

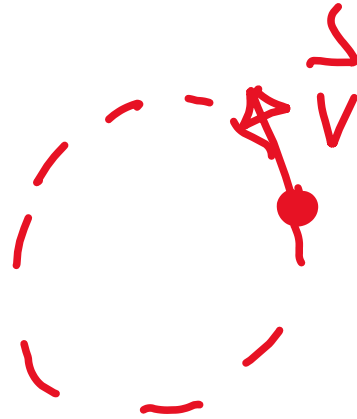
Today: Section 3.4

Today: Section 3.4

Motion in a
circle

Today: Section 3.4

Motion in a
Circle



Today: Section 3.4
HW#3 Due tonight

Today: Section 3.4

HW#3 Due tonight

Monday: Section 3.5

Today: Section 3.4

HW#3 Due tonight

Monday: Section 3.5

Relative
Motion

Today: Section 3.4

HW#3 Due tonight

Monday: Section 3.5

HW#4 Due Wednesday

Today: Section 3.4

HW#3 Due tonight

Monday: Section 3.5

HW#4 Due Wednesday

← Date of Exam #1

Uniform Circular Motion

Uniform Circular Motion

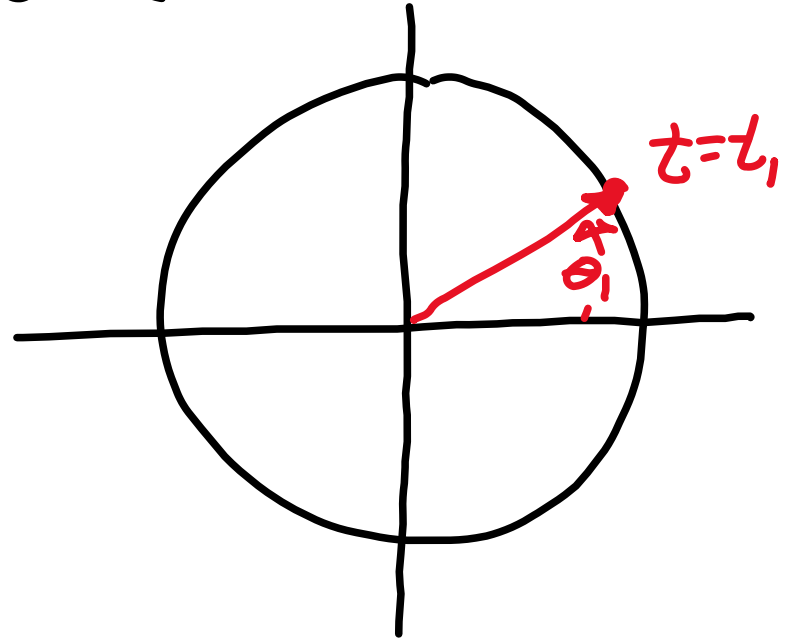
1st We will look at a
mathematic description
not given by the book

Uniform Circular Motion

1st We will look at a
mathematic description
not given by the book
& then revert back
to the treatment given
in the text

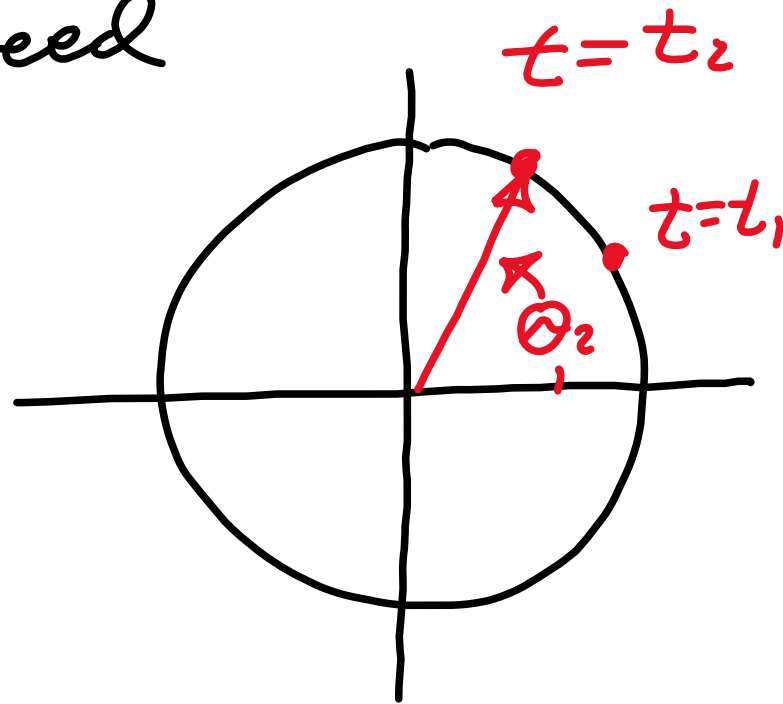
Uniform Circular Motion

Uniform \Rightarrow constant speed



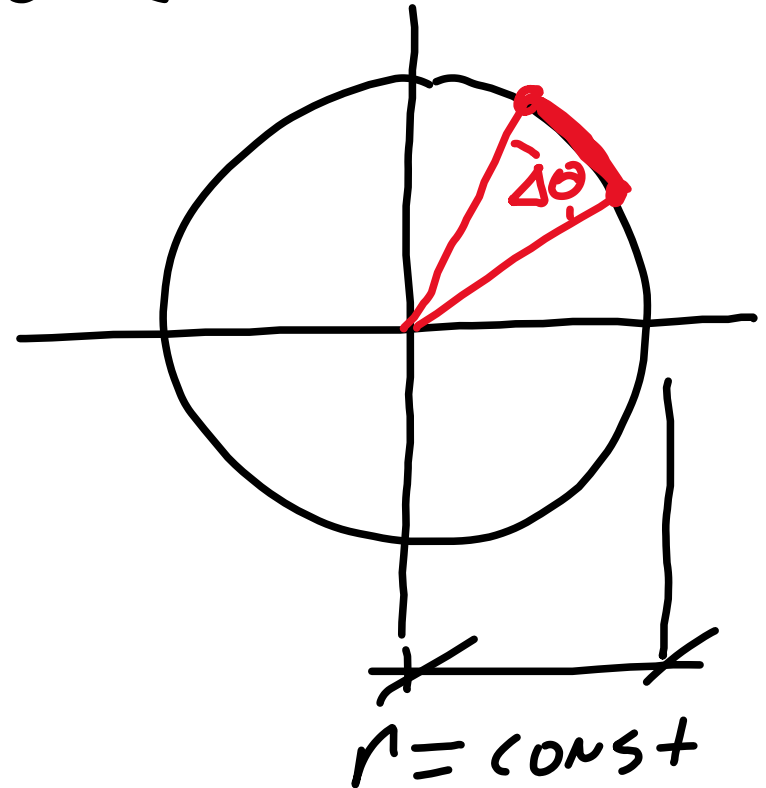
Uniform Circular Motion

Uniform \Rightarrow constant speed



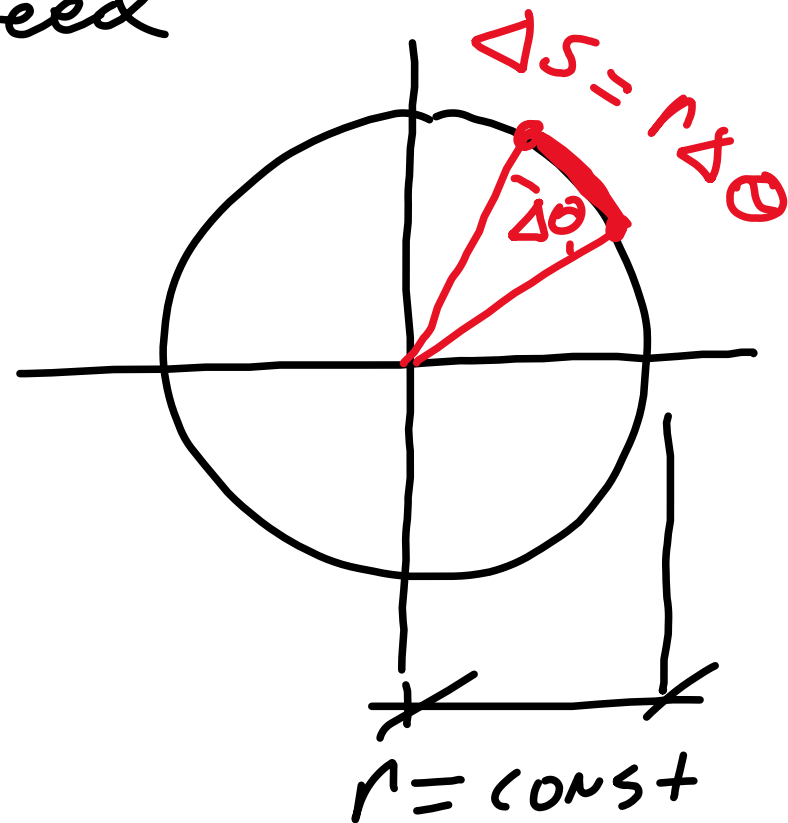
Uniform Circular Motion

Uniform \Rightarrow constant speed



Uniform Circular Motion

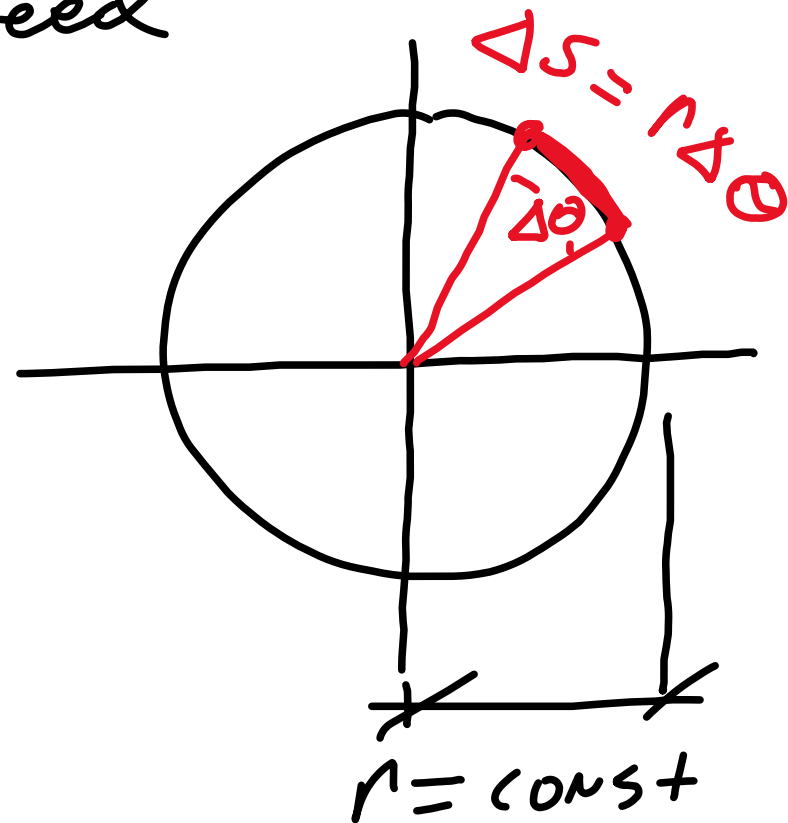
Uniform \Rightarrow constant speed



Uniform Circular Motion

Uniform \Rightarrow constant speed

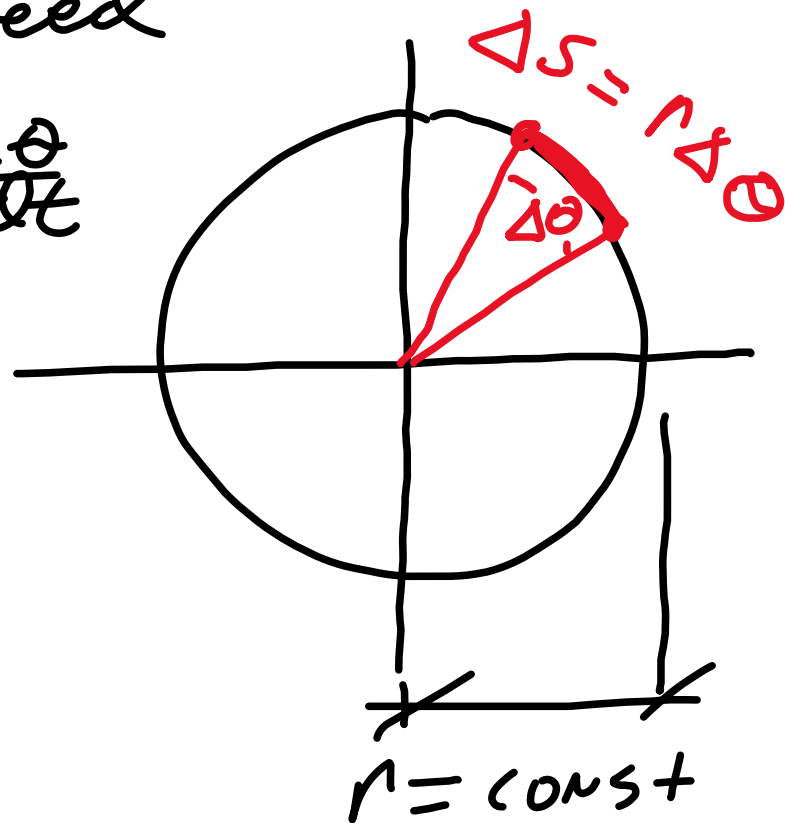
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t}$$



Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

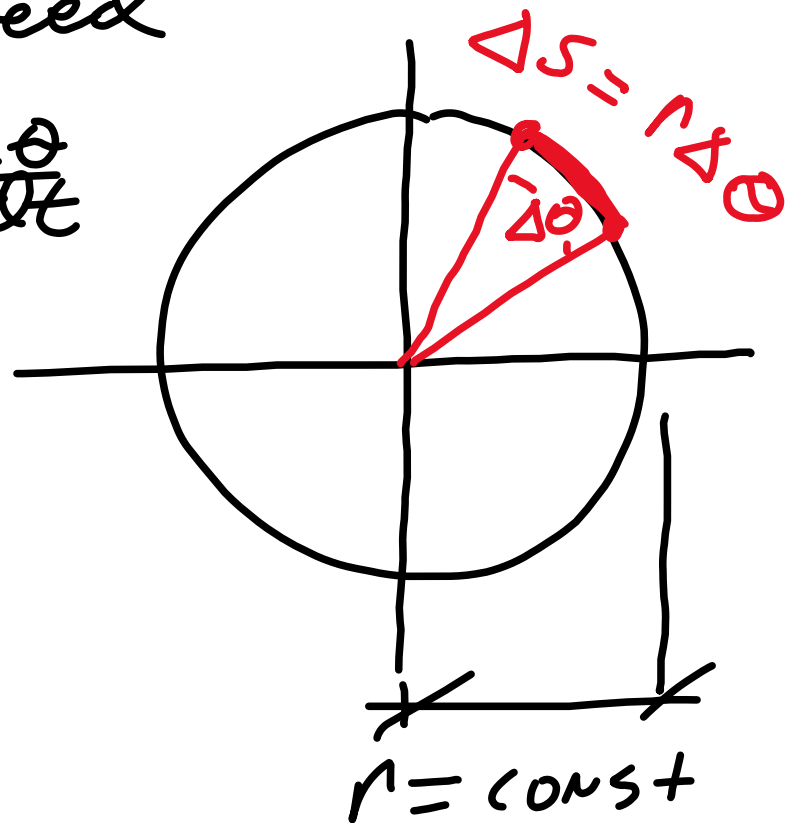


Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt}$$

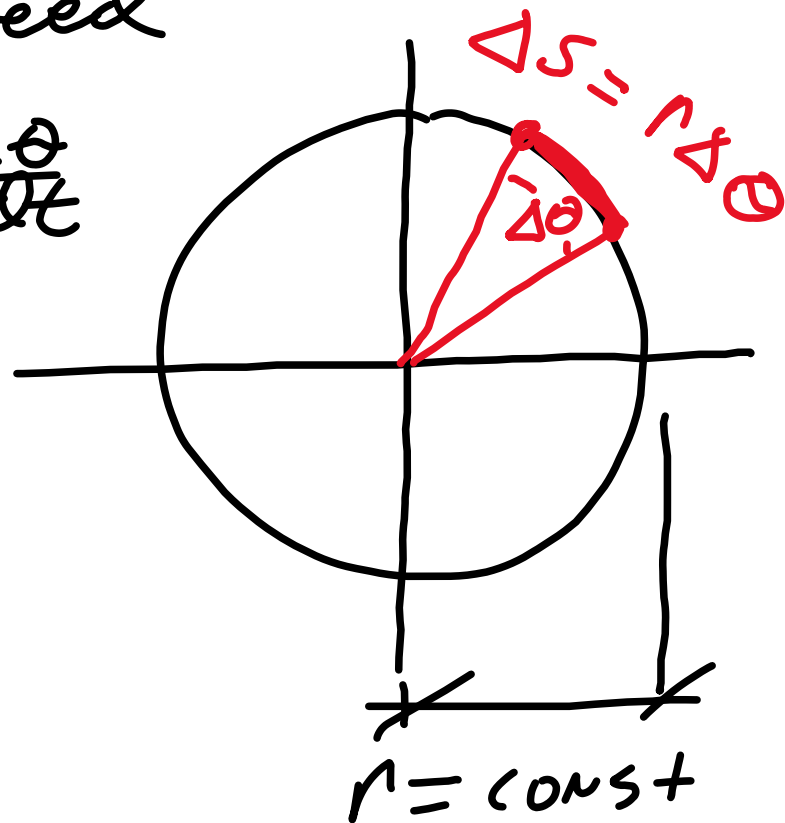


Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$



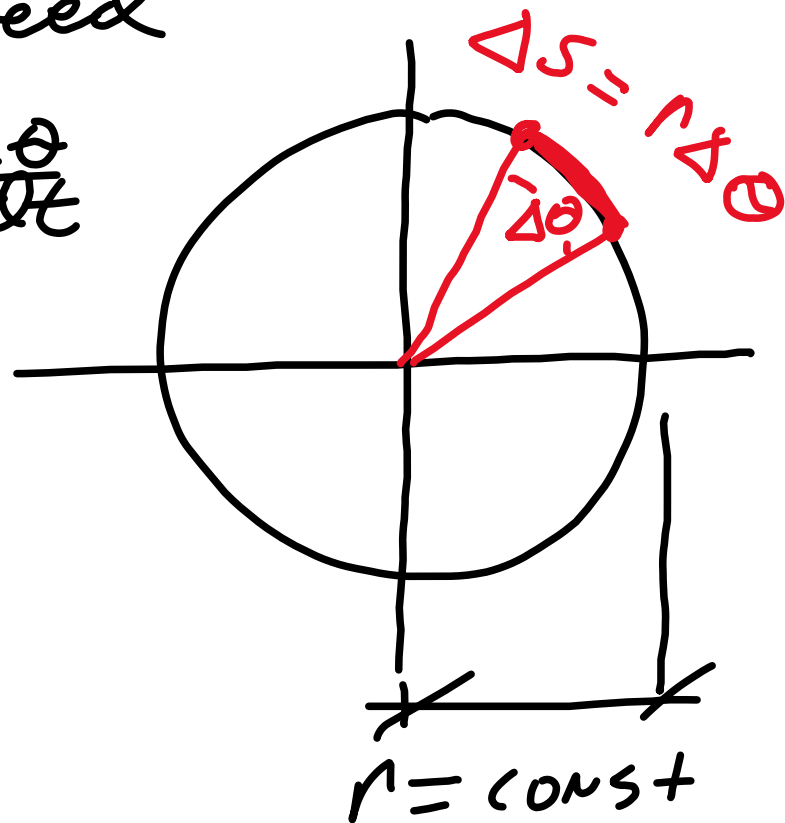
Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

Scalar



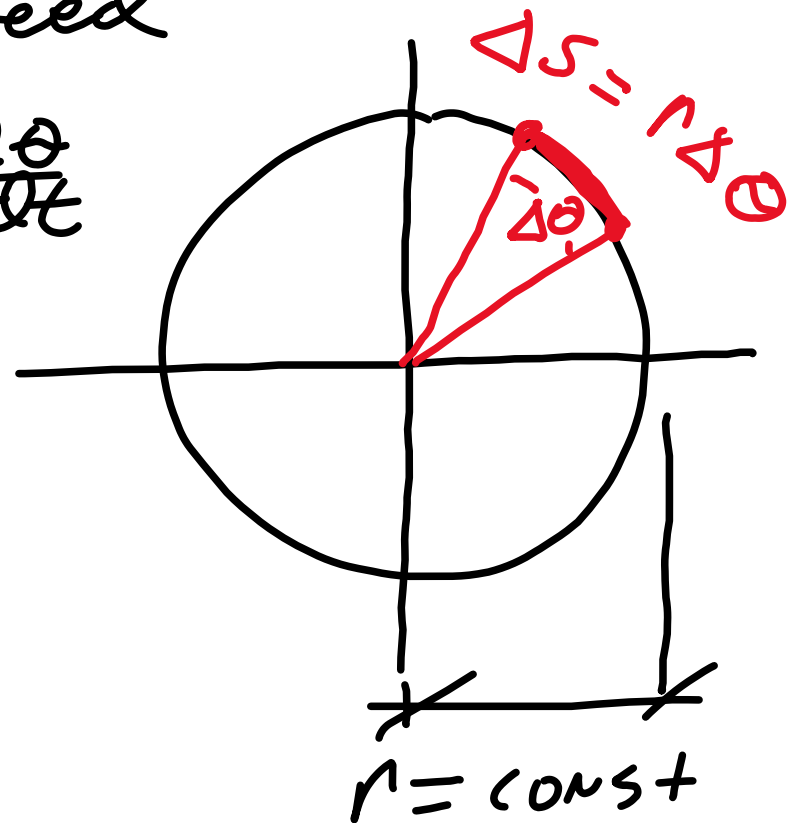
Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?



