Today: Sections 1.7 & 1.8

$\boxed{L2}$

Vectors

Today: Sections 1.7 & 1.8

$\boxed{L2}$

Wednesday: Sections 1.9 & 1.10

More vectors

$\boxed{L2}$

Today: Sections 1.7 & 1.8

Wednesday: Sections 1.9 & 1.10

HW #1: Due August 28th {Friday}

Scalars & vectors

* Scalars ≡ single number with unit.

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Temperature $T = 40^\circ C$

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Speed $v = 5\,mi/hr$

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Length $L = 2\,ft$

Scalars & Vectors

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* Vectors have magnitude and direction

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Examples:

Velocity $\vec{v} = 5 mi/hr$, west

position $\vec{r} = 2 ft$,

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- Speed $v = 5\,mi/hr$
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Examples:
- Velocity $\vec{v}$ = 5 mi/hr, west
- position $\vec{r}$ = 2 ft, x-direction

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Force $\vec{F} = 3\,lb$,

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Examples:

Velocity $\vec{V} = 5\,mi/hr$, west

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Force $\vec{F} = 3\,lb$, down

Torque $\vec{M} = 8\,ft\text{-}lb$, clockwise

Displacement

Displacement

We start with two points

Displacement

We start with two points we will call point P_1

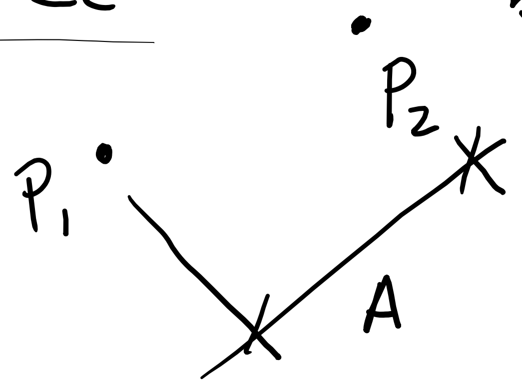
P_1 •

Displacement

We start with two points we will call point P_1 and point P_2.

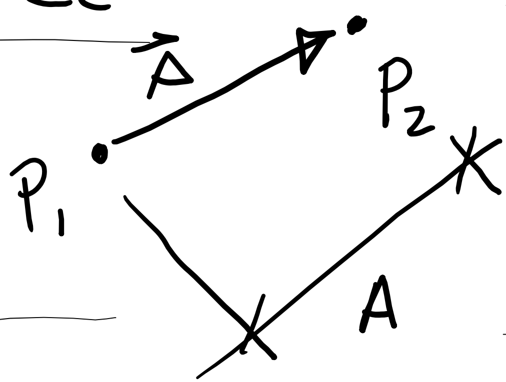
Displacement

We start with two points we will call point P_1 and point P_2. Let the distance between the points be A.



Displacement

We start with two points we will call point P_1 and point P_2. Let the distance between the points be A. And the Displacement Vector between the points be $\vec{A}$.



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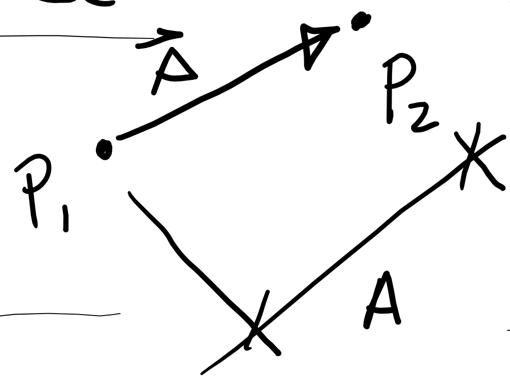

$A \equiv$ scalar distance

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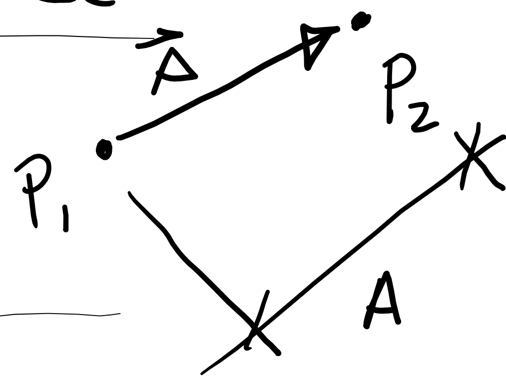

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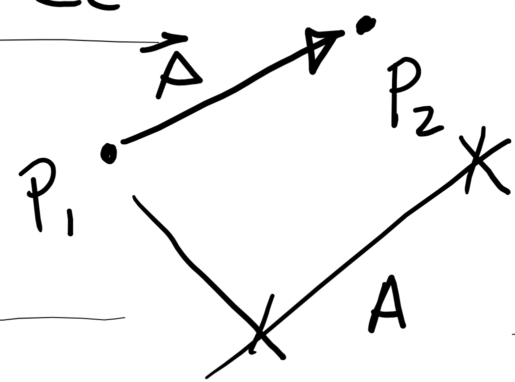

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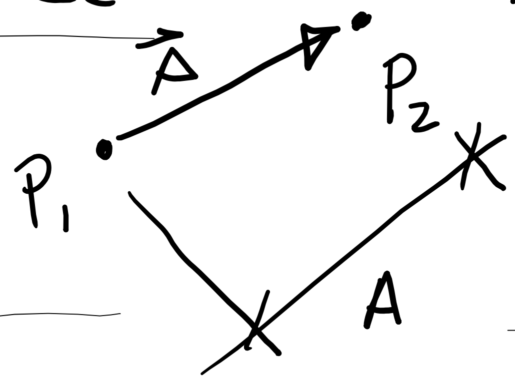
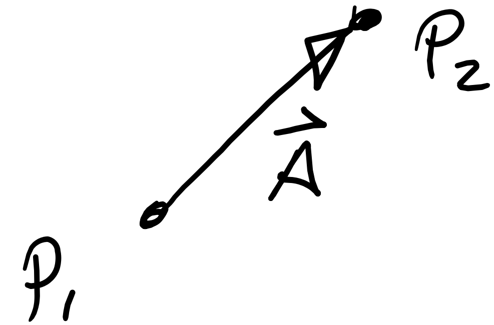


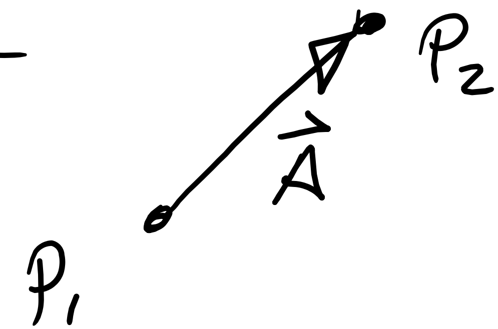
$A \equiv$ scalar distance {or magnitude of displacement}

& $\vec{A} \equiv$ Displacement vector {has magnitude & direction}

Displacement is a straight arrow from start to end.



