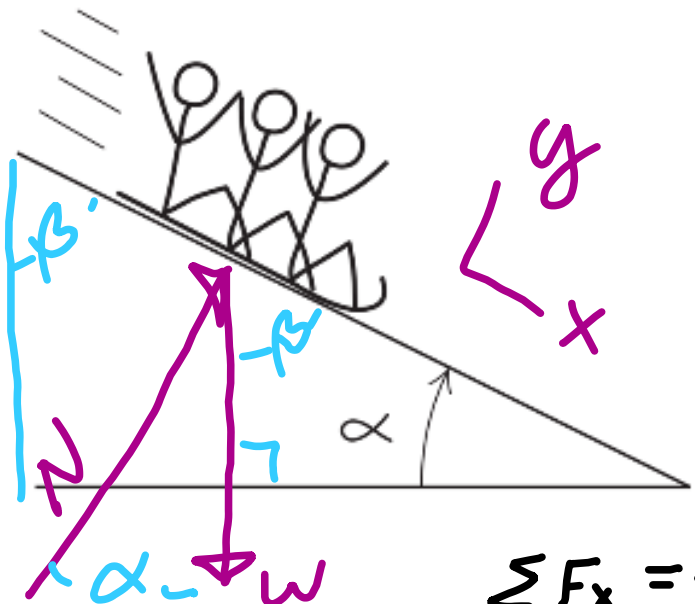
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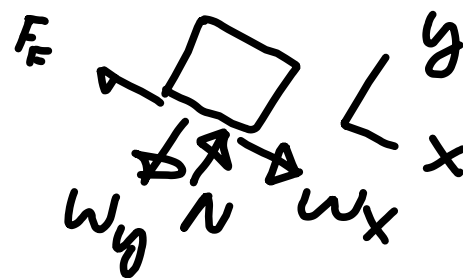
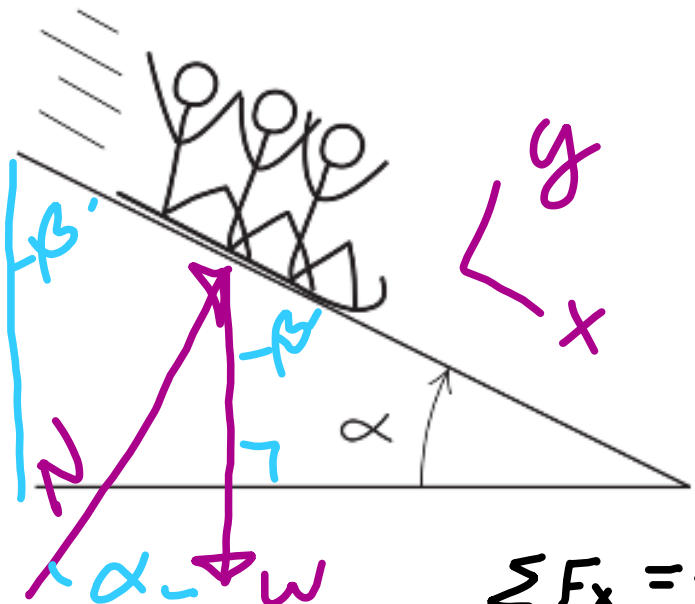
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