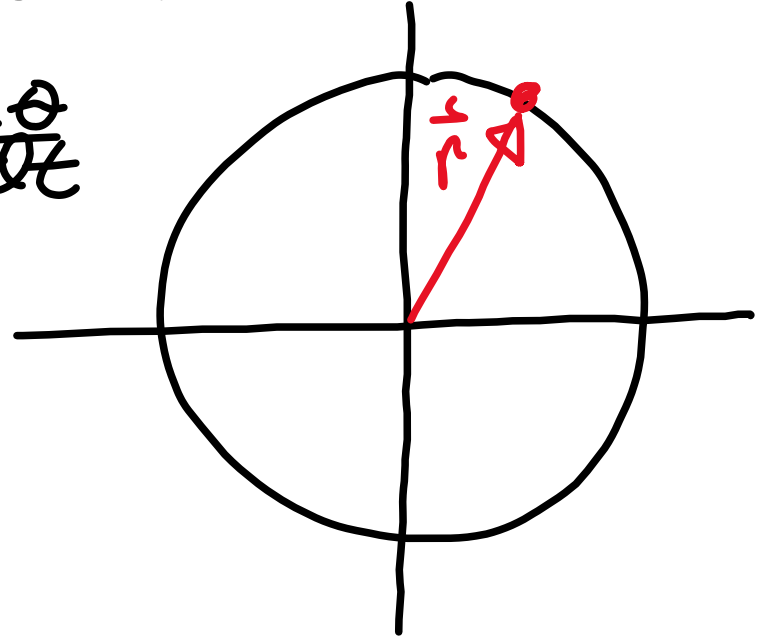
Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?



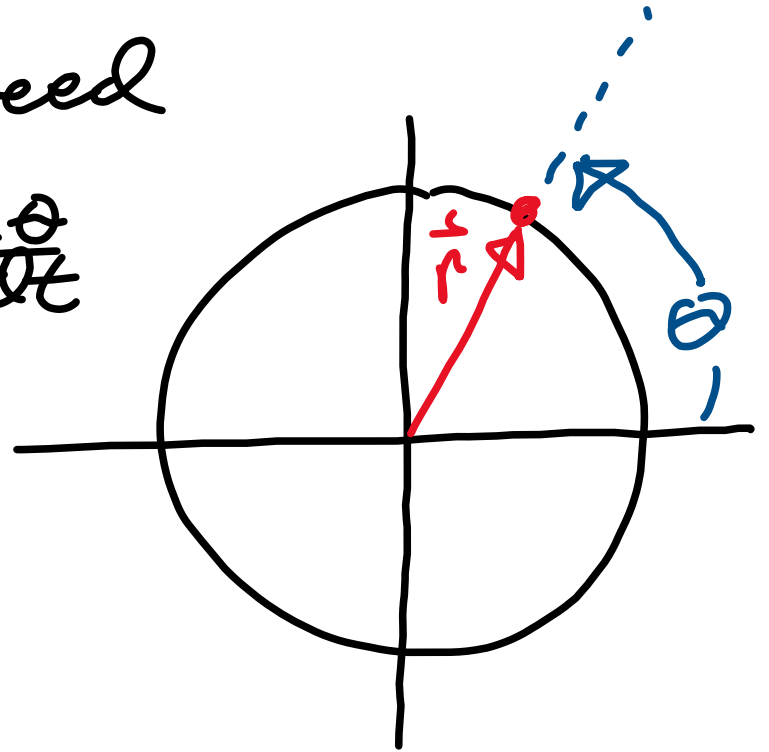
Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?



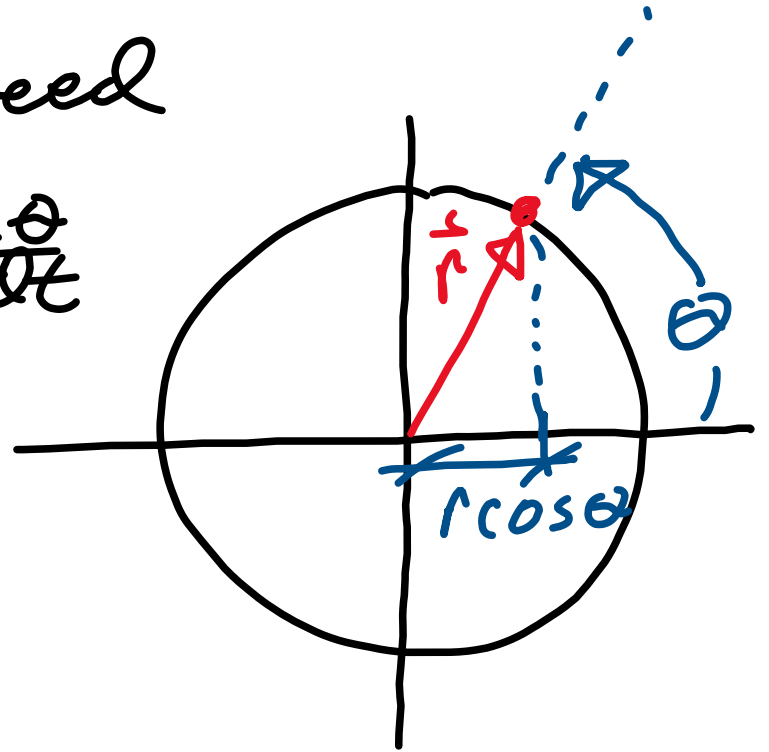
Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?



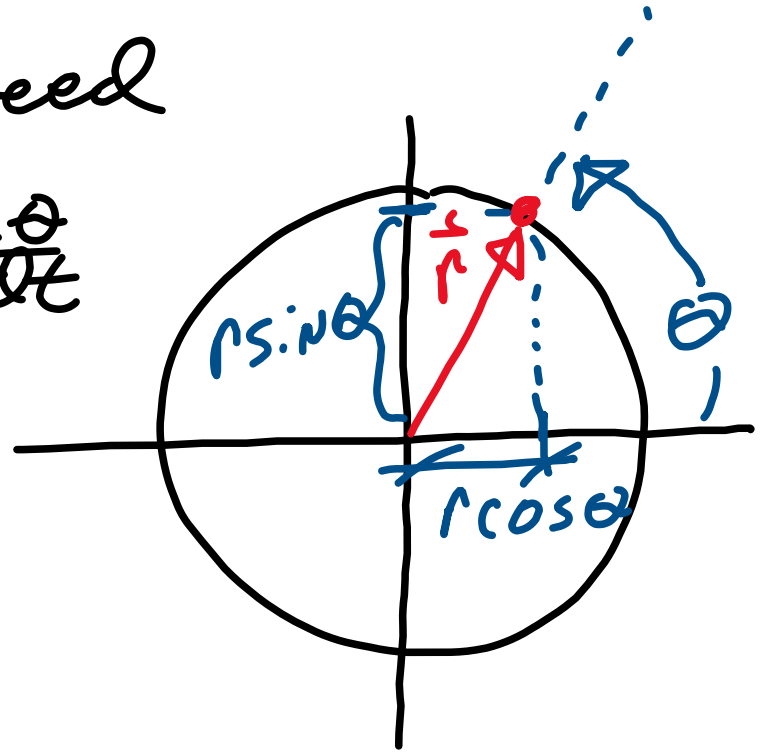
Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?



Uniform Circular Motion

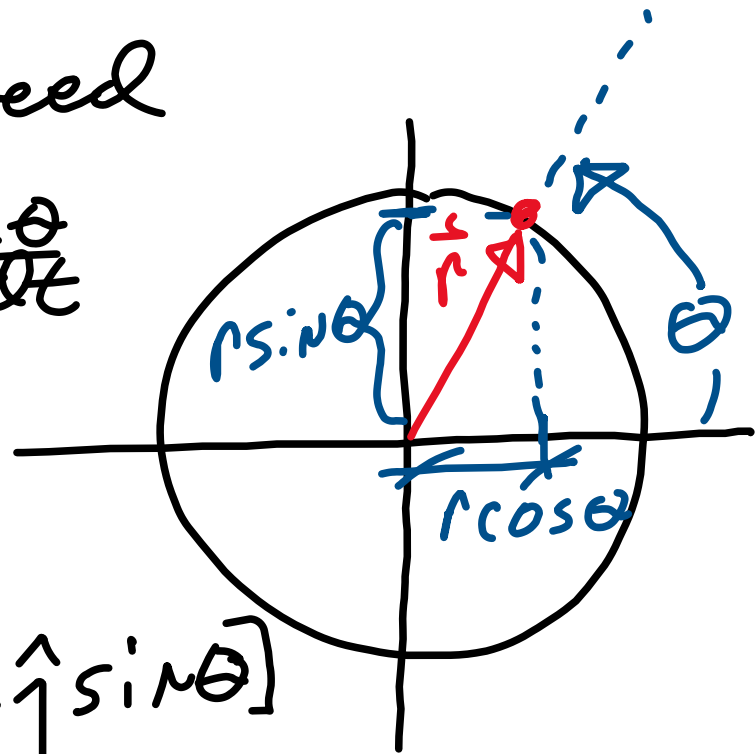
Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\vec{v} = \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta]$$



Uniform Circular Motion

Uniform \Rightarrow constant speed

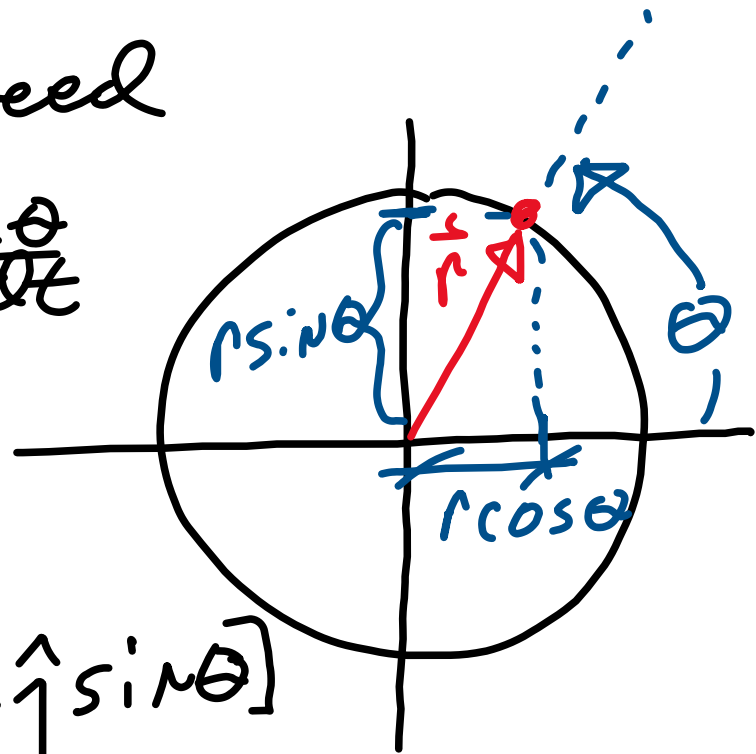
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\vec{v} = \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta]$$

$$\text{But } \frac{d \cos \theta}{dt} = [-\sin \theta] \frac{d\theta}{dt}$$



Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

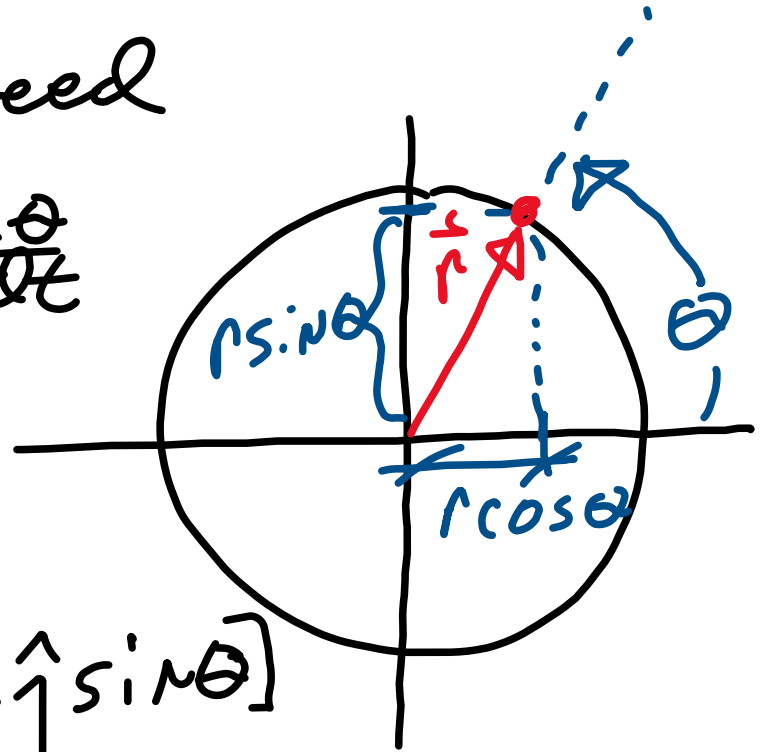
$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\vec{v} = \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta]$$

$$\text{But } \frac{d}{dt} \cos \theta = [-\sin \theta] \frac{d\theta}{dt} \quad \&$$

$$\frac{d}{dt} \sin \theta = [\cos \theta] \frac{d\theta}{dt}$$



Uniform Circular Motion

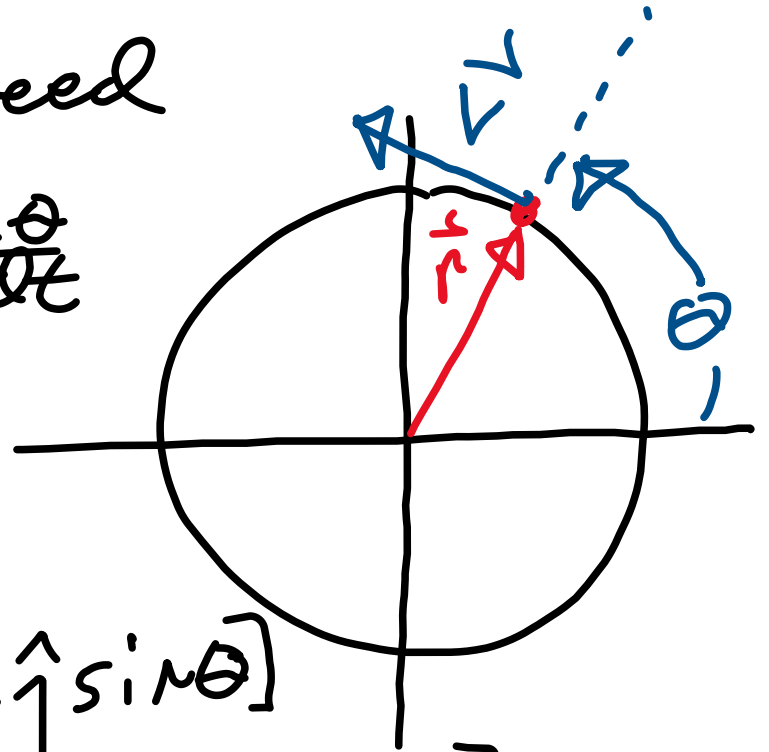
Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$



Uniform Circular Motion

Uniform \Rightarrow constant speed

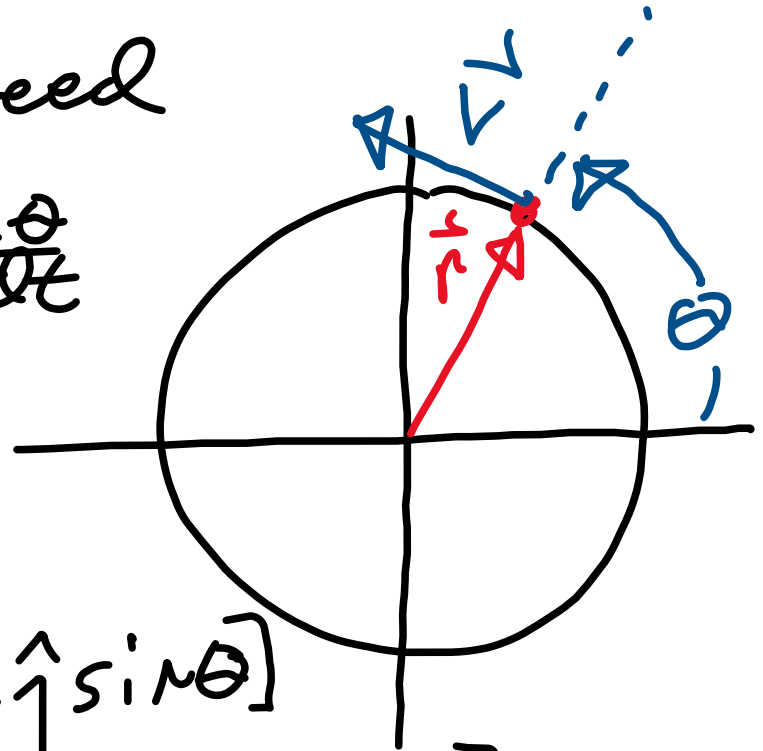
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

Note: $\cos(\theta + 90^\circ) =$



Uniform Circular Motion

Uniform \Rightarrow constant speed

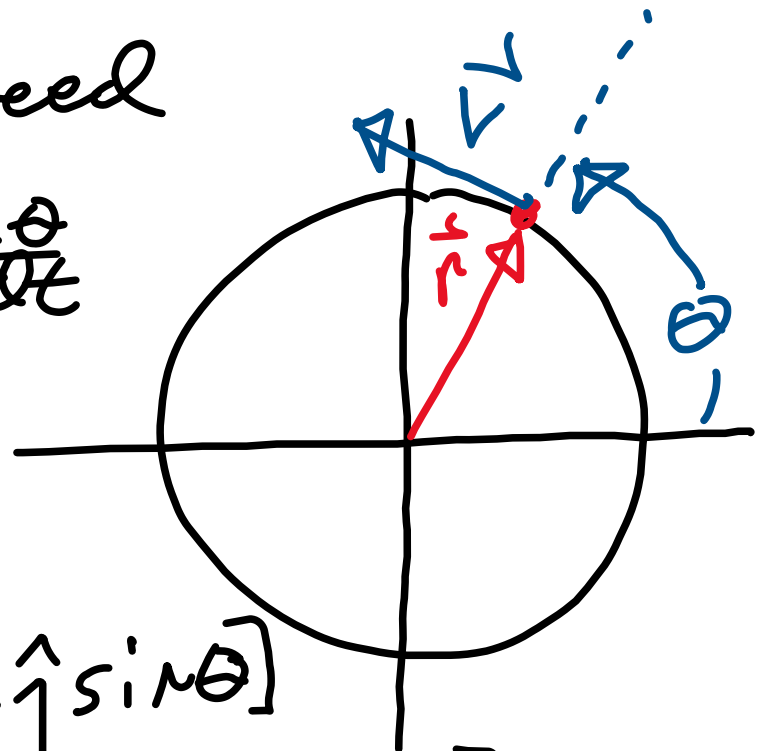
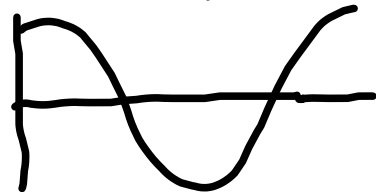
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