Displacement is a straight arrow from start to end. Does not depend on



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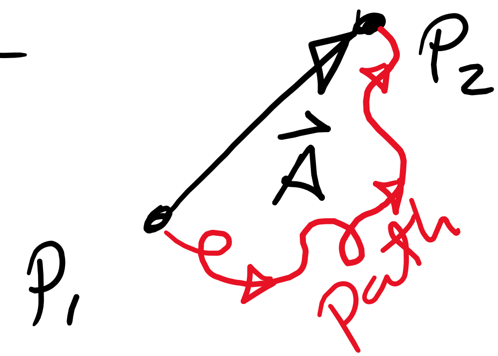

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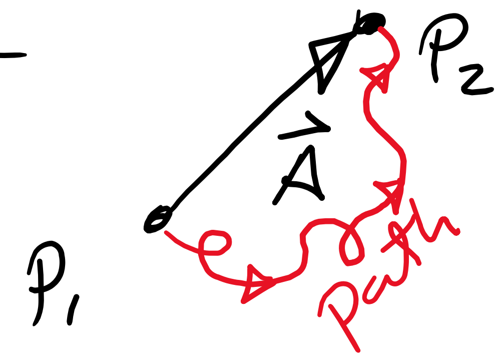
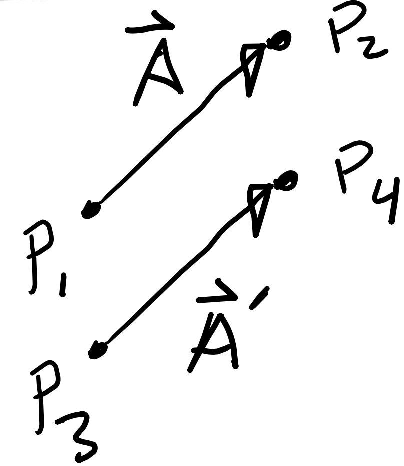


If two vectors have same direction, they are parallel.



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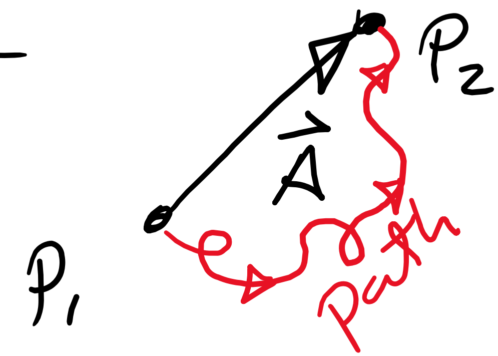
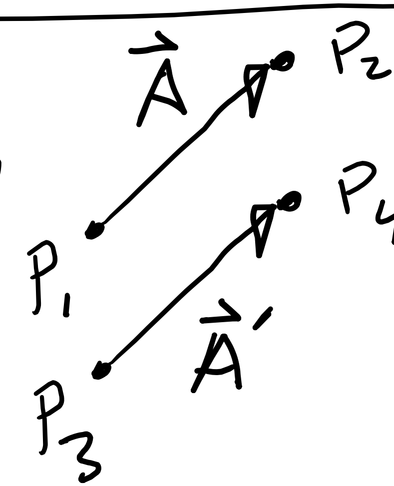


If two vectors have same direction, they are parallel.

If two vectors have same direction AND have same magnitude, they are equal



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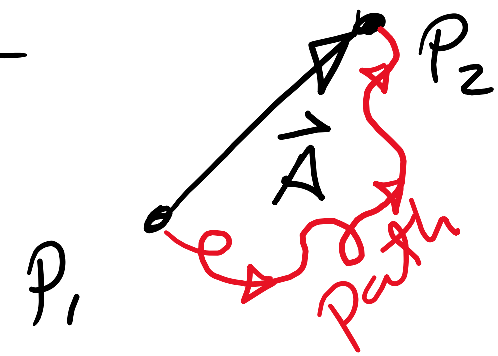

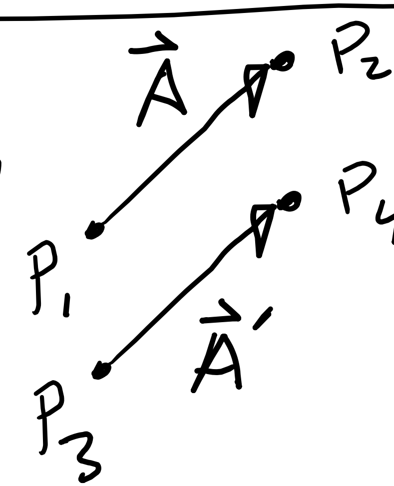

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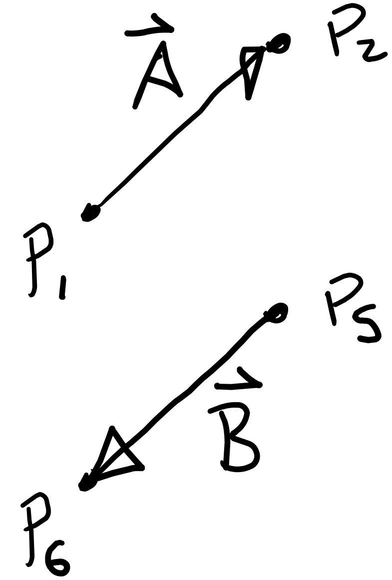
So $\vec{A} = \vec{A}'$

If same magnitude But opposite direction, then Negative : $\vec{A} = -\vec{B}$



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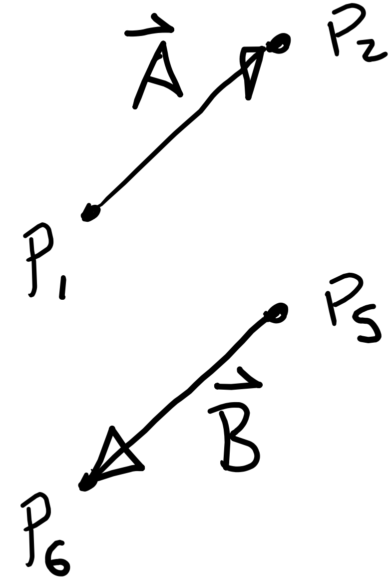

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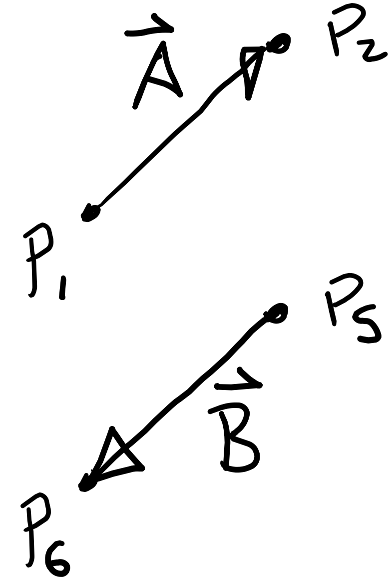

<u>Notation</u>: [Magnitude of $\vec{A}$] $\equiv |\vec{A}|$

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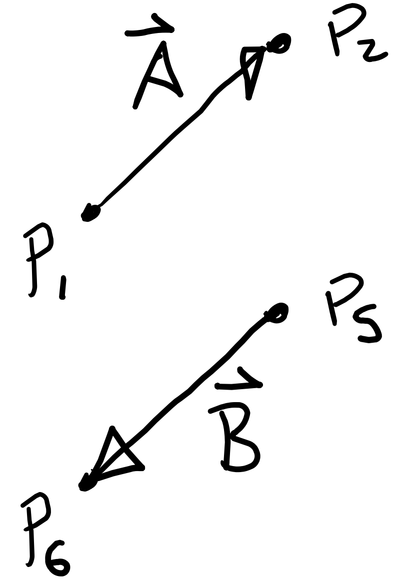

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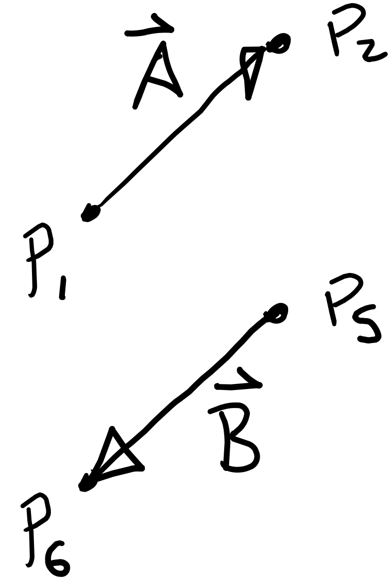

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$\vec{A}$ requires magnitude AND direction

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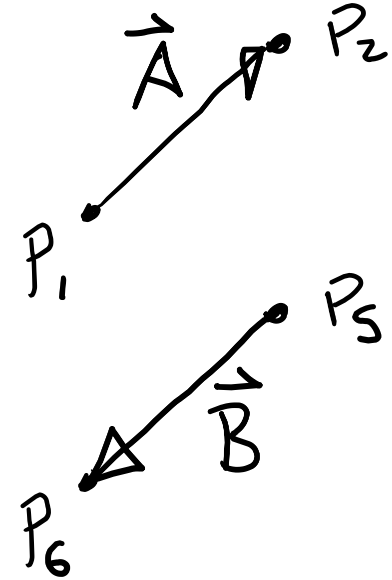

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$\vec{A}$ requires magnitude AND direction

A or $|\vec{A}|$ are scalars & require NO specification of direction

Multiplication of vector with scalar

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(a) Multiplication of vector with positive scalar changes magnitude but <u>Not</u> direction.

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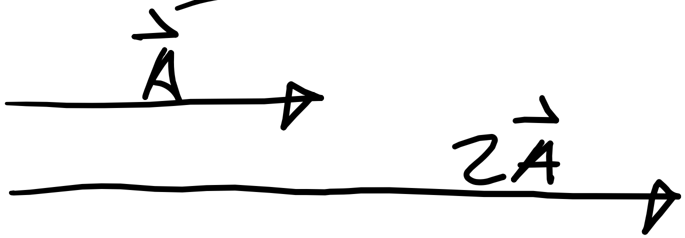

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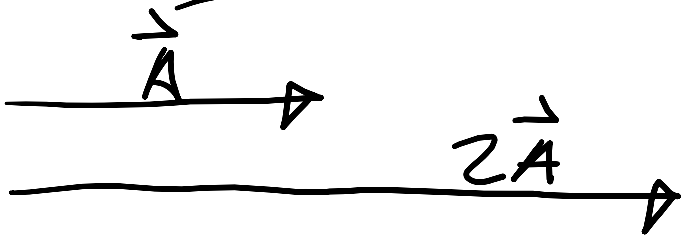

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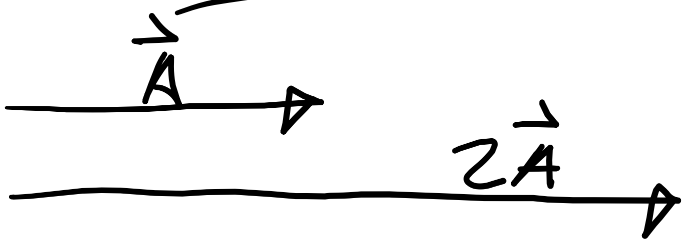

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(b) Multiplication of vector with a negative scalar changes magnitude and **reverses** direction

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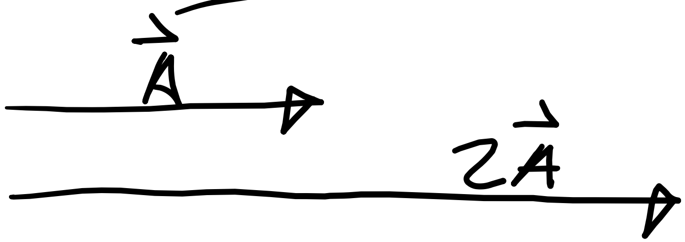

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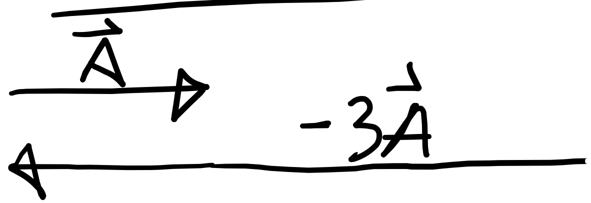

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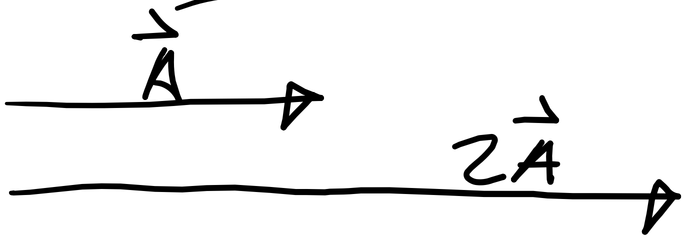