Note: $\cos(\theta + 90^\circ) =$



Uniform Circular Motion

Uniform \Rightarrow constant speed

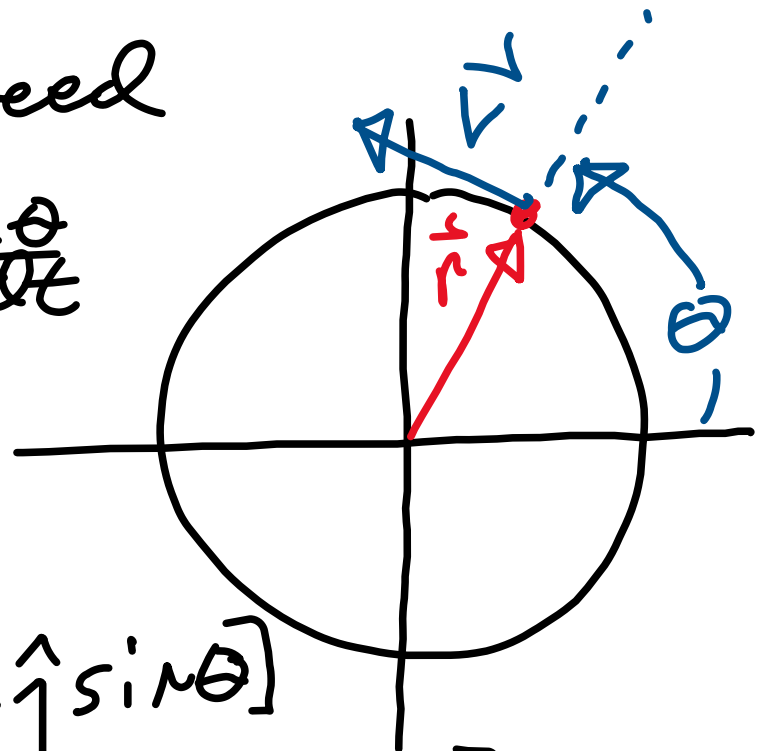
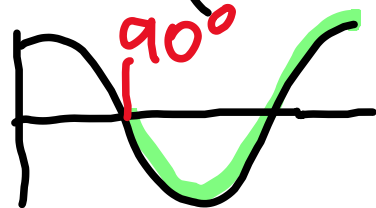
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

Note: $\cos(\theta + 90^\circ) =$



Uniform Circular Motion

Uniform \Rightarrow constant speed

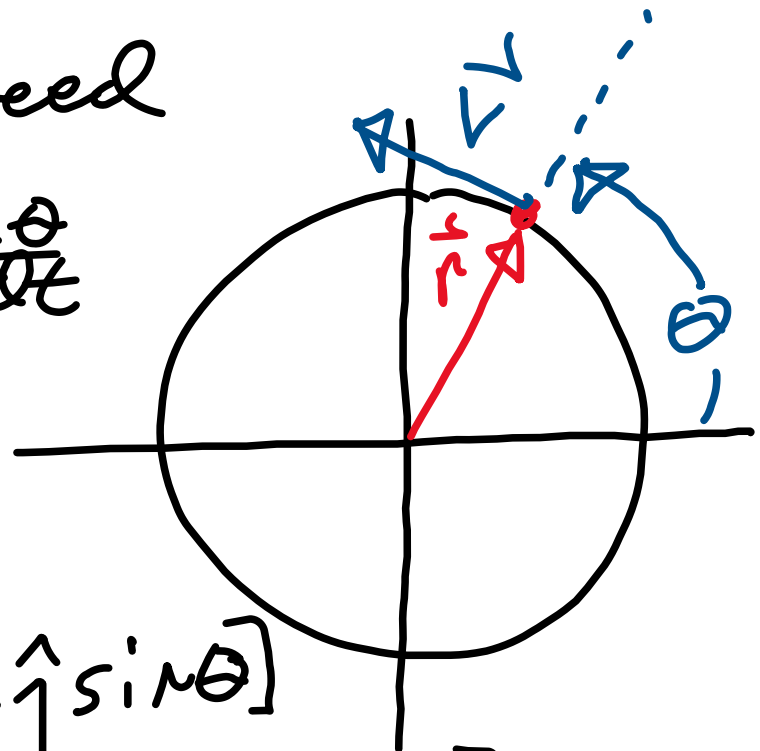
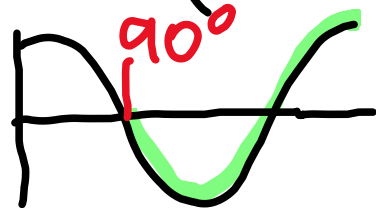
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= (r \frac{d\theta}{dt}) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

Note: $\cos(\theta + 90^\circ) = -\sin(\theta)$



Uniform Circular Motion

Uniform \Rightarrow constant speed

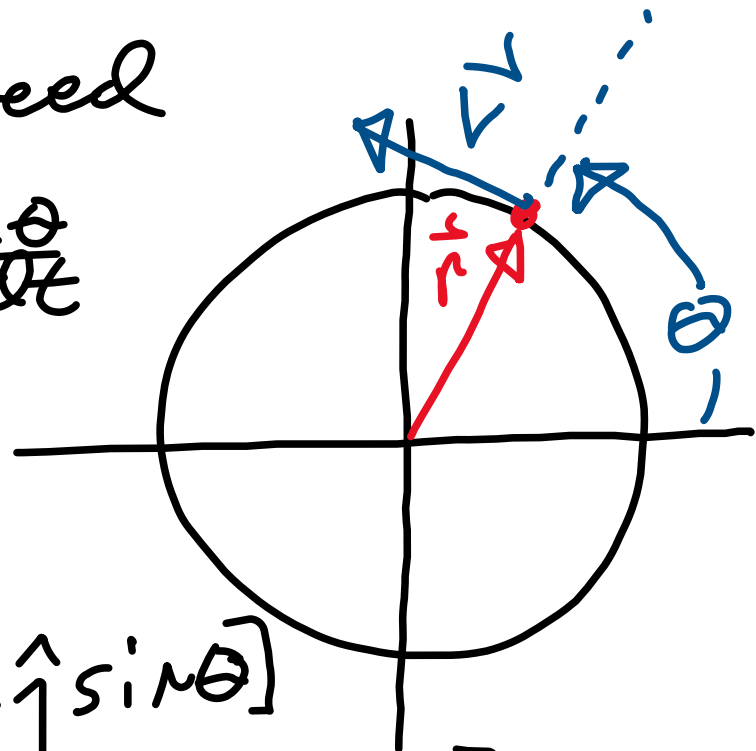
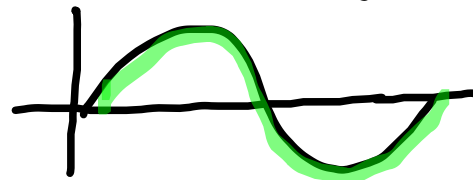
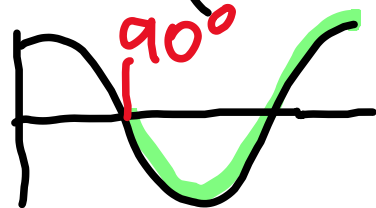
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

Note: $\cos(\theta + 90^\circ) = -\sin(\theta)$



Uniform Circular Motion

Uniform \Rightarrow constant speed

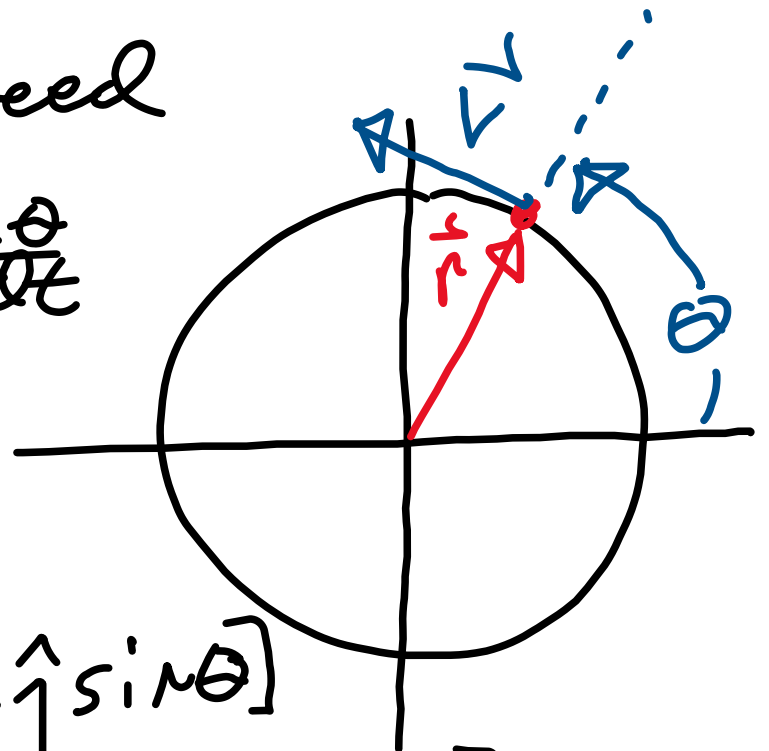
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

Note: $\sin(\theta + 90^\circ) =$



Uniform Circular Motion

Uniform \Rightarrow constant speed

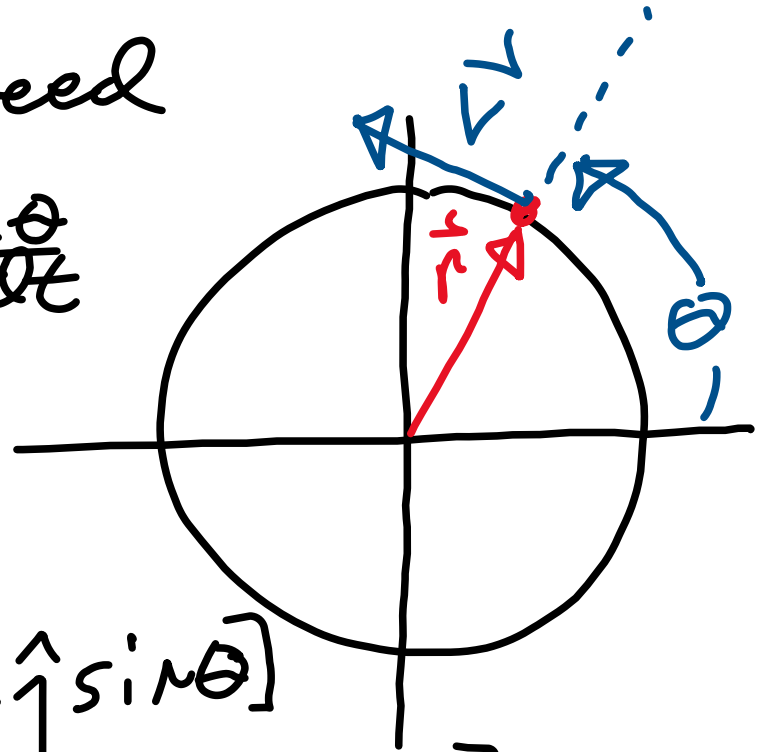
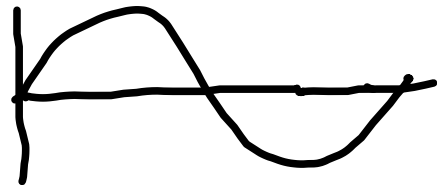
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

Note: $\sin(\theta + 90^\circ) =$



Uniform Circular Motion

Uniform \Rightarrow constant speed

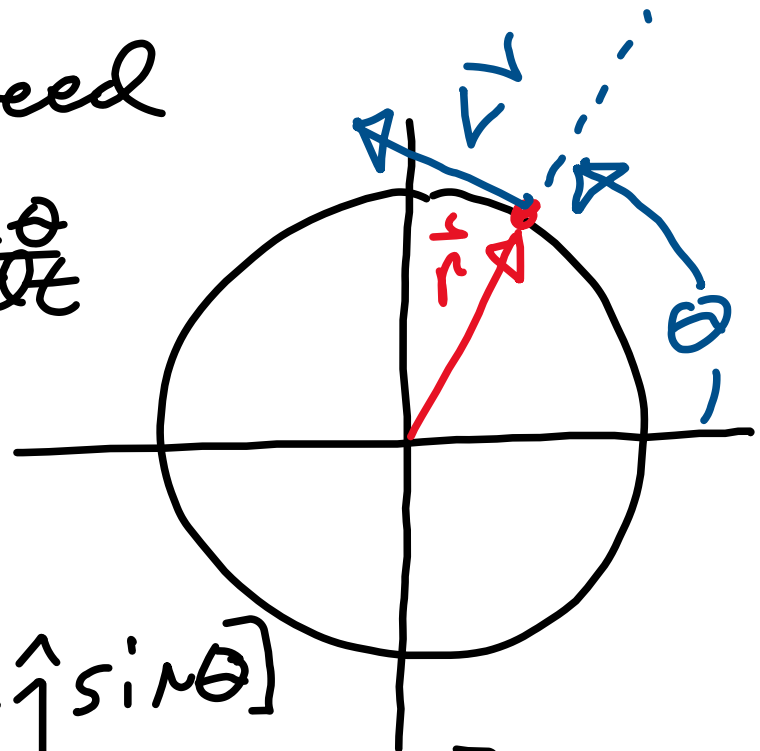
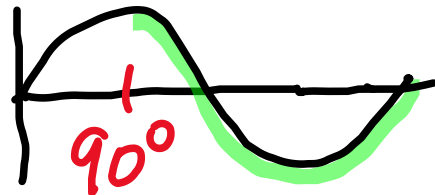
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

Note: $\sin(\theta + 90^\circ) =$



Uniform Circular Motion

Uniform \Rightarrow constant speed

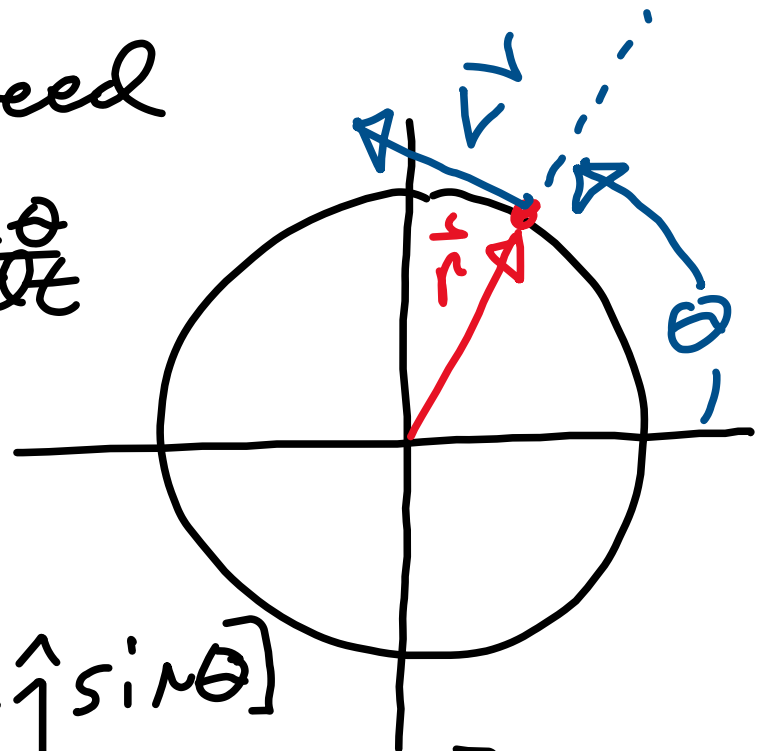
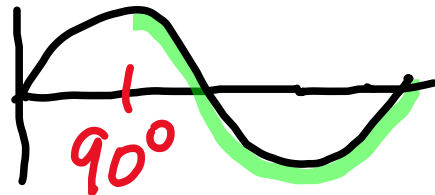
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= (r \frac{d\theta}{dt}) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

Note: $\sin(\theta + 90^\circ) = \cos(\theta)$



Uniform Circular Motion

Uniform \Rightarrow constant speed

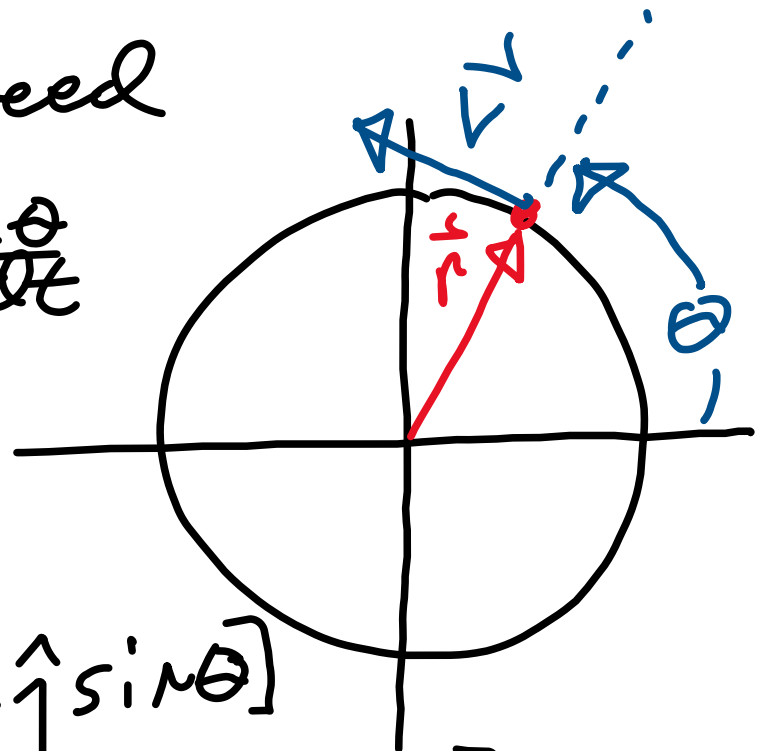
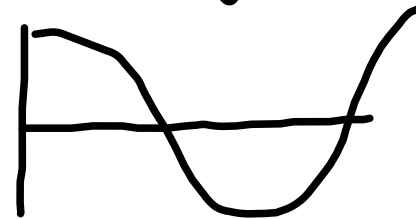
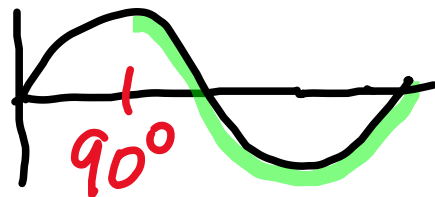
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= (r \frac{d\theta}{dt}) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

Note: $\sin(\theta + 90^\circ) = \cos(\theta)$



Uniform Circular Motion

Uniform \Rightarrow constant speed

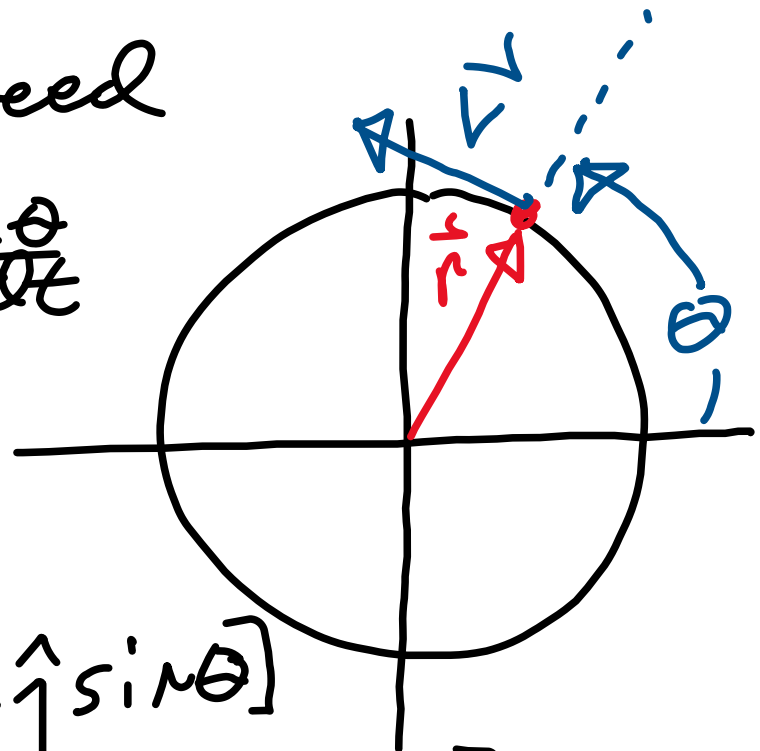
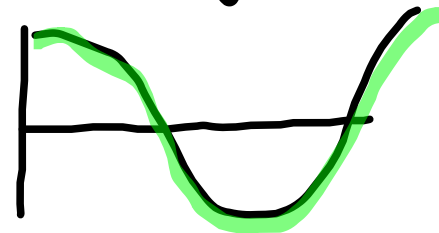
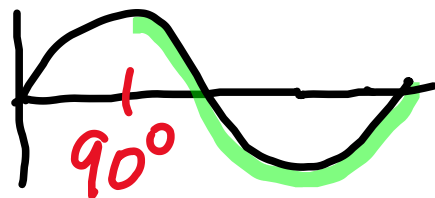
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