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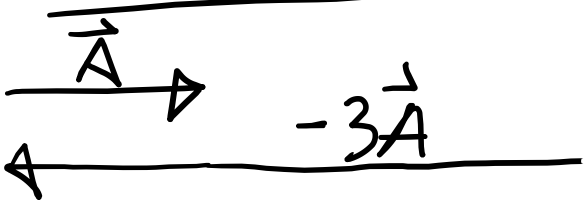
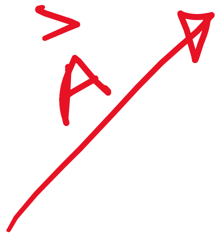


: 3 times as long as $\vec{A}$ & in opposite direction

Vector addition & subtraction

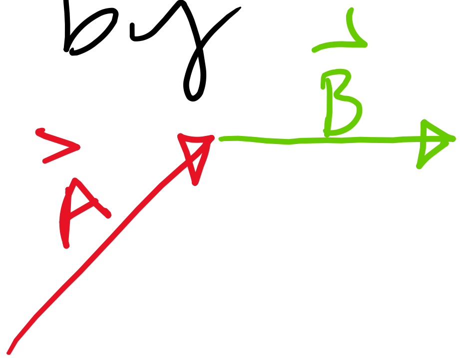
Vector addition & subtraction

Suppose particle undergoes displacement $\vec{A}$



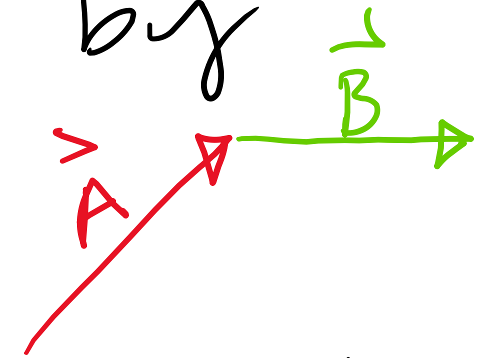
Vector addition & subtraction

Suppose particle undergoes displacement $\vec{A}$ followed by displacement $\vec{B}$.



Vector addition & subtraction

Suppose particle undergoes displacement $\vec{A}$ followed by displacement $\vec{B}$. This is the same as a vector $\vec{C}$ that starts at vector $\vec{A}$ origin & ends at vector $\vec{B}$ destination



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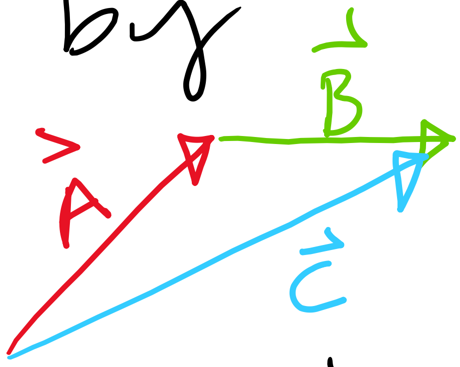
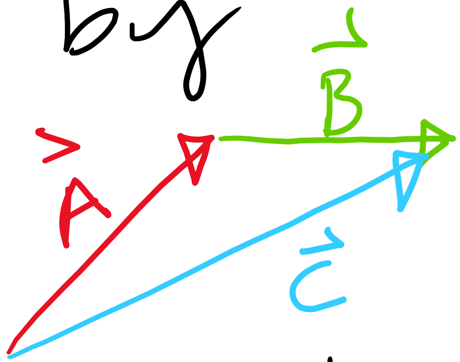


Vector addition & subtraction

Suppose particle undergoes displacement $\vec{A}$ followed by displacement $\vec{B}$. This is the <u>same</u> as a vector $\vec{C}$ that starts at vector $\vec{A}$ origin & ends at vector $\vec{B}$ destination. We can write $\vec{A}+\vec{B}=\vec{C}$ & $\vec{C}$ is called the "resultant" <u>or</u> Vector sum



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$\vec{C} = \vec{A} + \vec{B}$ Does NOT mean that $C = A + B$!

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! $\underbrace{\vec{C}=\vec{A}+\vec{B}}_{\text{Vector equation}}$ Does NOT mean that $\underbrace{C=A+B}_{\text{Scalar equation}}$!

Vector equation $\neq$ Scalar equation

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Important

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In general $C \leq A+B$

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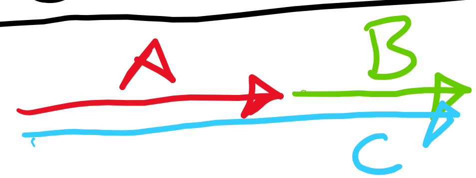

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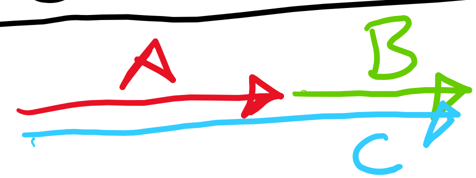

ONLY CASE WHERE $C=A+B$

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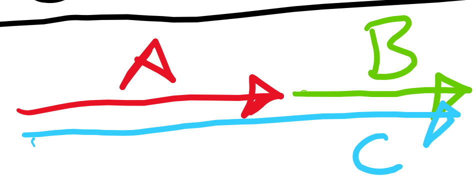

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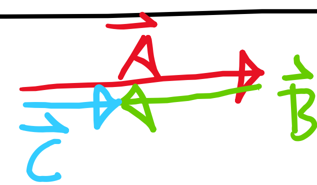

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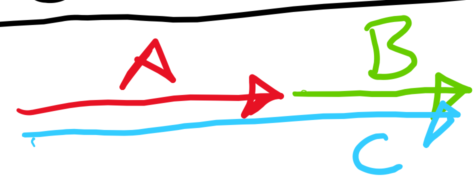

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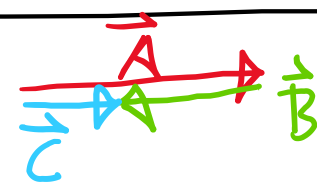
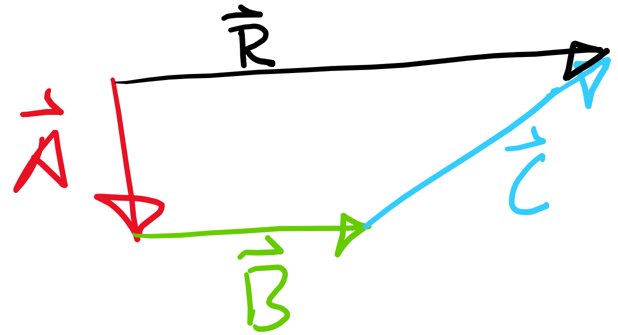