Note: $\sin(\theta + 90^\circ) = \cos(\theta)$



Uniform Circular Motion

Uniform \Rightarrow constant speed

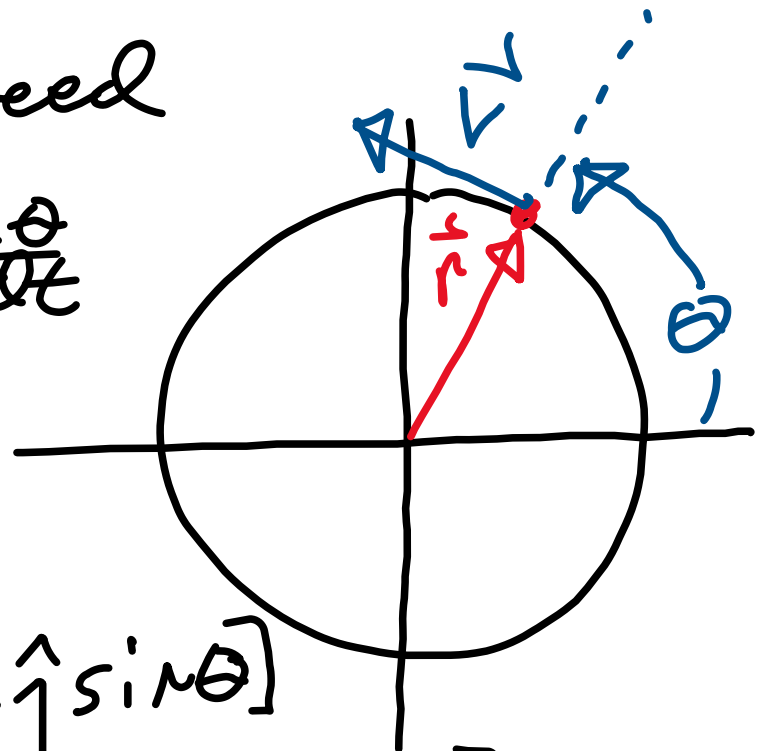
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

So \vec{v} is rotated 90°
relative to \vec{r}



Uniform Circular Motion

Uniform \Rightarrow constant speed

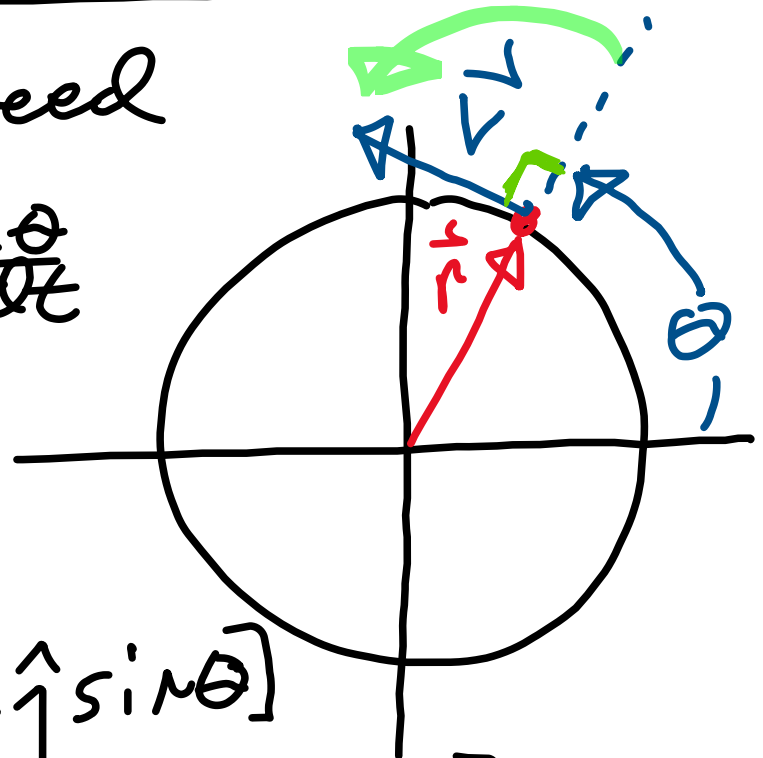
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

So \vec{v} is rotated 90°
relative to \vec{r}



Uniform Circular Motion

Uniform \Rightarrow constant speed

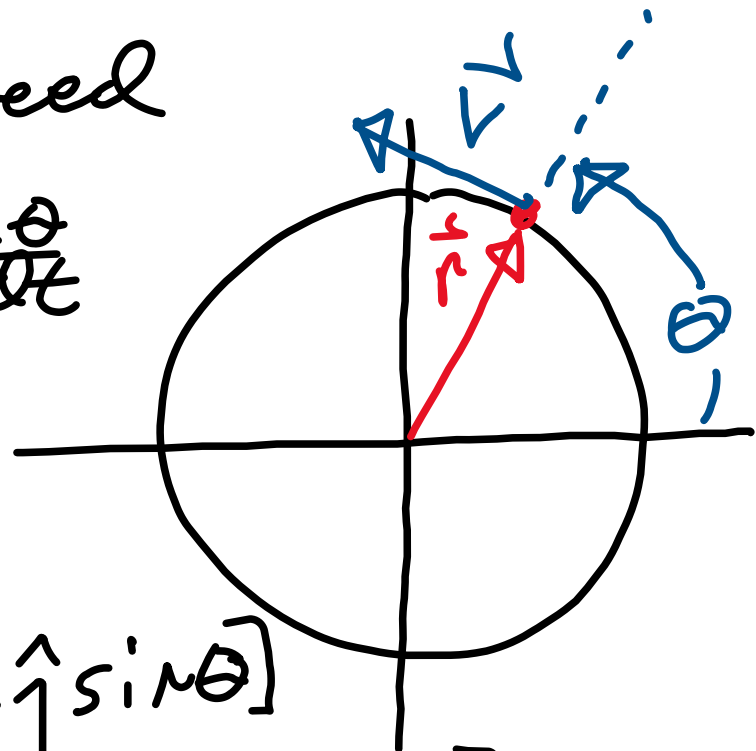
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

What about \vec{a} ?



Uniform Circular Motion

Uniform \Rightarrow constant speed

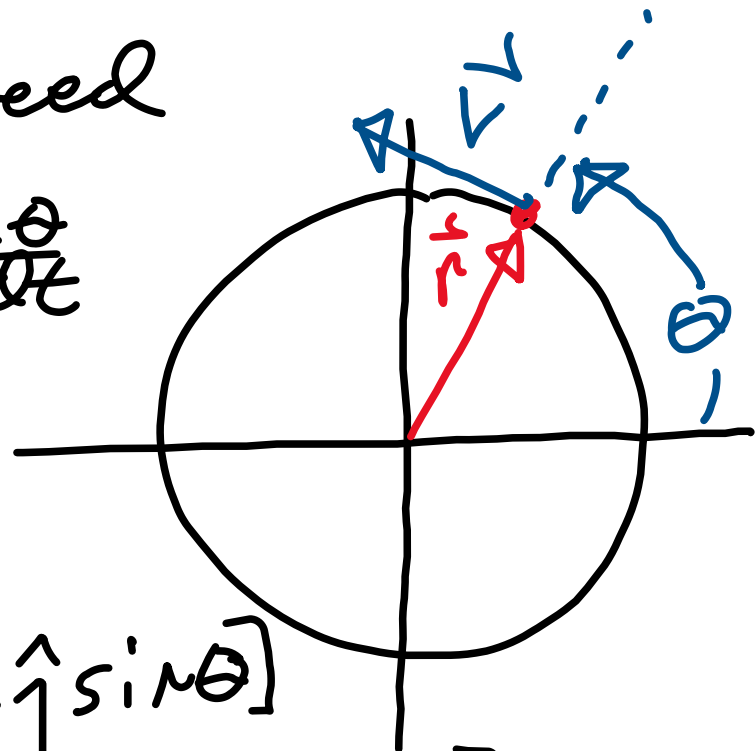
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$

What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \left[r \left(\frac{d\theta}{dt} \right)^2 \right] [-\hat{i} \cos \theta - \hat{j} \sin \theta]$$

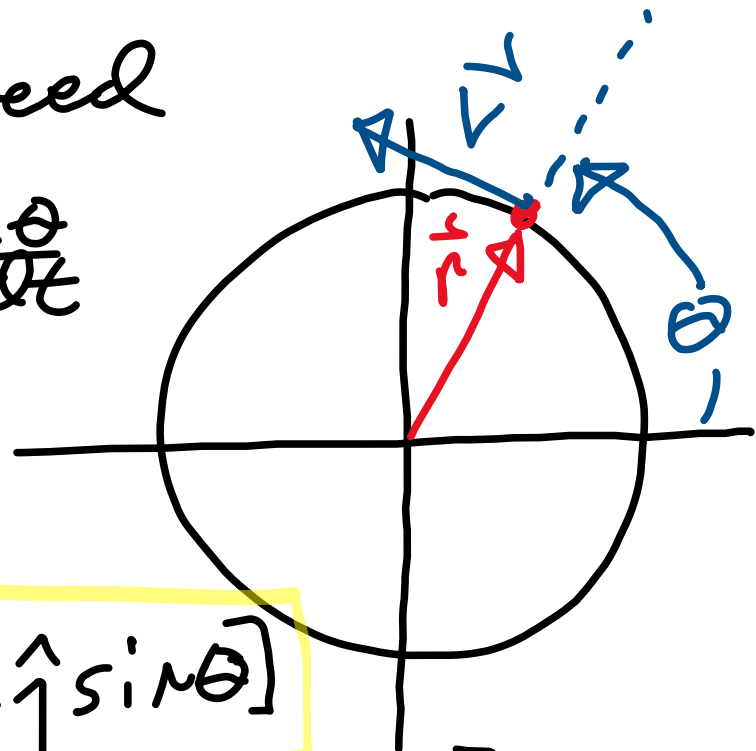


Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$



What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

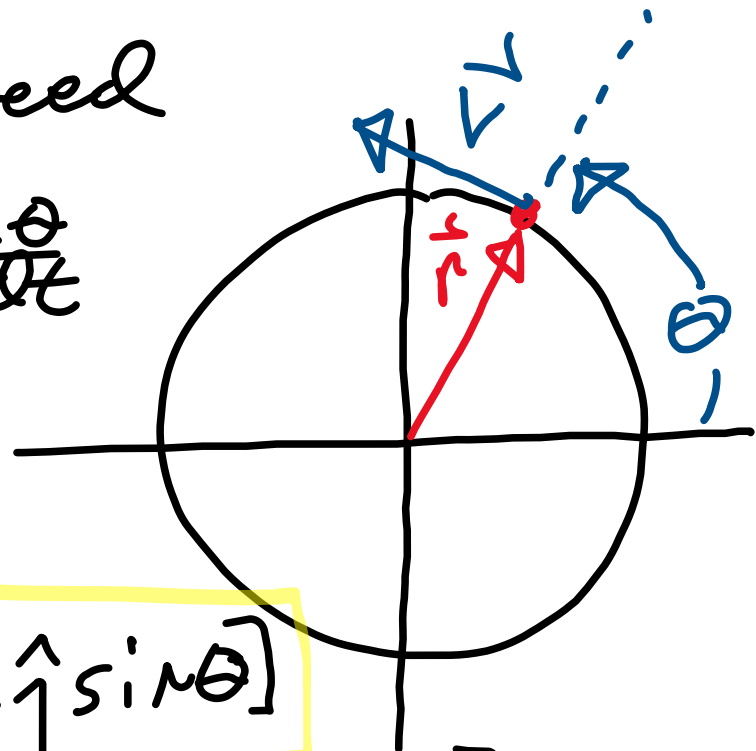
$$\vec{a} = \frac{d\vec{v}}{dt} = \left[r \left(\frac{d\theta}{dt} \right)^2 \right] [-\hat{i} \cos \theta - \hat{j} \sin \theta]$$

Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$



What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

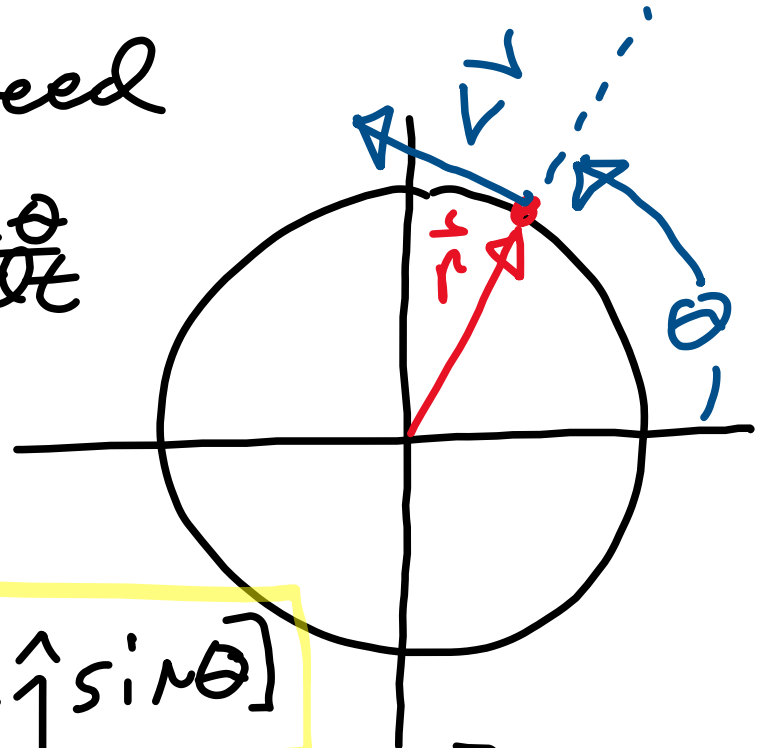
$$\vec{a} = \frac{d\vec{v}}{dt} = \left[r \left(\frac{d\theta}{dt} \right)^2 \right] [-\hat{i} \cos \theta - \hat{j} \sin \theta]$$

Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$



What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \left[r \left(\frac{d\theta}{dt} \right)^2 \right] [-\hat{i} \cos \theta - \hat{j} \sin \theta]$$

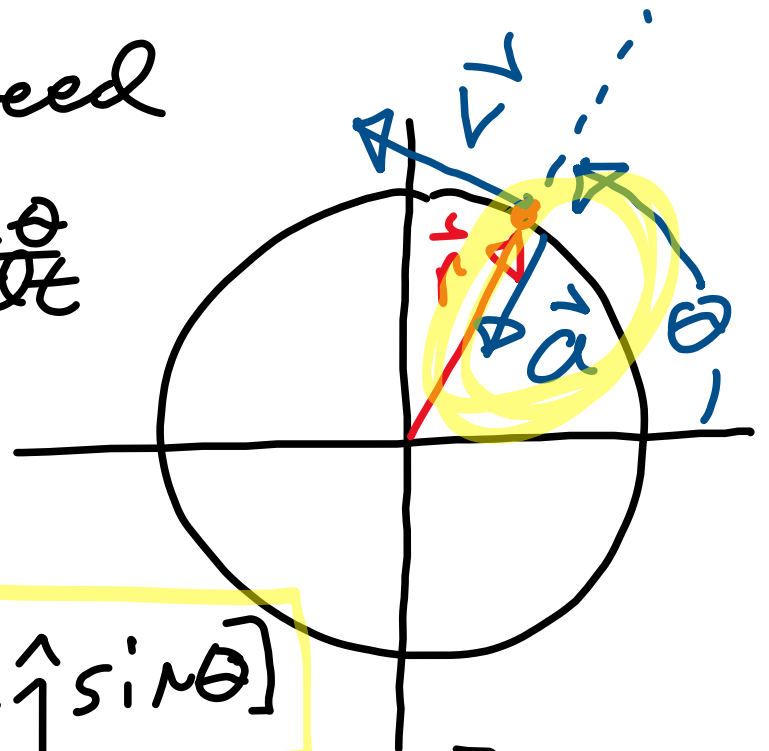
\vec{a} is rotated 90° relative to \vec{v}

Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$



What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

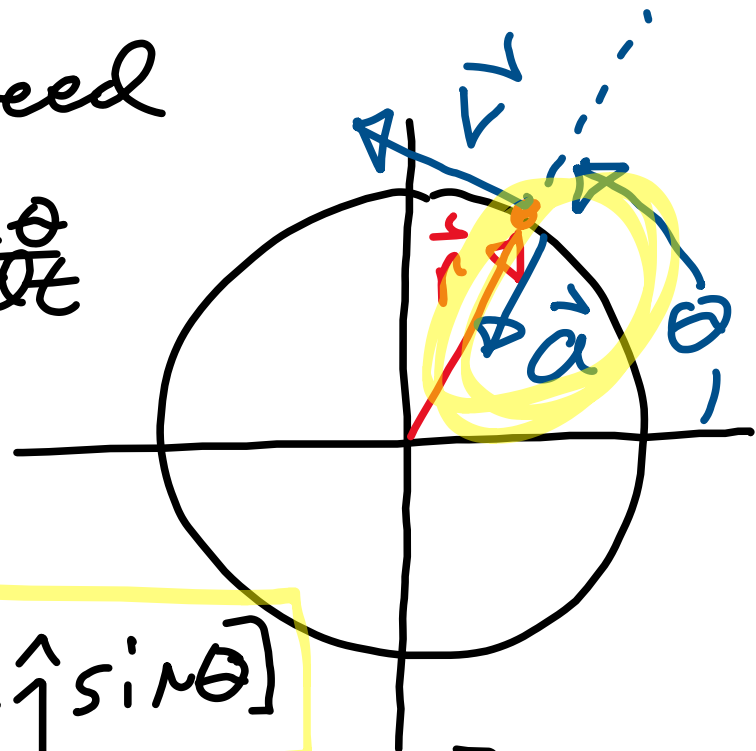
$$\vec{a} = \frac{d\vec{v}}{dt} = \left[r \left(\frac{d\theta}{dt} \right)^2 \right] [-\hat{i} \cos \theta - \hat{j} \sin \theta]$$

Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\& \quad v = \frac{ds}{dt} \Rightarrow v = r \frac{d\theta}{dt}$$



What about \vec{v} ?