ONLY CASE WHERE $C=|A-B|$

If $\vec{R} = \vec{A} + \vec{B} + \vec{C}$



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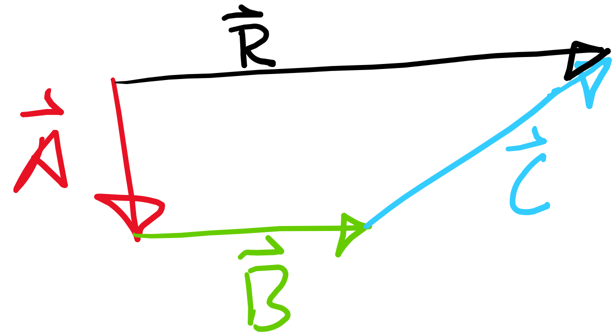
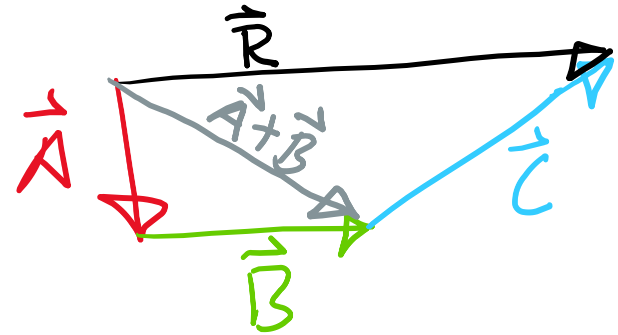


Then $\vec{R} = (\vec{A} + \vec{B}) + \vec{C}$



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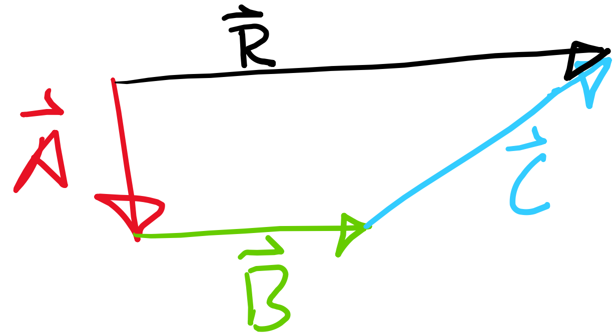

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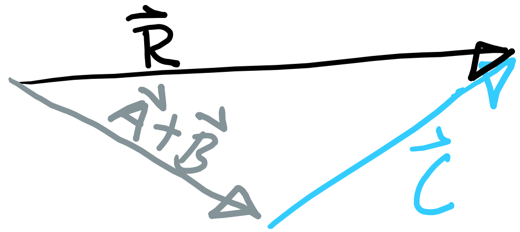

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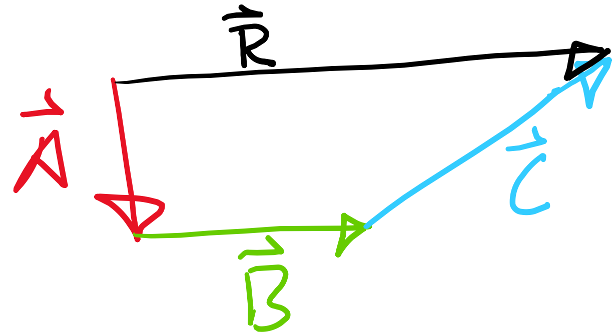

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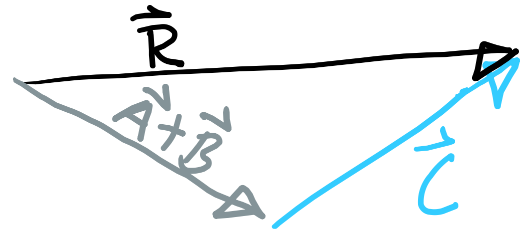

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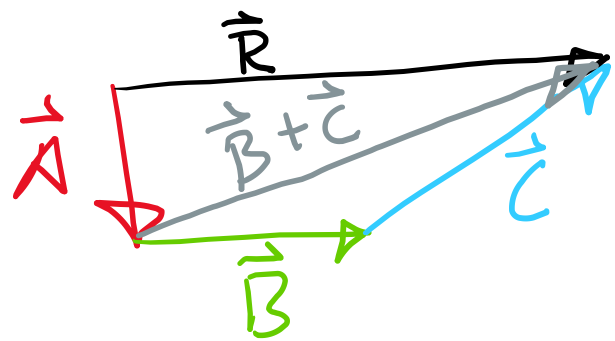

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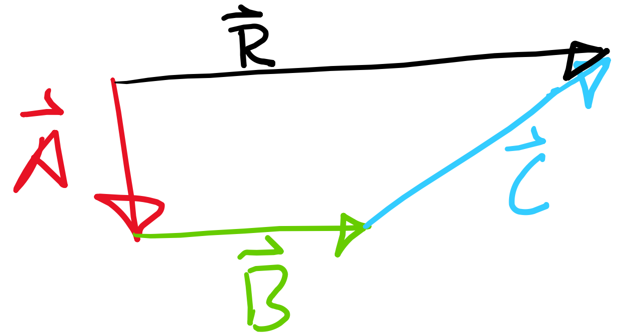

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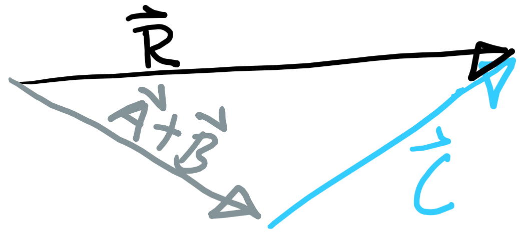

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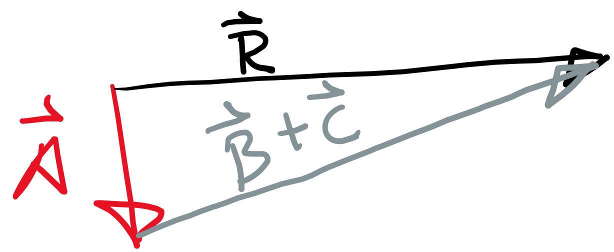

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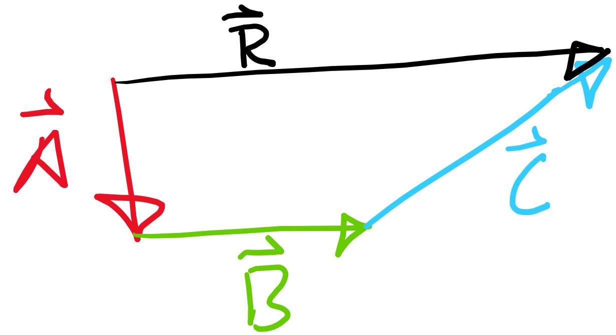

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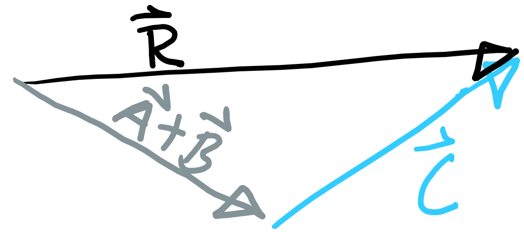

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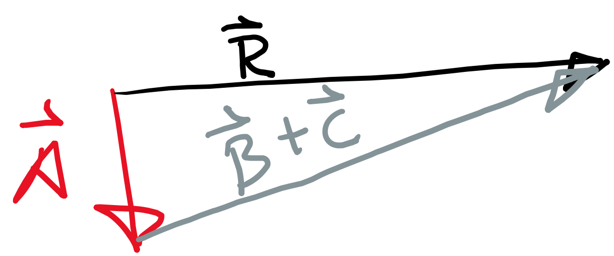

Vector addition obeys the associative law

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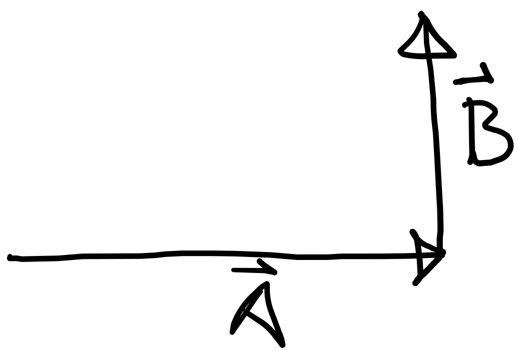

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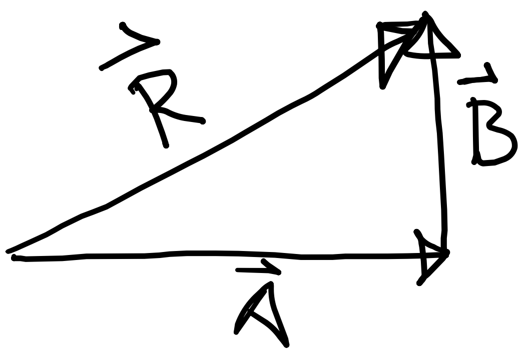

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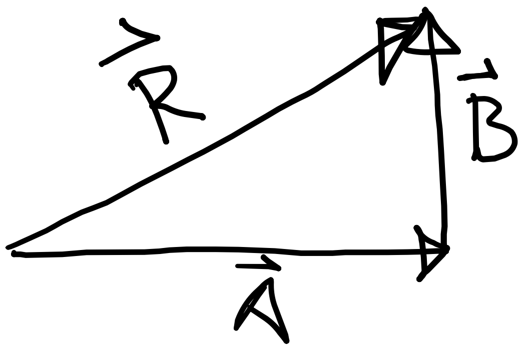

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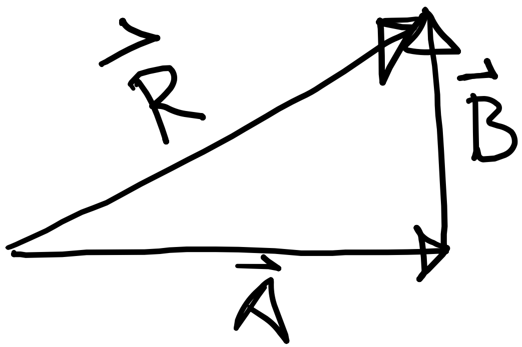

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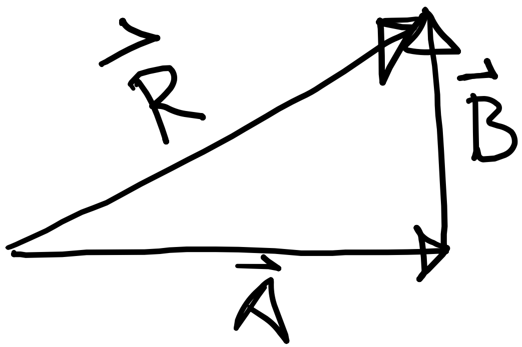

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$$\boxed{R = \sqrt{A^2 + B^2}}$$

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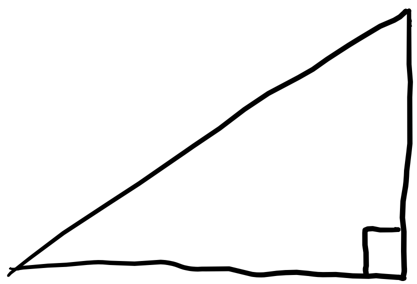

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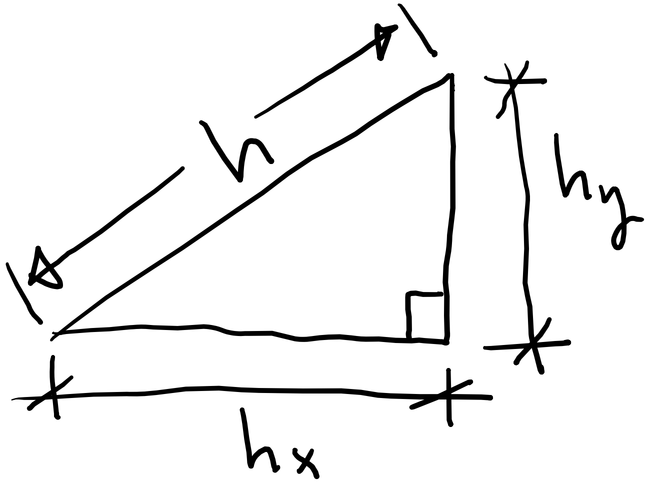

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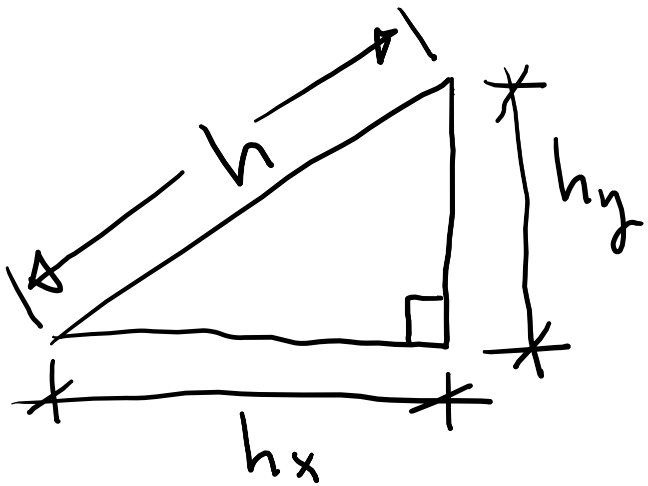