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = r \frac{d}{dt} [\hat{i} \cos \theta + \hat{j} \sin \theta] \\ &= \left(r \frac{d\theta}{dt} \right) [-\hat{i} \sin \theta + \hat{j} \cos \theta] \end{aligned}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \left[r \left(\frac{d\theta}{dt} \right)^2 \right] [-\hat{i} \cos \theta - \hat{j} \sin \theta]$$



\rightarrow points inward towards center

Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\text{so } a_{\perp} = r \left(\frac{d\theta}{dt} \right)^2 \left[\cos^2 \theta + \sin^2 \theta \right]^{\frac{1}{2}}$$

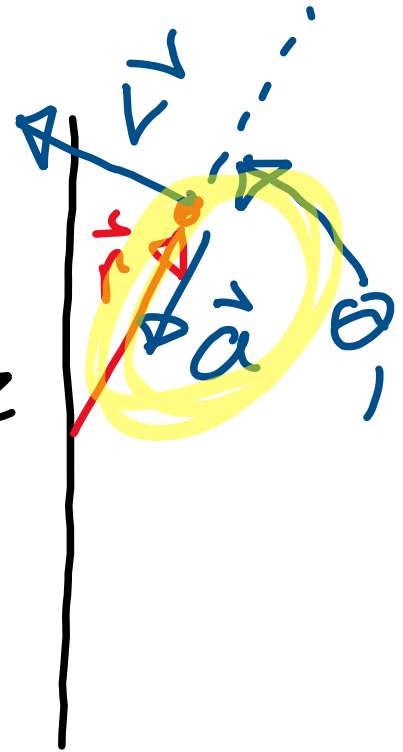


Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\text{so } a_{\perp} = r \left(\frac{d\theta}{dt} \right)^2 \underbrace{[\cos^2 \theta + \sin^2 \theta]}_1^{\frac{1}{2}}$$



Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\text{so } a_{\perp} = r \left(\frac{d\theta}{dt} \right)^2$$



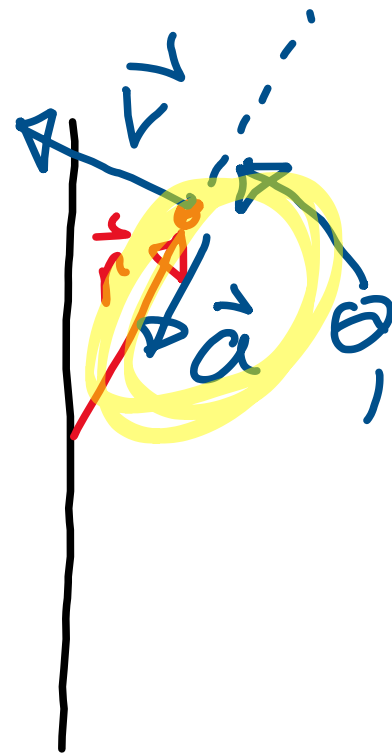
Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\text{so } a_{\perp} = r \left(\frac{d\theta}{dt} \right)^2$$

$$\text{But } r \frac{d\theta}{dt} = v$$



Uniform Circular Motion

Uniform \Rightarrow constant speed

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\text{so } a_{\perp} = r \left(\frac{d\theta}{dt} \right)^2$$

$$\text{But } r \frac{d\theta}{dt} = v$$

$$\text{so } \frac{v^2}{r} = r \left(\frac{d\theta}{dt} \right)^2$$



Uniform Circular Motion

Uniform \Rightarrow constant speed

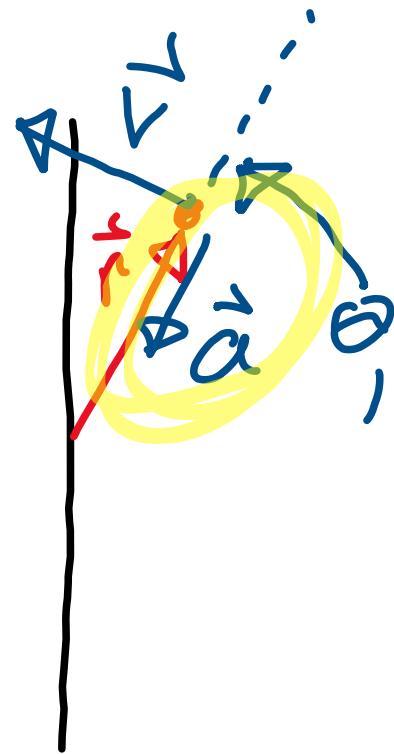
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\text{so } a_{\perp} = r \left(\frac{d\theta}{dt} \right)^2$$

But $r \frac{d\theta}{dt} = v$

$$\text{so } \frac{v^2}{r} = r \left(\frac{d\theta}{dt} \right)^2$$

$$\Rightarrow \boxed{a_{\perp} = \frac{v^2}{r}}$$



Uniform Circular Motion

Uniform \Rightarrow constant speed

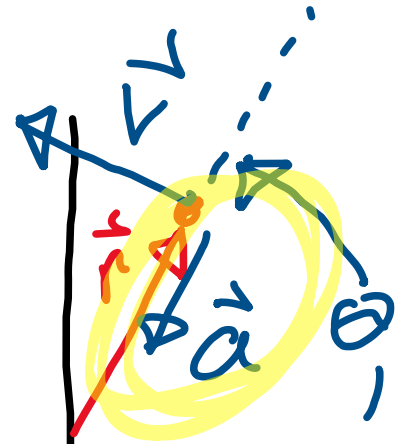
$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt}$$

$$\text{so } a_{\perp} = r \left(\frac{d\theta}{dt} \right)^2$$

But $r \frac{d\theta}{dt} = v$

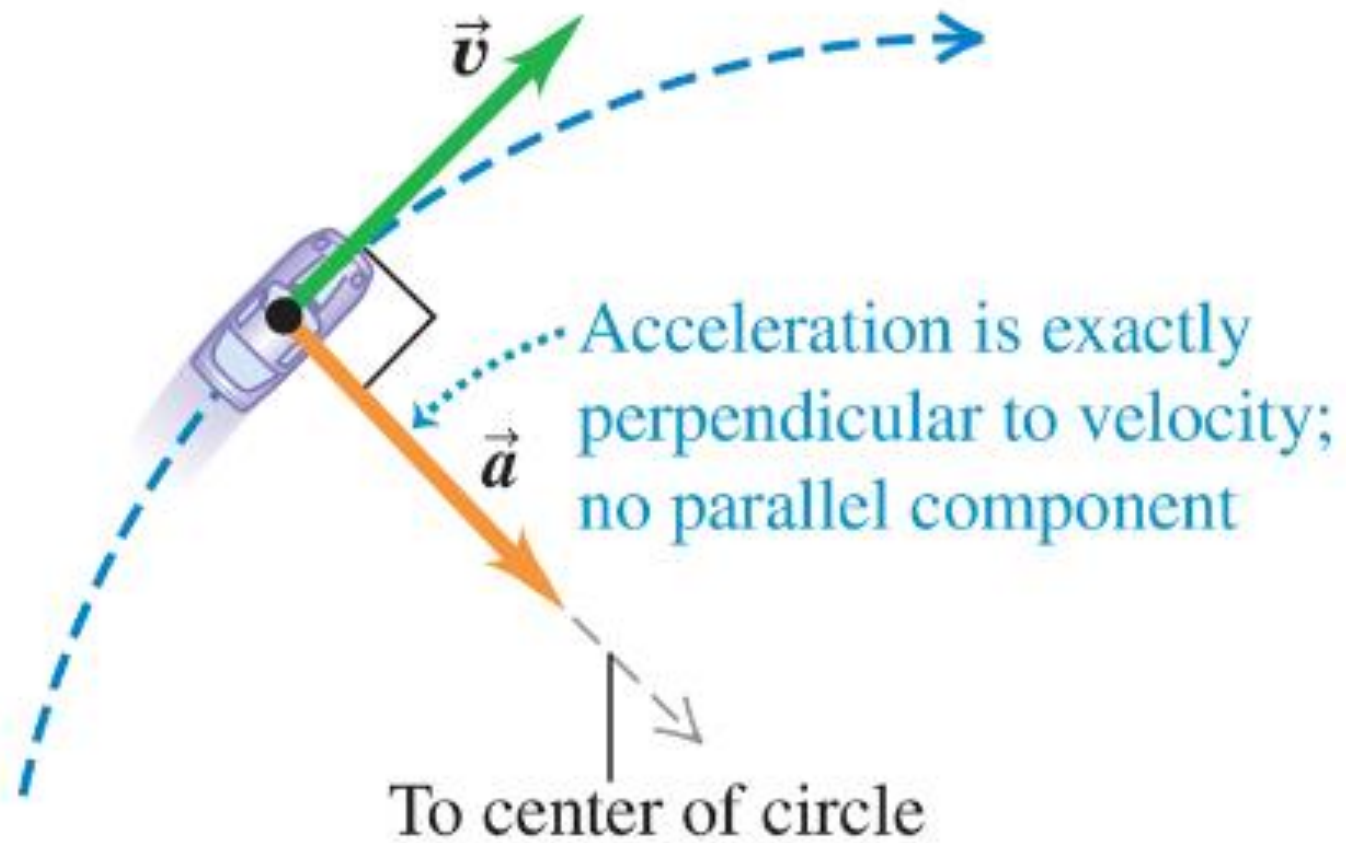
$$\text{so } \frac{v^2}{r} = r \left(\frac{d\theta}{dt} \right)^2$$

$$\Rightarrow \boxed{a_{\perp} = \frac{v^2}{r}}$$



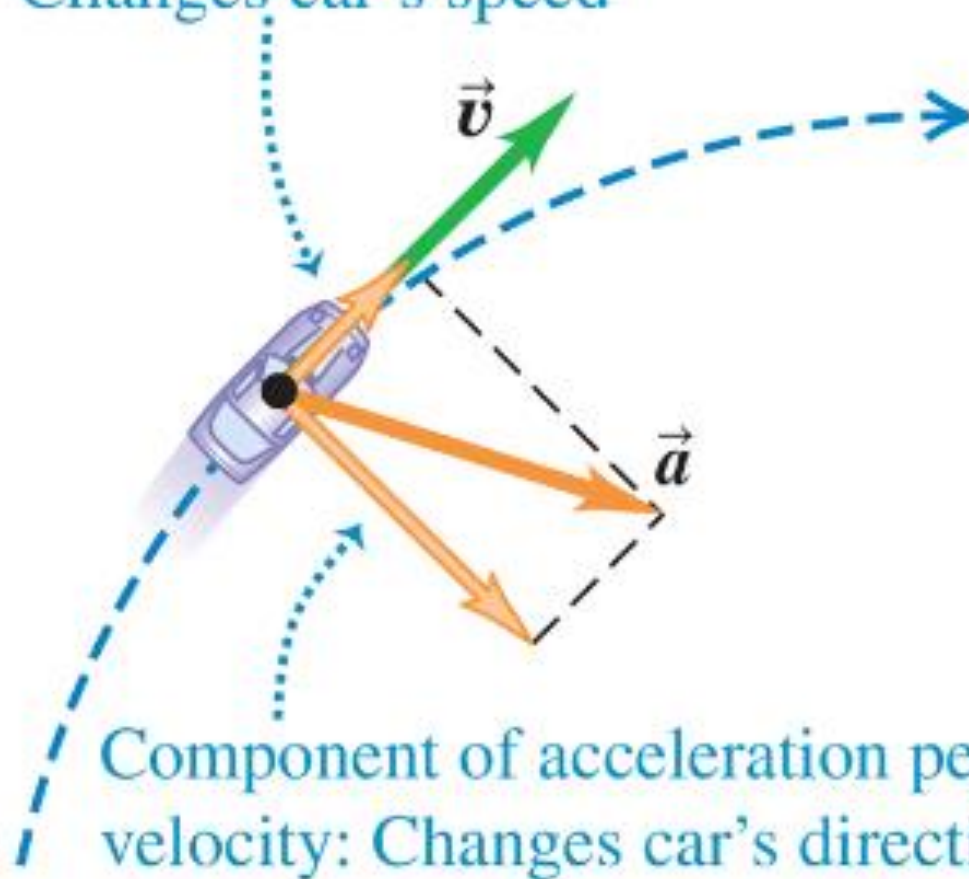
Back to
the text

(a) Uniform circular motion: Constant speed along a circular path



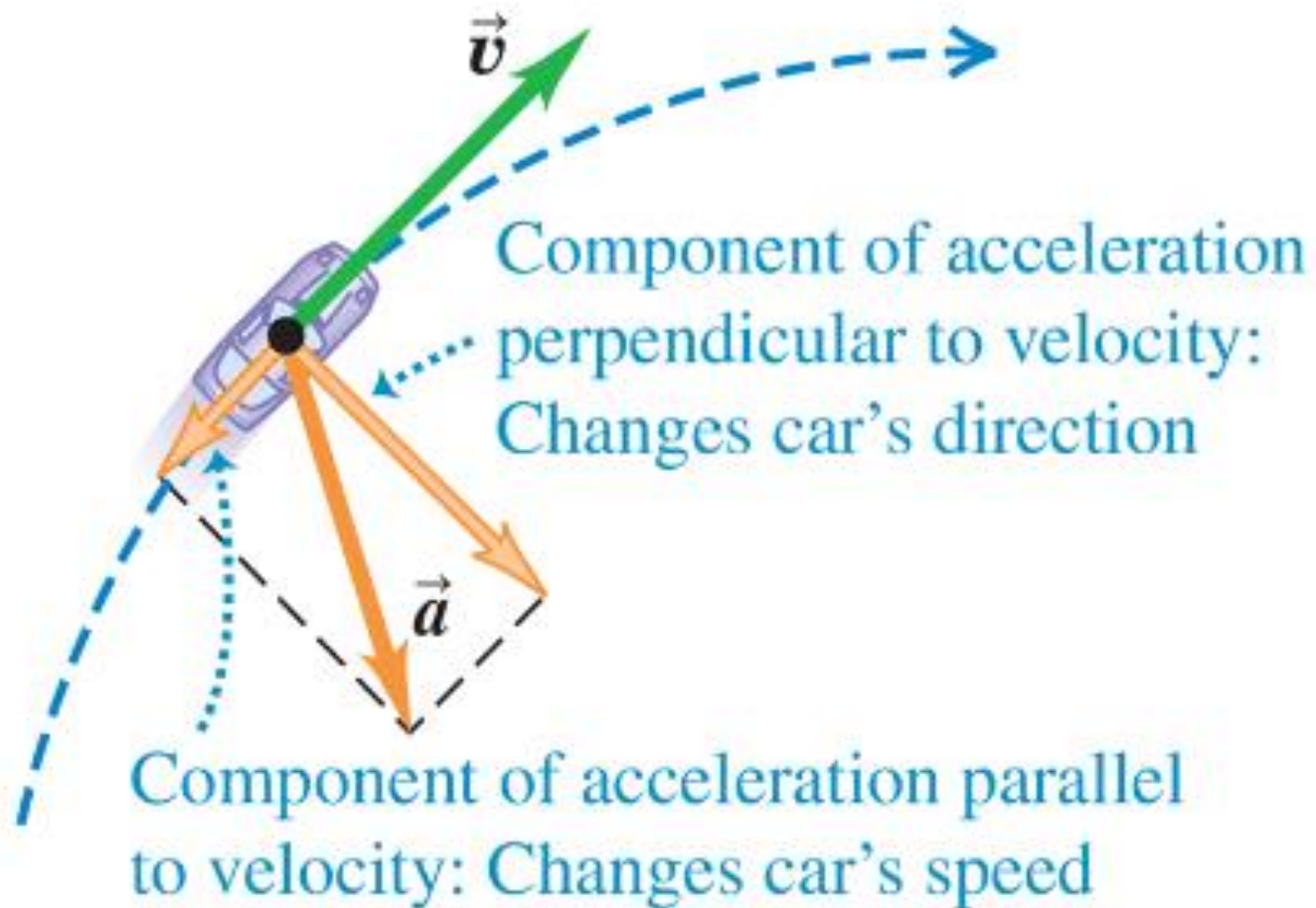
(b) Car speeding up along a circular path

Component of acceleration parallel to velocity:
Changes car's speed

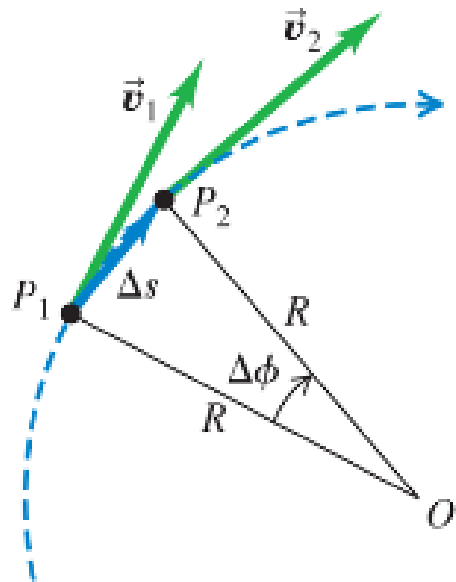


Component of acceleration perpendicular to
velocity: Changes car's direction

(c) Car slowing down along a circular path

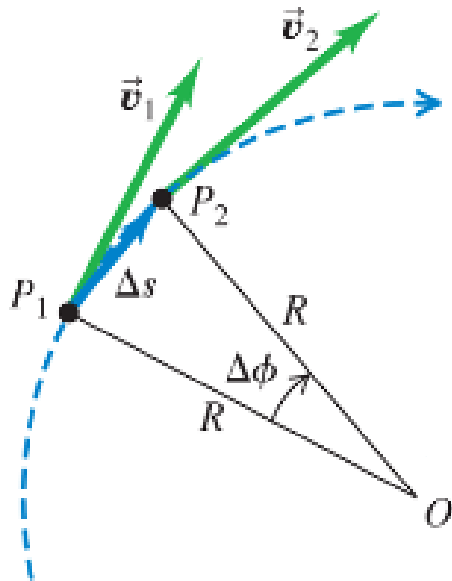


(a) A particle moves a distance Δs at constant speed along a circular path.



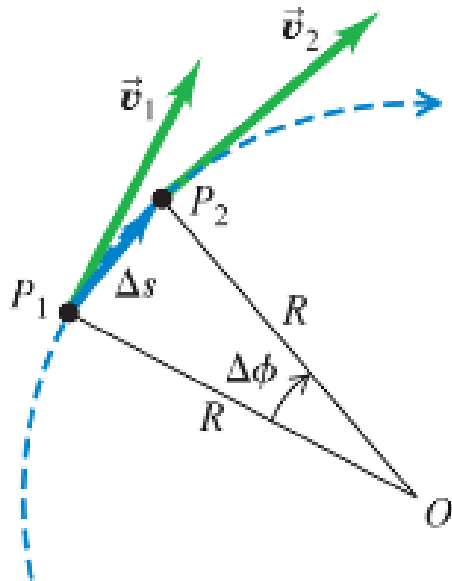
(a) A particle moves a distance Δs at constant speed along a circular path.

path length
 Δs

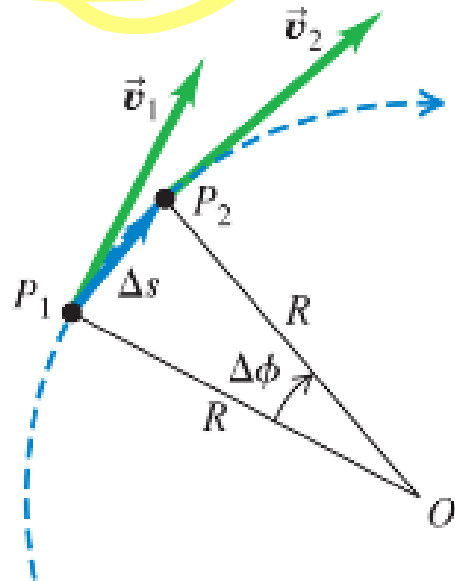


(a) A particle moves a distance Δs at constant speed along a circular path.

path length
 $\Delta s = R \Delta \phi$



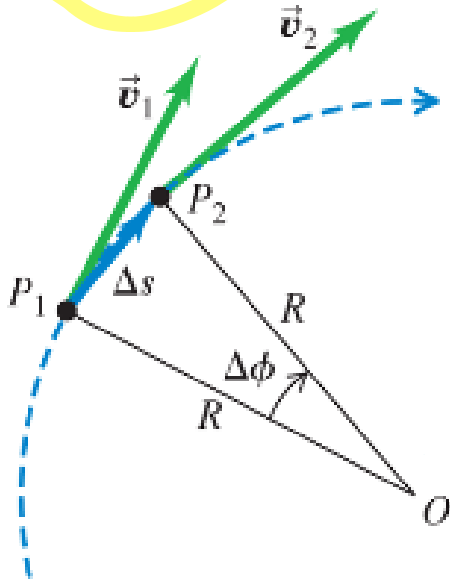
(a) A particle moves a distance Δs at constant speed along a circular path.



path length
 $\Delta s = R \Delta\phi$

Constant speed

(a) A particle moves a distance Δs at constant speed along a circular path.

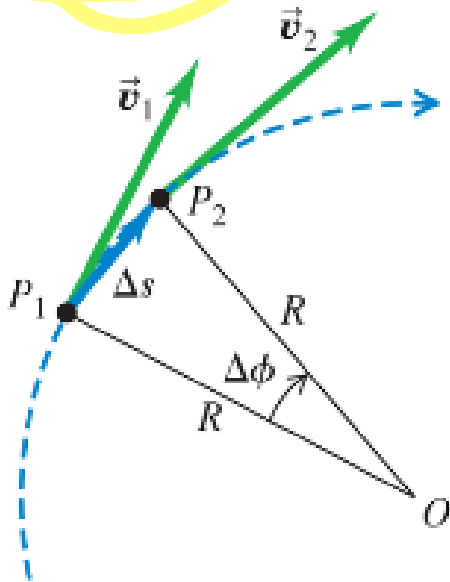


path length
 $\Delta s = R \Delta\phi$

Constant speed so

$$v = \frac{\Delta s}{\Delta t} = \text{constant}$$

(a) A particle moves a distance Δs at constant speed along a circular path.



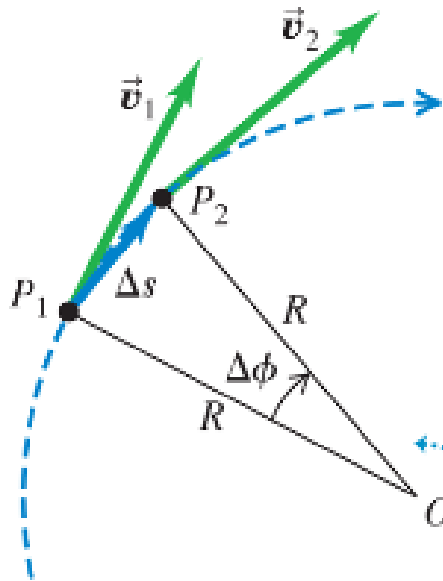
path length
 $\Delta s = R\Delta\phi$

Constant speed so

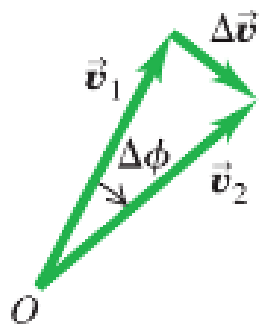
$$v = \frac{\Delta s}{\Delta t} = \text{constant}$$

But

(a) A particle moves a distance Δs at constant speed along a circular path.

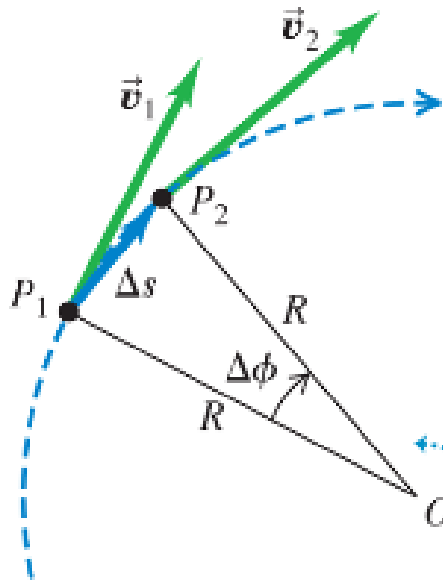


(b) The corresponding change in velocity $\Delta\vec{v}$. The average acceleration is in the same direction as $\Delta\vec{v}$.



These two triangles are similar.

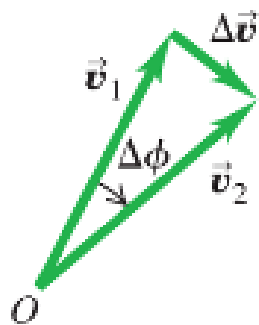
(a) A particle moves a distance Δs at constant speed along a circular path.



Since

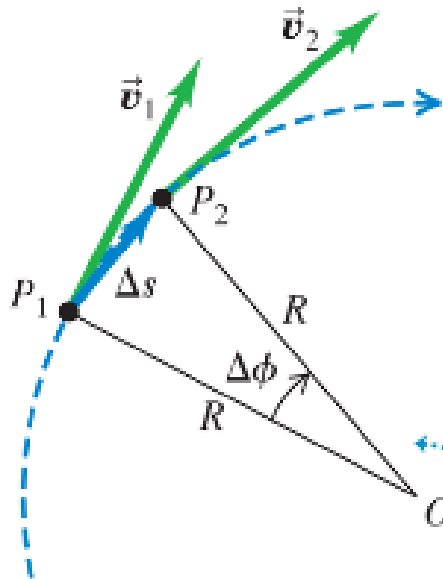
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.

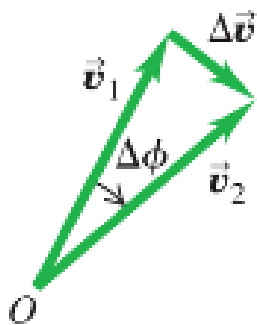


These two triangles are similar.

(a) A particle moves a distance Δs at constant speed along a circular path.



(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



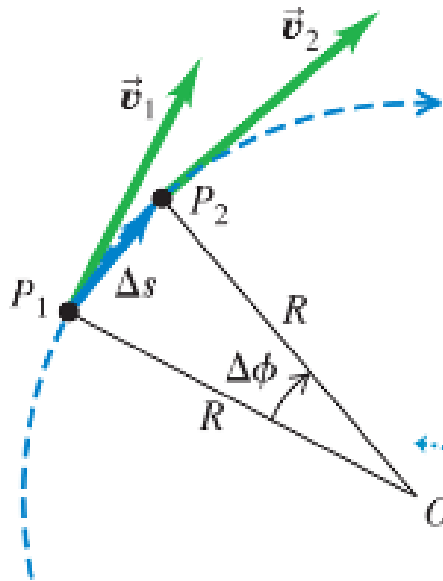
These two triangles are similar.

Since

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

scalar

(a) A particle moves a distance Δs at constant speed along a circular path.

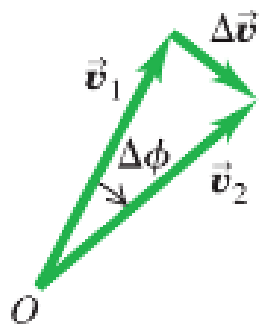


Since

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$
 then

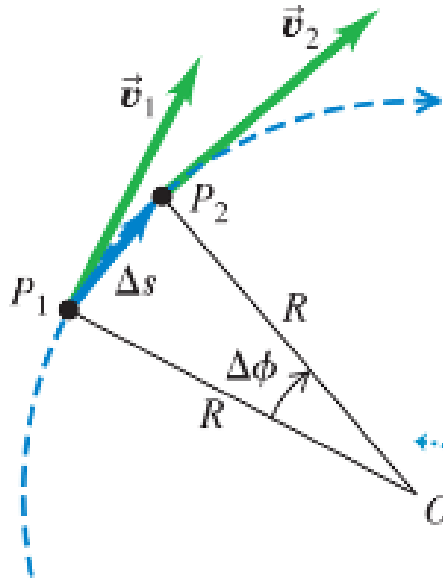
\vec{a} is in direction
of $\Delta \vec{v}$

(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



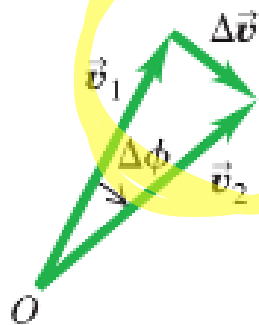
These two triangles
are similar.

(a) A particle moves a distance Δs at constant speed along a circular path.



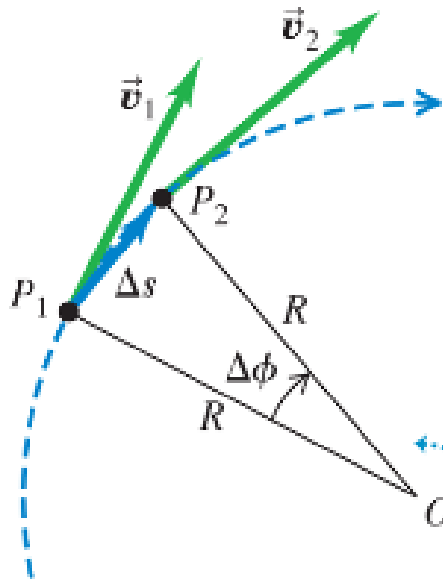
Since
 $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$ then
 \vec{a} is in direction
of $\Delta \vec{v}$

(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.

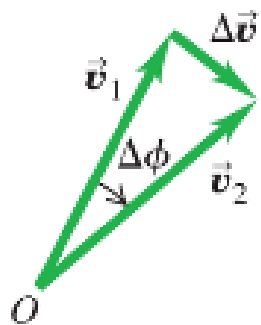


These two triangles are similar.

(a) A particle moves a distance Δs at constant speed along a circular path.

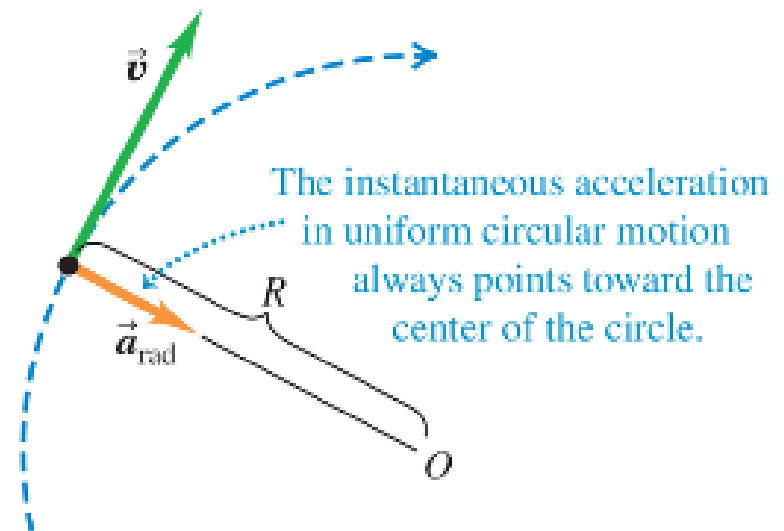


(b) The corresponding change in velocity $\Delta\vec{v}$. The average acceleration is in the same direction as $\Delta\vec{v}$.



These two triangles are similar.

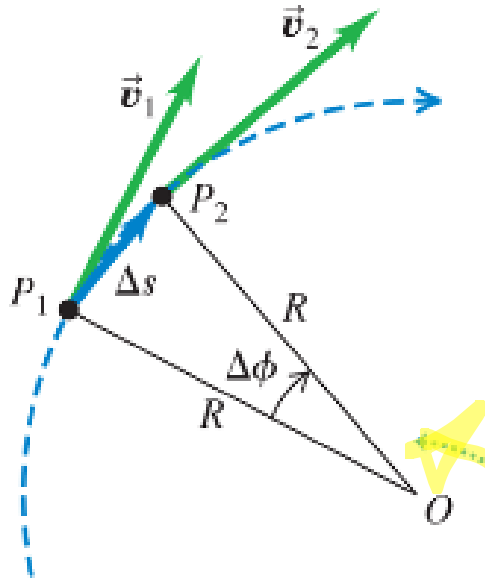
(c) The instantaneous acceleration



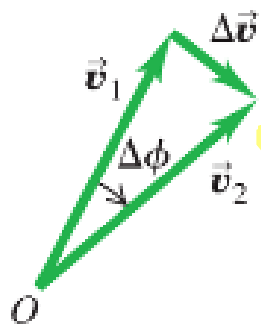
The instantaneous acceleration in uniform circular motion always points toward the center of the circle.

Similar triangles

(a) A particle moves a distance Δs at constant speed along a circular path.



(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.

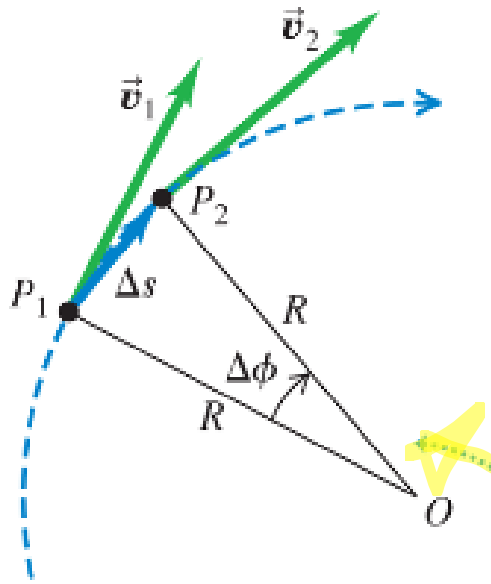


These two triangles are similar.

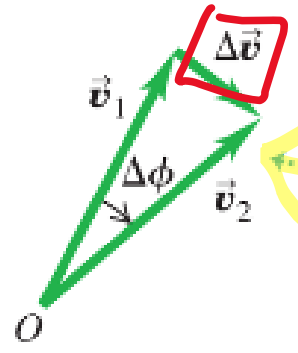
Similar triangles

so $\Delta \vec{v}$

(a) A particle moves a distance Δs at constant speed along a circular path.



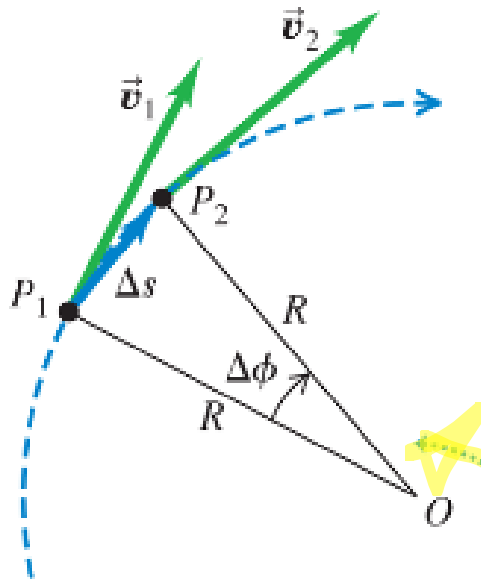
(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



These two triangles are similar.

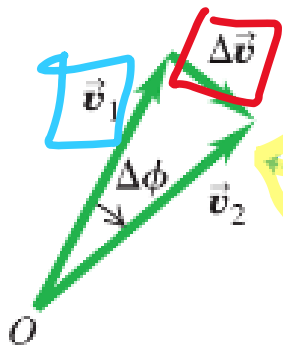
Similar triangles

(a) A particle moves a distance Δs at constant speed along a circular path.



$$\text{SO } \frac{\boxed{|\Delta\vec{v}|}}{\boxed{v_1}}$$

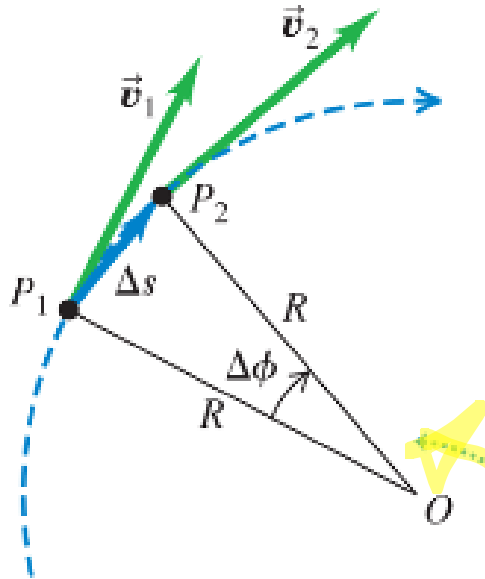
(b) The corresponding change in velocity $\Delta\vec{v}$. The average acceleration is in the same direction as $\Delta\vec{v}$.



These two triangles are similar.

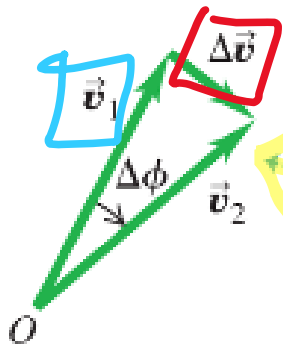
Similar triangles

(a) A particle moves a distance Δs at constant speed along a circular path.



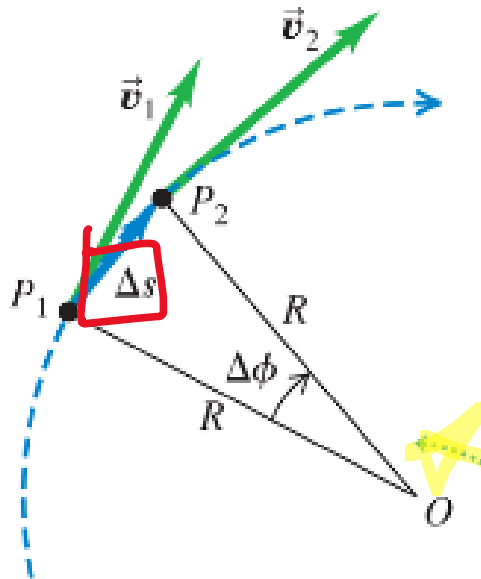
$$\text{SO } \frac{|\Delta \vec{v}|}{v_1} =$$

(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



These two triangles are similar.

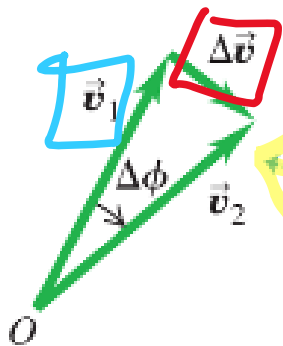
(a) A particle moves a distance Δs at constant speed along a circular path.



Similar triangles

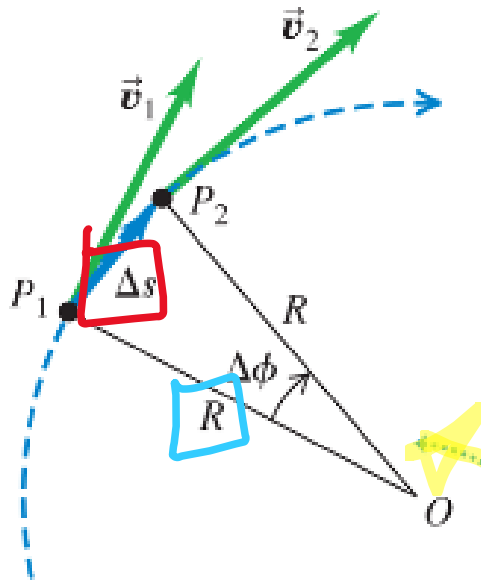
$$\text{so } \frac{\boxed{|\Delta \vec{v}|}}{\boxed{v_1}} = \boxed{\Delta s}$$

(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



These two triangles are similar.

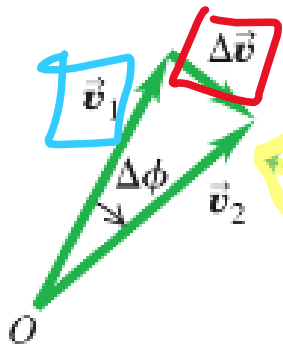
(a) A particle moves a distance Δs at constant speed along a circular path.



Similar triangles

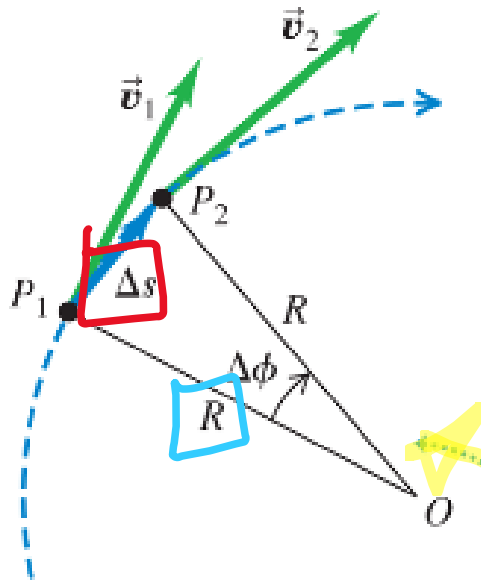
$$\text{SO } \frac{\boxed{\Delta \vec{v}}}{\boxed{v_1}} = \frac{\boxed{\Delta s}}{\boxed{R}}$$

(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



These two triangles are similar.

(a) A particle moves a distance Δs at constant speed along a circular path.

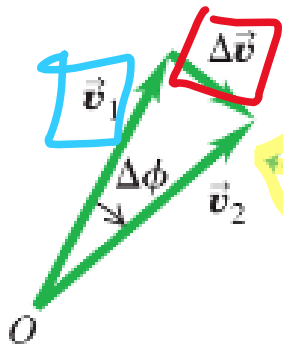


Similar triangles

$$\text{SO } \frac{|\Delta \vec{v}|}{v_1} = \frac{\Delta s}{R} \Rightarrow$$

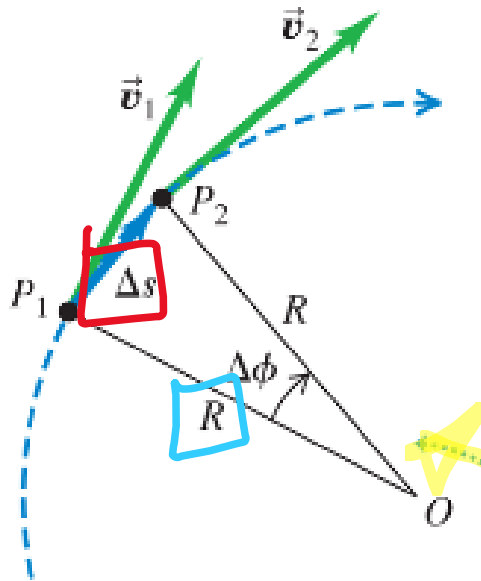
$$|\Delta \vec{v}| = v_1 \Delta s / R$$

(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



These two triangles are similar.

(a) A particle moves a distance Δs at constant speed along a circular path.