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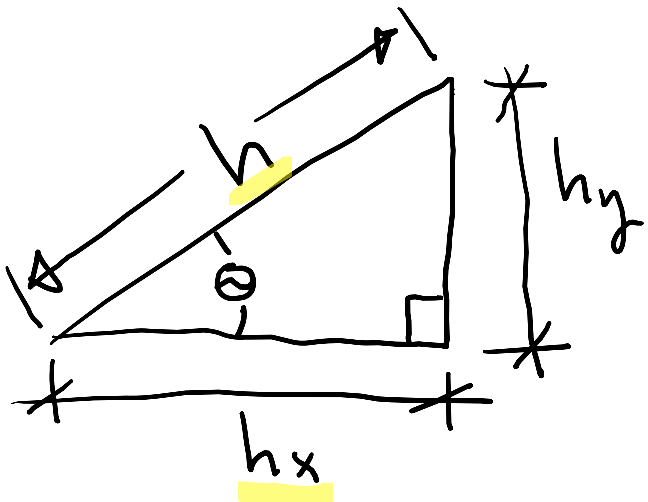

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ASU

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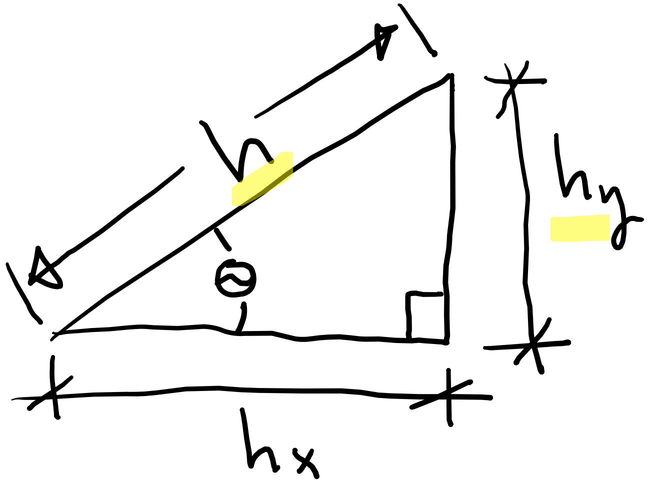

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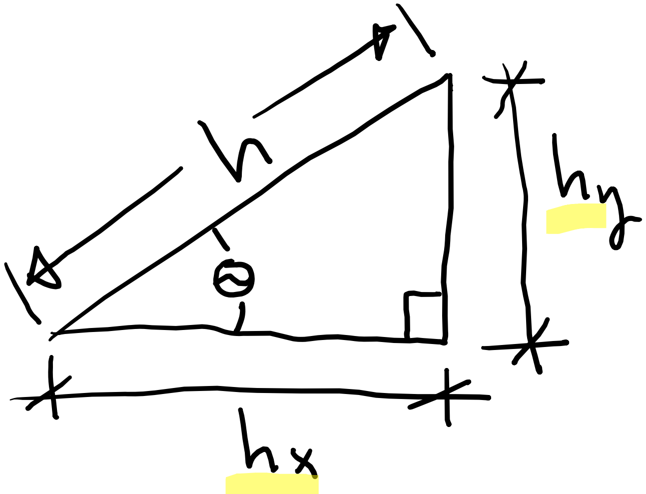

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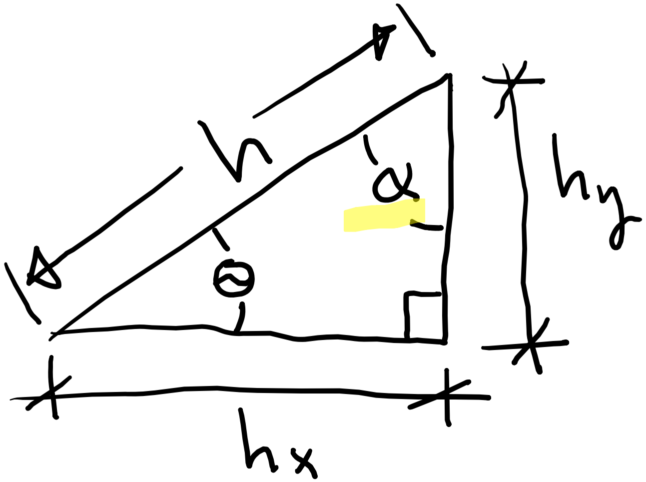

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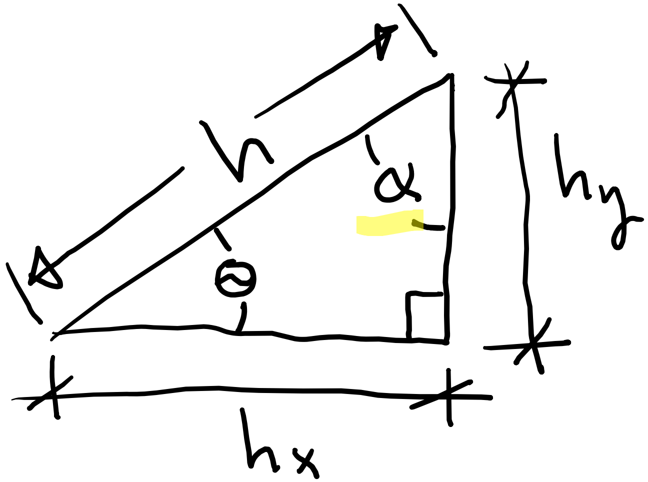

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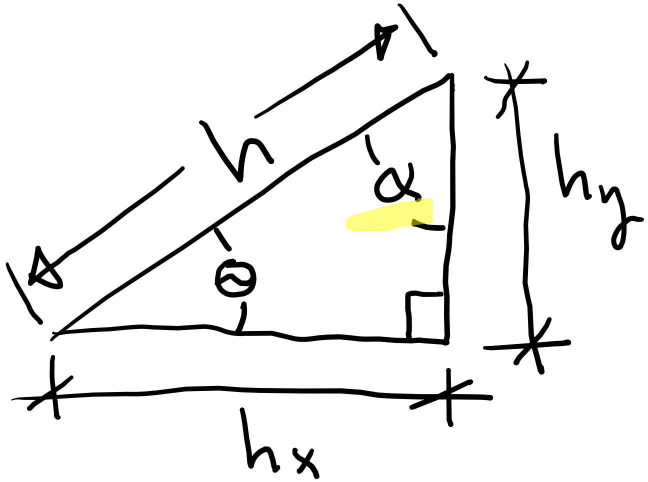