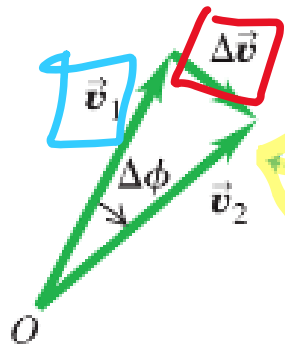
Similar triangles

$$\text{so } \frac{|\Delta \vec{v}|}{v_1} = \frac{\Delta s}{R} \Rightarrow$$

$$|\Delta \vec{v}| = v_1 \Delta s / R \quad \& \text{ since}$$

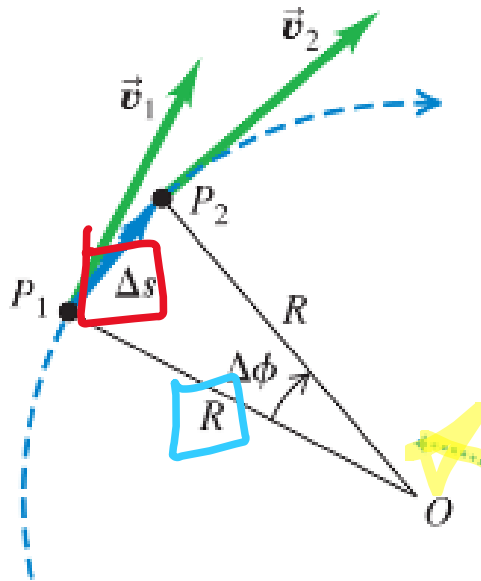
$$a = \frac{|\Delta \vec{v}|}{\Delta t}$$

(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



These two triangles are similar.

(a) A particle moves a distance Δs at constant speed along a circular path.



Similar triangles

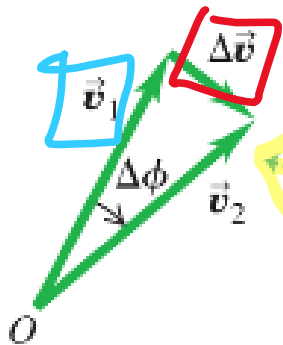
$$\text{SO } \frac{|\Delta \vec{v}|}{v_1} = \frac{\Delta s}{R} \Rightarrow$$

$$|\Delta \vec{v}| = v_1 \Delta s / R \quad \& \quad \text{since}$$

$$a = \frac{|\Delta \vec{v}|}{\Delta t} \quad \text{then}$$

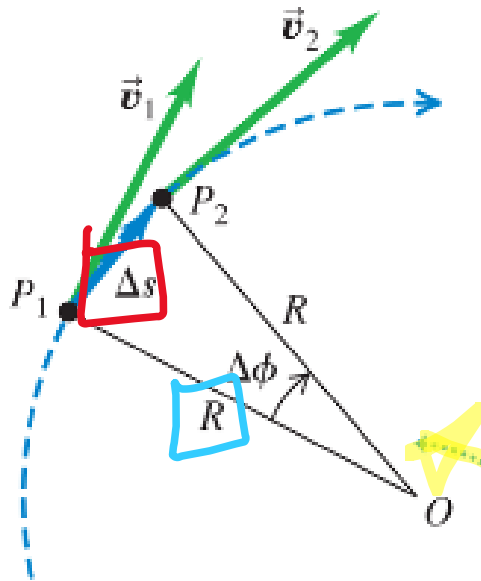
$$a = \frac{v_1 \Delta s}{R \Delta t}$$

(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



These two triangles are similar.

(a) A particle moves a distance Δs at constant speed along a circular path.



Similar triangles

$$\text{so } \frac{|\Delta \vec{v}|}{v_1} = \frac{\Delta s}{R} \Rightarrow$$

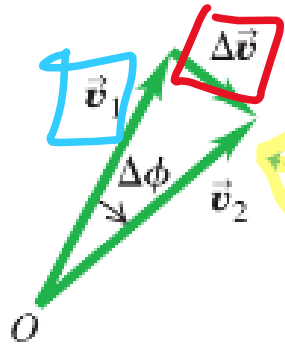
$$|\Delta \vec{v}| = v_1 \Delta s / R \quad \& \quad \text{since}$$

$$a = \frac{|\Delta \vec{v}|}{\Delta t} \quad \text{then}$$

$$a = \frac{v_1 \Delta s}{R \Delta t}$$

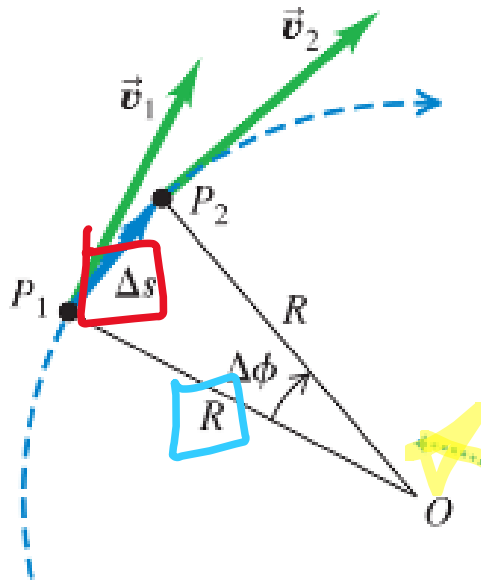
$$\text{But } \frac{\Delta s}{\Delta t} = v$$

(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.

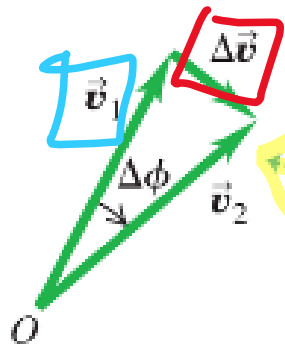


These two triangles are similar.

(a) A particle moves a distance Δs at constant speed along a circular path.



(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



These two triangles are similar.

Similar triangles

$$\text{so } \frac{|\Delta \vec{v}|}{v_1} = \frac{\Delta s}{R} \Rightarrow$$

$$|\Delta \vec{v}| = v_1 \Delta s / R \quad \& \quad \text{since}$$

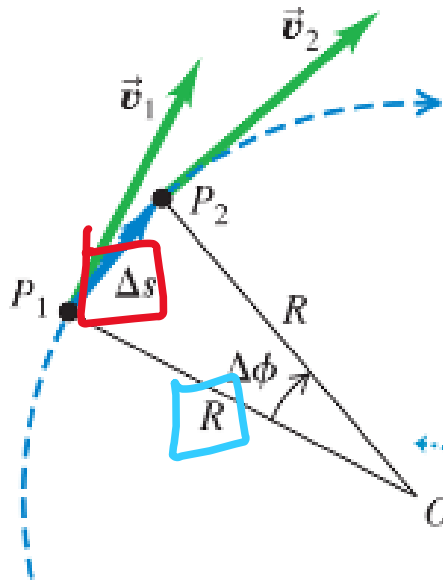
$$a = \frac{|\Delta \vec{v}|}{\Delta t} \quad \text{then}$$

$$a = \frac{v_1 \Delta s}{R \Delta t}$$

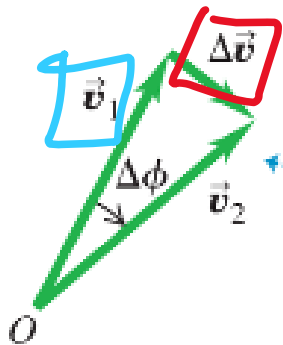
$$\text{But } \frac{\Delta s}{\Delta t} = v$$

$$\text{so } a = \frac{v^2}{R}$$

(a) A particle moves a distance Δs at constant speed along a circular path.



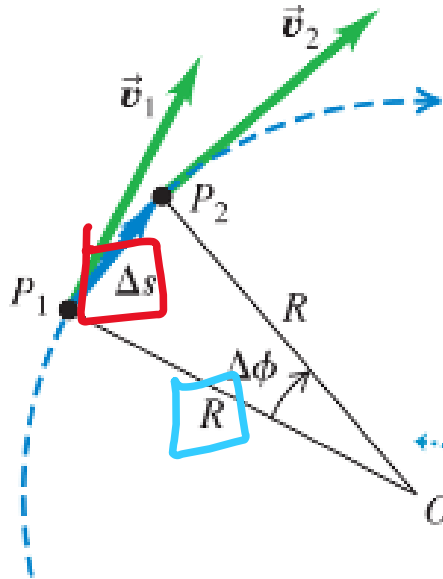
(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



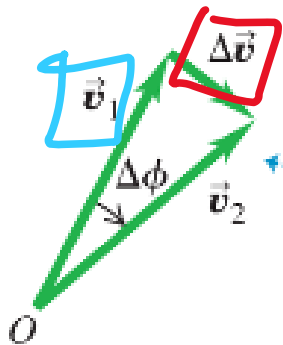
These two triangles are similar.

Note: we will write $a_{\text{rad}} = \frac{v^2}{R}$ to remind us that this is only for the radial [a_{rad} or a_{\perp}] part of the acceleration

(a) A particle moves a distance Δs at constant speed along a circular path.



(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



These two triangles are similar.

Note: we will

$$\text{write } a_{\text{rad}} = \frac{v^2}{R}$$

to remind us that this is only for the radial [a_{rad} or a_{\perp}] part

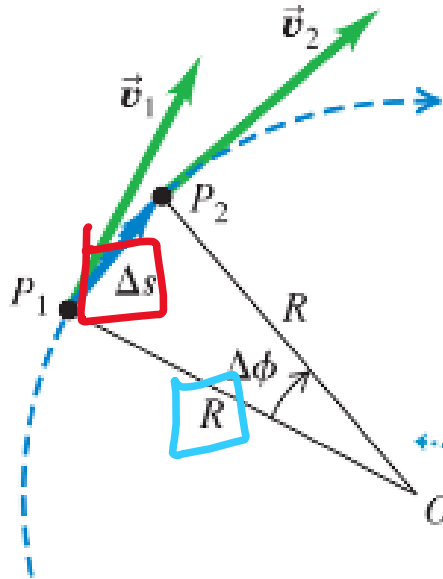
of the acceleration

When uniform circular

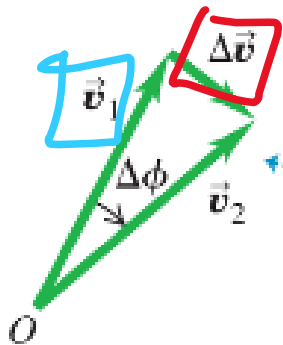
motion the acceleration

is 100% a_{rad}

(a) A particle moves a distance Δs at constant speed along a circular path.



(b) The corresponding change in velocity $\Delta \vec{v}$. The average acceleration is in the same direction as $\Delta \vec{v}$.



These two triangles are similar.

Note: we will

$$\text{write } a_{\text{rad}} = \frac{v^2}{R}$$

to remind us that this is only for the radial [a_{rad} or a_{\perp}] part

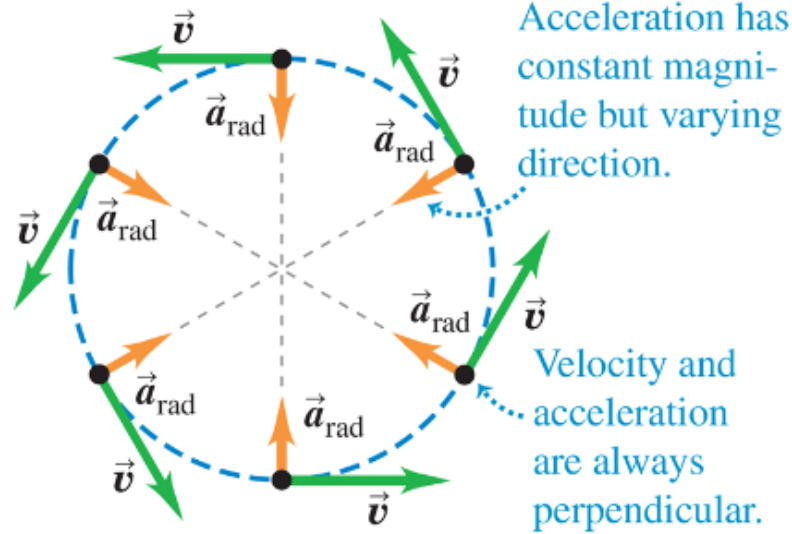
of the acceleration

When uniform circular

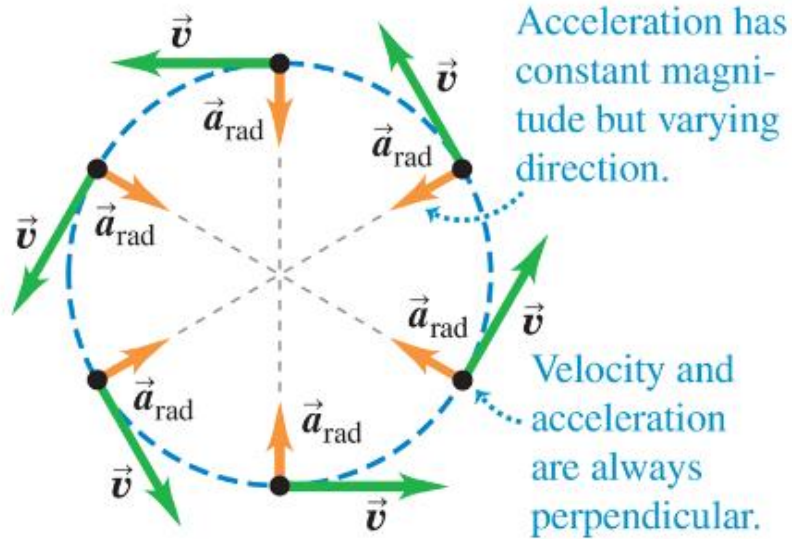
motion the acceleration

is 100% a_{rad} {or a_{\perp} }

(a) Uniform circular motion

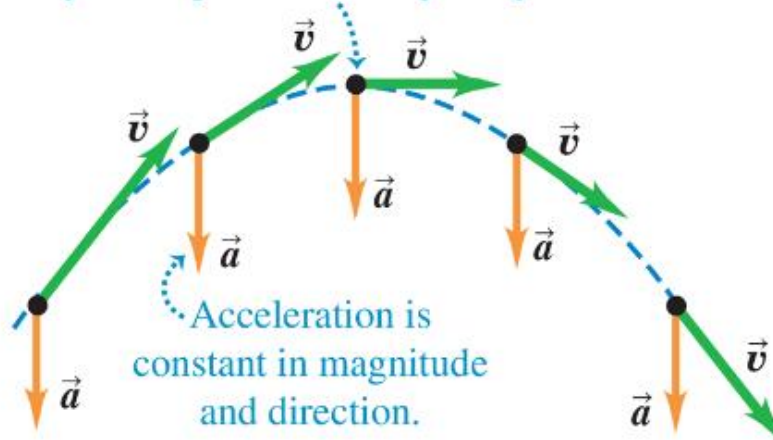


(a) Uniform circular motion



(b) Projectile motion

Velocity and acceleration are perpendicular only at the peak of the trajectory.



Uniform Circular Motion

From previous $v = \frac{\Delta s}{\Delta t}$

Uniform Circular Motion

From previous $v = \frac{\Delta s}{\Delta t}$ If Δs is one full revolution

Uniform Circular Motion

From previous $v = \frac{\Delta s}{\Delta t}$ If Δs is one full revolution then $\Delta s = 2\pi R$

Uniform Circular Motion

From previous $v = \frac{\Delta s}{\Delta t}$ If Δs is one full revolution then $\Delta s = 2\pi R$.

Let $T \equiv$ time for one full revolution

Uniform Circular Motion

From previous $v = \frac{\Delta s}{\Delta t}$ If Δs is one full revolution then $\Delta s = 2\pi R$.

Let $T \equiv$ time for one full revolution \equiv period of motion

Uniform Circular Motion

From previous $v = \frac{\Delta s}{\Delta t}$ If Δs is one full revolution then $\Delta s = 2\pi R$.

Let $T \equiv$ time for one full revolution \equiv period of motion

$$\Rightarrow v = \frac{2\pi R}{T}$$

Uniform Circular Motion

From previous $v = \frac{\Delta s}{\Delta t}$ If Δs is one full revolution then $\Delta s = 2\pi R$.

Let $T \equiv$ time for one full revolution \equiv period of motion

$$\Rightarrow v = \frac{2\pi R}{T}$$

Uniform Circular Motion

From previous $v = \frac{\Delta s}{\Delta t}$ If Δs is one full revolution then $\Delta s = 2\pi R$.

Let $T \equiv$ time for one full revolution \equiv period of motion

$$\Rightarrow v = \frac{2\pi R}{T} \quad \& \text{ since}$$

$$a_{\text{rad}} = \frac{v^2}{R}$$

Uniform Circular Motion

From previous $v = \frac{\Delta s}{\Delta t}$ If Δs is one full revolution then $\Delta s = 2\pi R$.

Let $T \equiv$ time for one full revolution \equiv period of motion

$$\Rightarrow v = \frac{2\pi R}{T} \quad \& \text{ since}$$

$$a_{\text{rad}} = \frac{v^2}{R} \quad \text{then} \quad a_{\text{rad}} = \frac{4\pi^2 R}{T^2}$$

Uniform Circular Motion

From previous $v = \frac{\Delta s}{\Delta t}$ If Δs is one full revolution then $\Delta s = 2\pi R$.

Let $T \equiv$ time for one full revolution \equiv period of motion

$$\Rightarrow v = \frac{2\pi R}{T} \quad \& \text{ since}$$

$$a_{\text{rad}} = \frac{v^2}{R} \text{ then}$$

$$a_{\text{rad}} = \frac{4\pi^2 R}{T^2}$$

An Aston Martin V12 Vantage sports car has a “lateral acceleration” of $0.97g = (0.97)(9.8 \text{ m/s}^2) = 9.5 \text{ m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h , or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?



An Aston Martin V12 Vantage sports car has a “lateral acceleration” of $0.97g = (0.97)(9.8 \text{ m/s}^2) = 9.5 \text{ m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h , or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

An Aston Martin V12 Vantage sports car has a "lateral acceleration" of $0.97g = (0.97)(9.8 \text{ m/s}^2) = 9.5 \text{ m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h , or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

$$a_{\text{rad}} = 0.97g = 9.5 \text{ m/s}^2$$

An Aston Martin V12 Vantage sports car has a “lateral acceleration” of $0.97g = (0.97)(9.8 \text{ m/s}^2) = 9.5 \text{ m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h , or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

$$a_{\text{rad}} = 0.97g = 9.5 \text{ m/s}^2, v = 40 \text{ m/s}$$

An Aston Martin V12 Vantage sports car has a “lateral acceleration” of $0.97g = (0.97)(9.8 \text{ m/s}^2) = 9.5 \text{ m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h , or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

$$a_{\text{rad}} = 0.97g = 9.5 \text{ m/s}^2, v = 40 \text{ m/s}$$

An Aston Martin V12 Vantage sports car has a “lateral acceleration” of $0.97g = (0.97)(9.8 \text{ m/s}^2) = 9.5 \text{ m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h , or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

$$a_{\text{rad}} = 0.97g = 9.5 \text{ m/s}^2, v = 40 \text{ m/s}$$

Find R_{min} :

An Aston Martin V12 Vantage sports car has a "lateral acceleration" of $0.97g = (0.97)(9.8 \text{ m/s}^2) = 9.5 \text{ m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h , or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

$$a_{\text{rad}} = 0.97g = 9.5 \text{ m/s}^2, v = 40 \text{ m/s}$$

Find R_{min} : $a_{\text{rad}} = \frac{v^2}{R}$

An Aston Martin V12 Vantage sports car has a "lateral acceleration" of $0.97g = (0.97)(9.8 \text{ m/s}^2) = 9.5 \text{ m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h , or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

$$a_{\text{rad}} = 0.97g = 9.5 \text{ m/s}^2, v = 40 \text{ m/s}$$

Find R_{min} : $a_{\text{rad}} = \frac{v^2}{R} \Rightarrow$

$$R = \frac{v^2}{a_{\text{rad}}}$$

An Aston Martin V12 Vantage sports car has a "lateral acceleration" of $0.97g = (0.97)(9.8 \text{ m/s}^2) = 9.5 \text{ m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h , or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

$$a_{\text{rad}} = 0.97g = 9.5 \text{ m/s}^2, v = 40 \text{ m/s}$$

Find R_{min} : $a_{\text{rad}} = \frac{v^2}{R} \Rightarrow$

$$R = \frac{v^2}{a_{\text{rad}}} = \frac{40 \text{ m/s}}{9.5 \text{ m/s}^2}$$

An Aston Martin V12 Vantage sports car has a "lateral acceleration" of $0.97g = (0.97)(9.8 \text{ m/s}^2) = 9.5 \text{ m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h , or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

$$a_{\text{rad}} = 0.97g = 9.5 \text{ m/s}^2, v = 40 \text{ m/s}$$

Find R_{min} : $a_{\text{rad}} = \frac{v^2}{R} \Rightarrow$

$$R = \frac{v^2}{a_{\text{rad}}} = \frac{40^2 \text{ m/s}^2}{9.5 \text{ m/s}^2} = 170 \text{ m}$$

An Aston Martin V12 Vantage sports car has a "lateral acceleration" of $0.97g = (0.97)(9.8 \text{ m/s}^2) = 9.5 \text{ m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h , or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

$$a_{\text{rad}} = 0.97g = 9.5 \text{ m/s}^2, v = 40 \text{ m/s}$$

Find R_{min} : $a_{\text{rad}} = \frac{v^2}{R} \Rightarrow$

$$R = \frac{v^2}{a_{\text{rad}}} = \frac{40^2 \text{ m/s}^2}{9.5 \text{ m/s}^2} = 170 \text{ m}$$

So $R_{\text{min}} = 170 \text{ m}$

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration?



Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration?

$$R = 5\text{ m}$$

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration?

$$R = 5\text{m}$$

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration?

$$R = 5\text{ m}, T = 4\text{ s}$$

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration?

$$R = 5\text{ m}, T = 4\text{ s}$$

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration?

$$R = 5\text{ m}, T = 4\text{ s}$$

Find a :

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration? $R = 5\text{ m}, T = 4\text{ s}$

Find a : Since uniform circular
motion $a = a_{\text{rad}}$

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration? $R = 5\text{ m}, T = 4\text{ s}$

Find a : Since uniform circular
motion $a = a_{\text{rad}} = \frac{v^2}{R}$

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration? $R = 5\text{ m}, T = 4\text{ s}$

Find a : Since uniform circular motion $a = a_{\text{rad}} = \frac{v^2}{R}$ But $v = \frac{2\pi R}{T}$

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration? $R = 5\text{ m}, T = 4\text{ s}$

Find a : Since uniform circular motion $a = a_{\text{rad}} = \frac{v^2}{R}$ But $v = \frac{2\pi R}{T}$

$$\text{so } a = \frac{4\pi^2 R}{T^2}$$

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration? $R = 5\text{ m}, T = 4\text{ s}$

Find a : Since uniform circular motion $a = a_{\text{rad}} = \frac{v^2}{R}$ But $v = \frac{2\pi R}{T}$

$$\text{so } a = \frac{4\pi^2 R}{T^2} = \frac{4 \cdot 3.14^2 \cdot 5\text{ m}}{16\text{ s}^2}$$

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration? $R = 5\text{ m}, T = 4\text{ s}$

Find a : Since uniform circular motion $a = a_{\text{rad}} = \frac{v^2}{R}$ But $v = \frac{2\pi R}{T}$

$$\text{so } a = \frac{4\pi^2 R}{T^2} = \frac{4 \cdot 3.14^2 \cdot 5\text{ m}}{16\text{ s}^2}$$

$$\Rightarrow a = 12\text{ m/s}^2$$

Nonuniform circular motion

Nonuniform circular motion

Nonuniform circular motion

* Radial component is still $a_{rad} = \frac{v^2}{R}$

Nonuniform circular motion

* Radial component is still $a_{rad} = \frac{v^2}{R}$

* $a_{tan} = \frac{d|\vec{v}|}{dt}$

Nonuniform circular motion

* Radial component is still $a_{rad} = \frac{v^2}{R}$

* $a_{tan} = \frac{d|\vec{v}|}{dt} = \frac{dv}{dt}$

Nonuniform circular motion

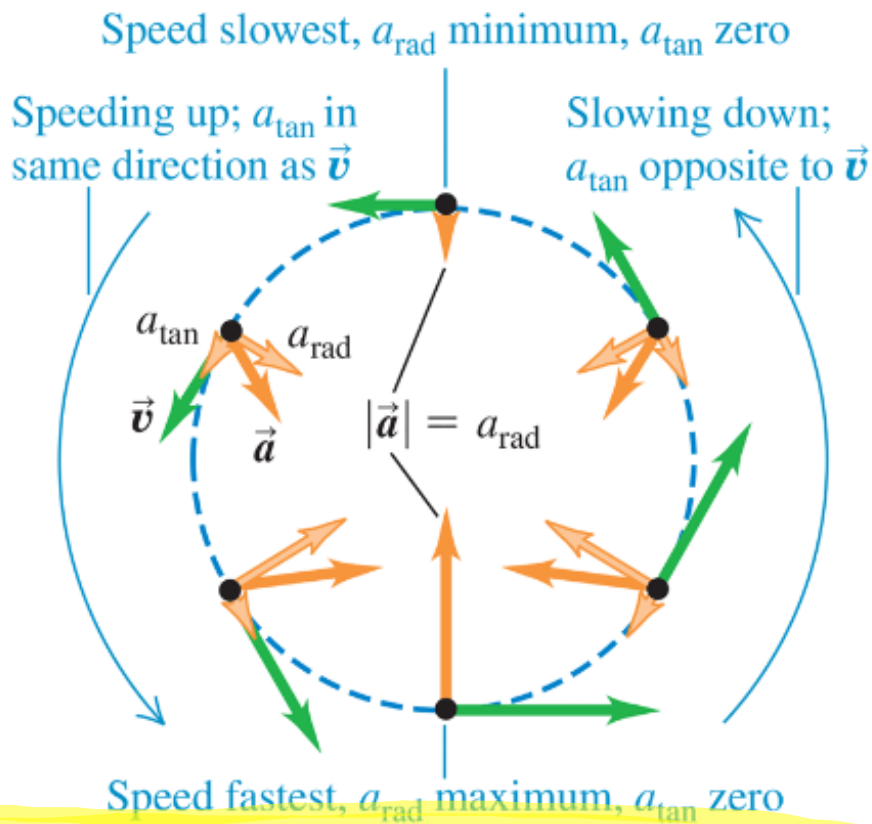
* Radial component is still $a_{rad} = \frac{v^2}{R}$

$$* a_{tan} = \underbrace{\frac{d|\vec{v}|}{dt}}_{\text{scalar}} = \frac{dv}{dt}$$

Nonuniform circular motion

* Radial component is still $a_{\text{rad}} = \frac{v^2}{R}$

* $a_{\text{tan}} = \frac{d|\vec{v}|}{dt} = \frac{dv}{dt} = \frac{ds}{dt}$



A particle moving in a vertical loop with a varying speed, like a roller coaster car.

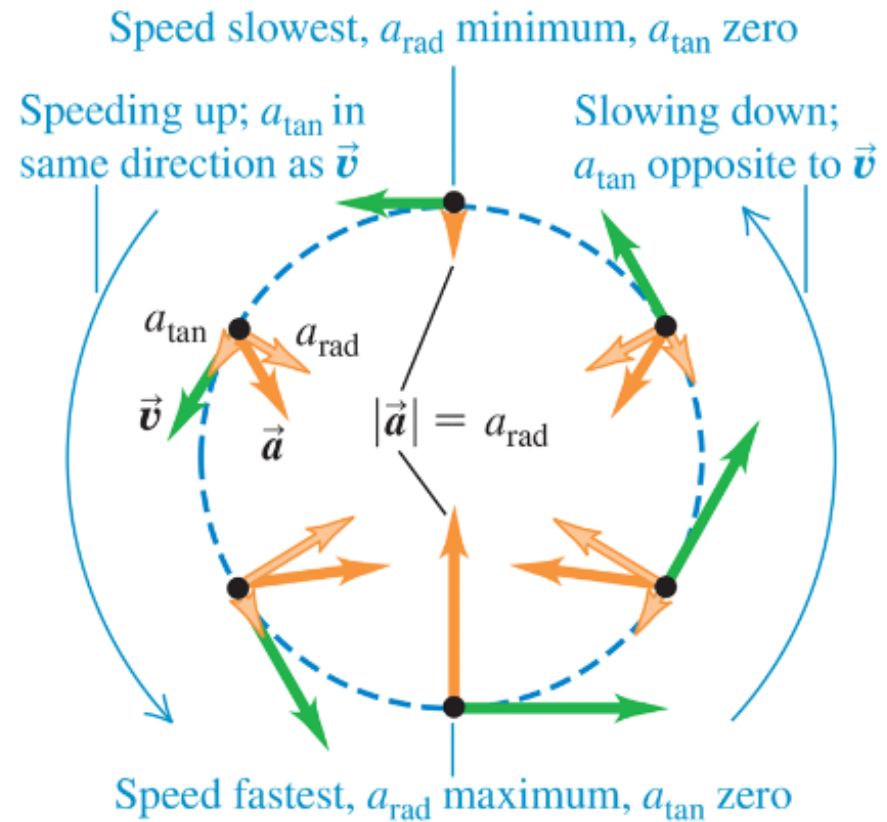
Nonuniform circular motion

* Radial component is still $a_{\text{rad}} = \frac{v^2}{R}$

* $a_{\text{tan}} = \frac{d|\vec{v}|}{dt} = \frac{dv}{dt} = \frac{ds}{dt}$

Note:

$$\frac{d|\vec{v}|}{dt} \neq \left| \frac{d\vec{v}}{dt} \right|$$



A particle moving in a vertical loop with a varying speed, like a roller coaster car.

Nonuniform circular motion

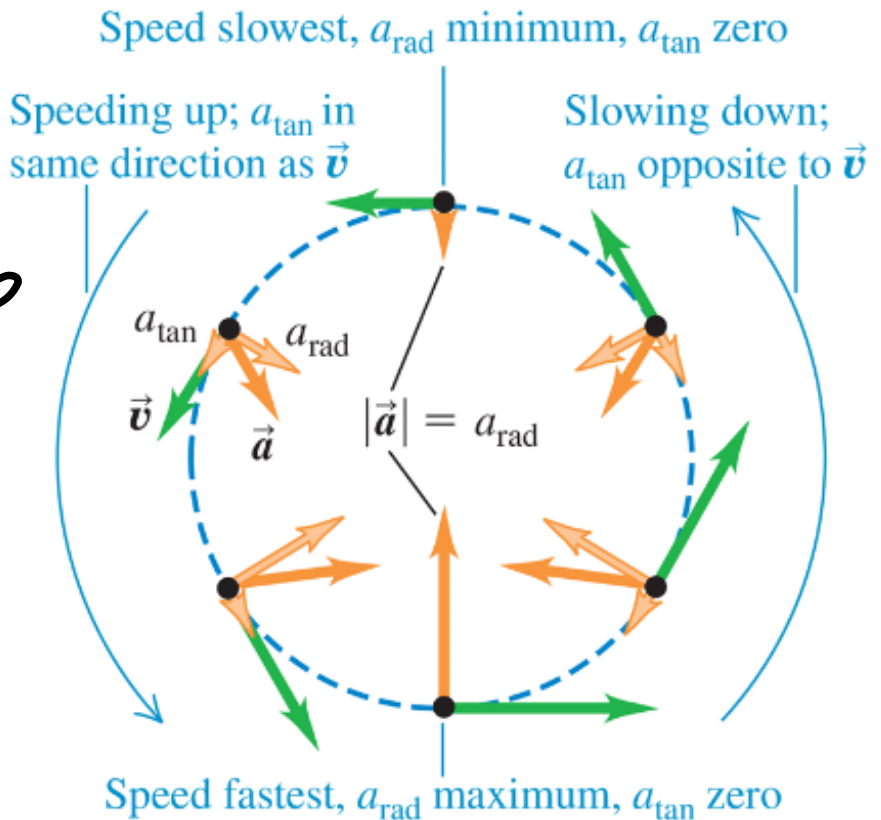
* Radial component is still $a_{\text{rad}} = \frac{v^2}{R}$

* $a_{\text{tan}} = \frac{d|\vec{v}|}{dt} = \frac{dv}{dt} = \frac{ds}{dt}$

Note:

$\frac{d|\vec{v}|}{dt} \neq \left| \frac{d\vec{v}}{dt} \right|$ since

$$\frac{d|\vec{v}|}{dt} = \frac{ds}{dt}$$



A particle moving in a vertical loop with a varying speed, like a roller coaster car.

Nonuniform circular motion

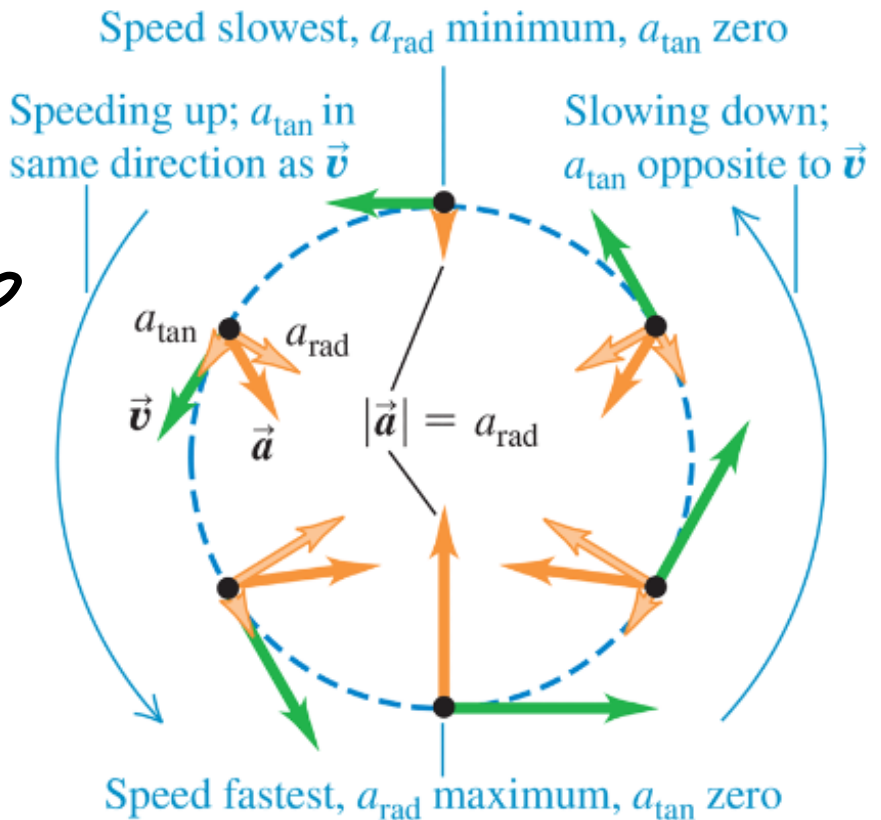
* Radial component is still $a_{\text{rad}} = \frac{v^2}{R}$

* $a_{\text{tan}} = \frac{d|\vec{v}|}{dt} = \frac{dv}{dt} = \frac{ds}{dt}$

Note:

$\frac{d|\vec{v}|}{dt} \neq \left| \frac{d\vec{v}}{dt} \right|$ since

$$\frac{d|\vec{v}|}{dt} = \frac{ds}{dt} = a_{\text{tan}}$$



A particle moving in a vertical loop with a varying speed, like a roller coaster car.

Nonuniform circular motion

* Radial component is still $a_{rad} = \frac{v^2}{R}$

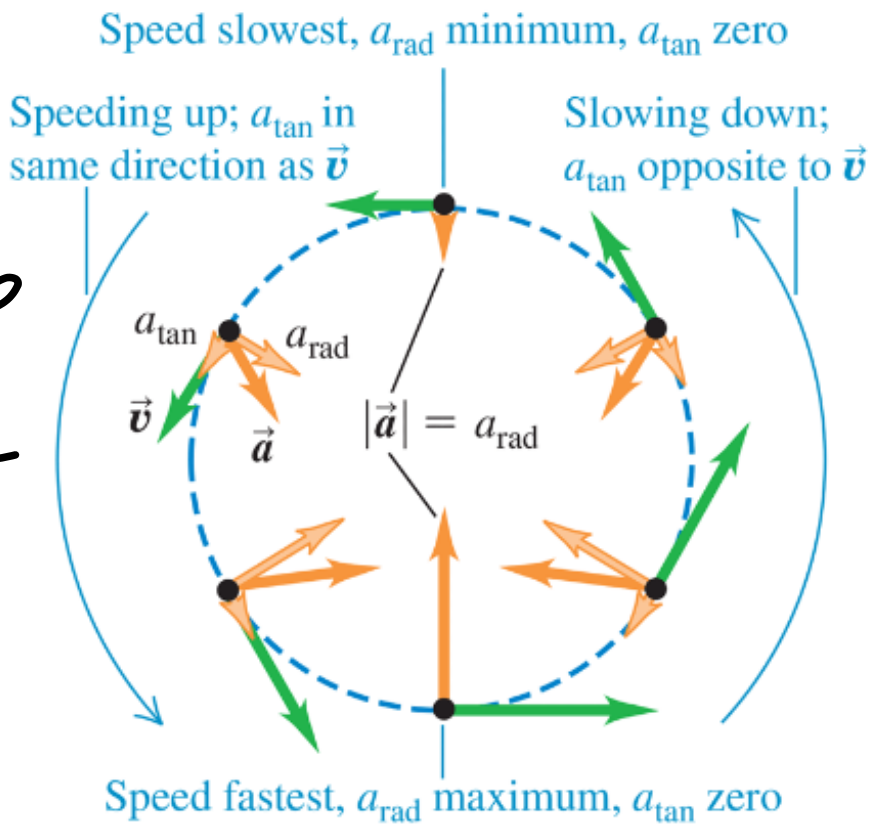
* $a_{tan} = \frac{d|\vec{v}|}{dt} = \frac{dv}{dt} = \frac{ds}{dt}$

Note:

$\frac{d|\vec{v}|}{dt} \neq \left| \frac{d\vec{v}}{dt} \right|$ since

$\frac{d|\vec{v}|}{dt} = \frac{ds}{dt} = a_{tan}$ But

$\left| \frac{d\vec{v}}{dt} \right| = |\vec{a}|$



A particle moving in a vertical loop with a varying speed, like a roller coaster car.

Nonuniform circular motion

* Radial component is still $a_{rad} = \frac{v^2}{R}$

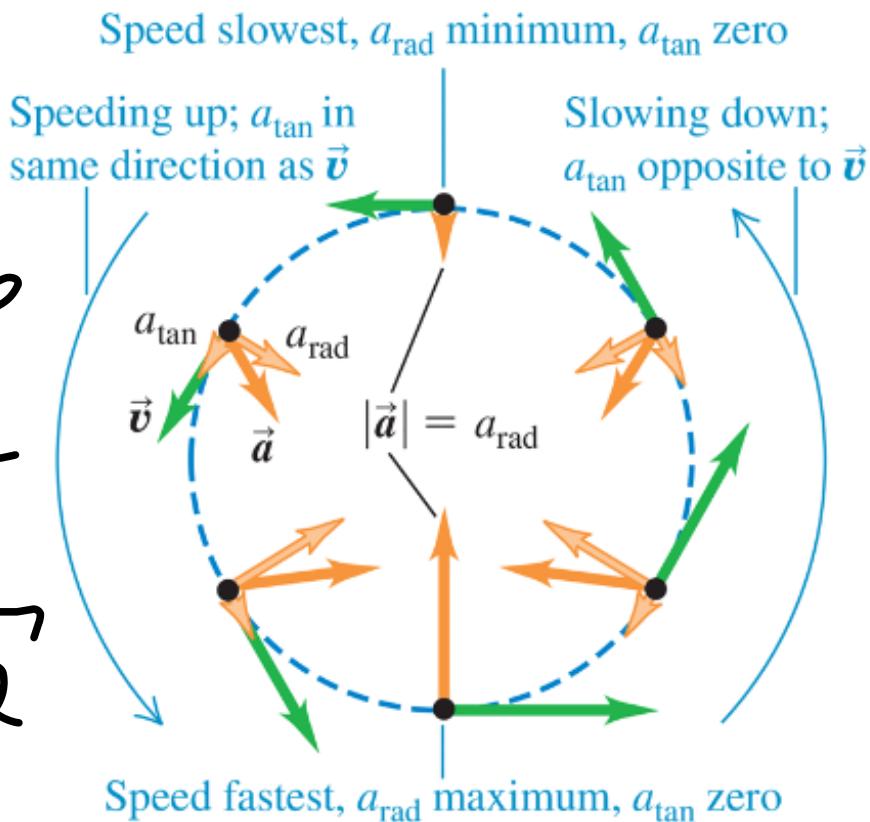
* $a_{tan} = \frac{d|\vec{v}|}{dt} = \frac{dv}{dt} = \frac{ds}{dt}$

Note:

$\frac{d|\vec{v}|}{dt} \neq \left| \frac{d\vec{v}}{dt} \right|$ since

$\frac{d|\vec{v}|}{dt} = \frac{ds}{dt} = a_{tan}$ But

$\left| \frac{d\vec{v}}{dt} \right| = |\vec{a}| = \sqrt{a_{tan}^2 + a_{rad}^2}$



A particle moving in a vertical loop with a varying speed, like a roller coaster car.