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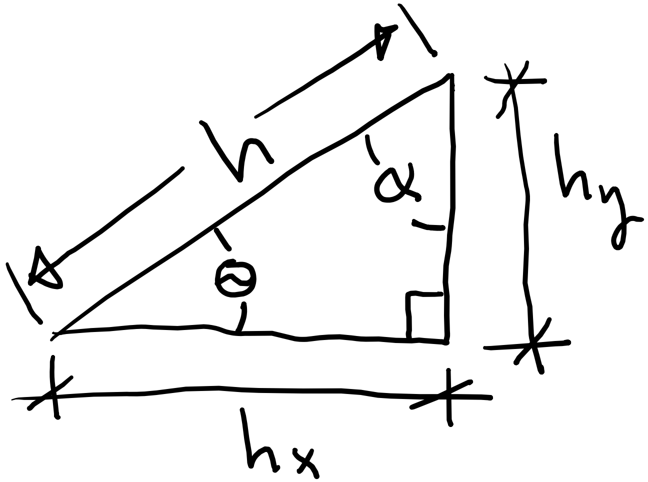

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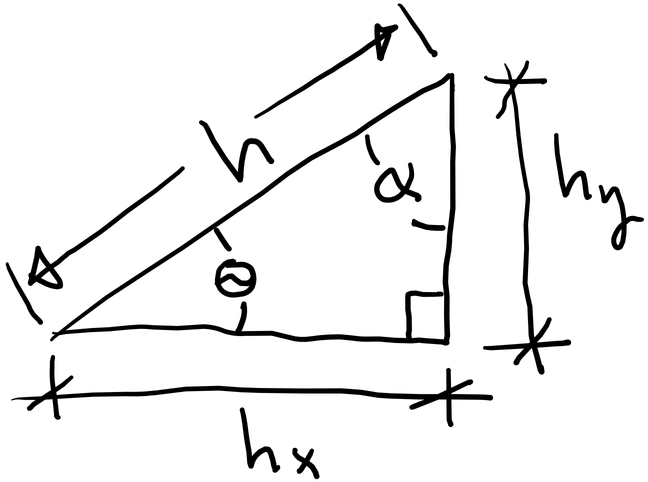

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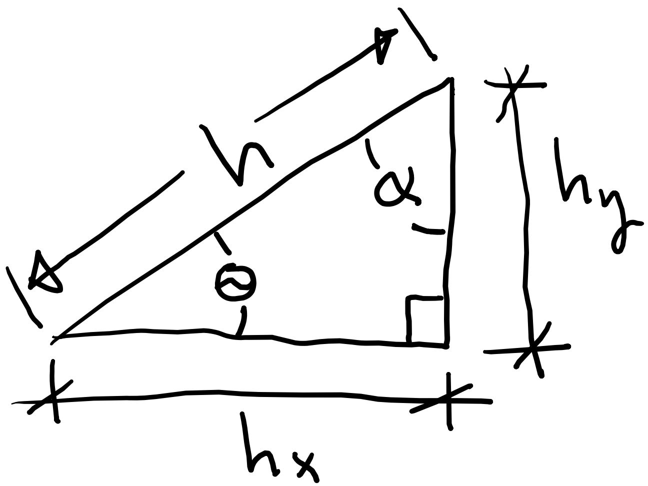

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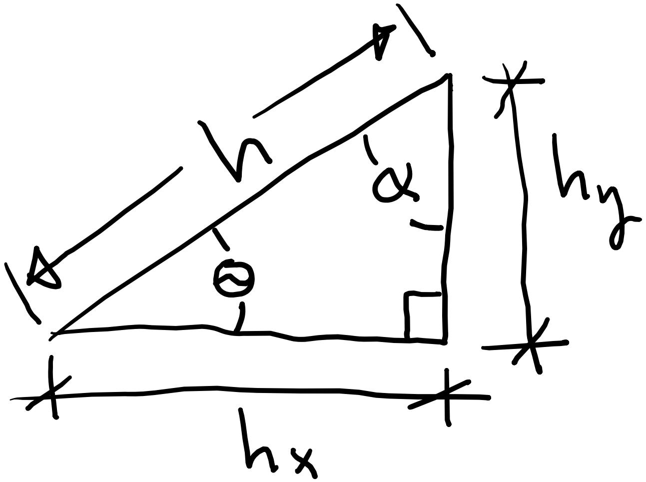

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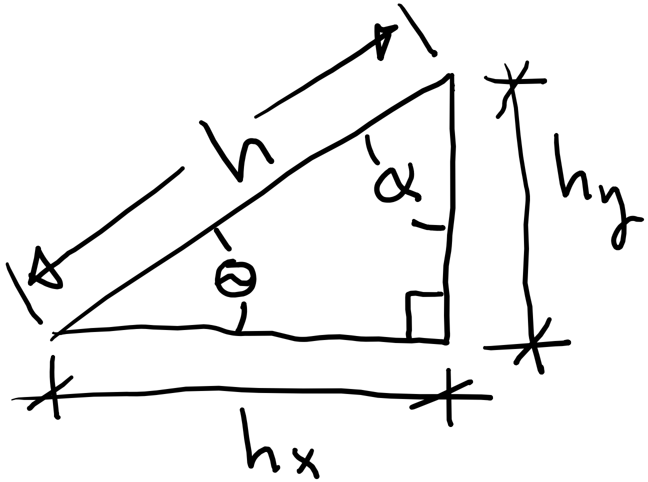

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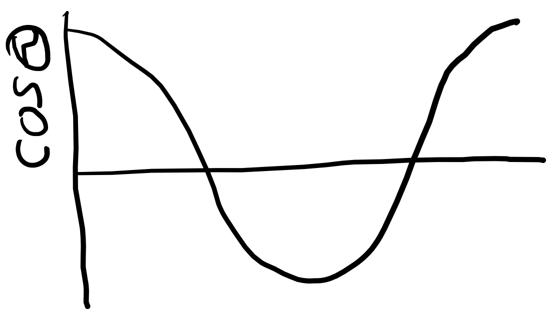

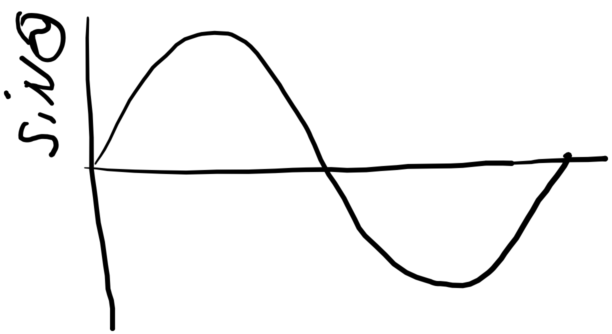

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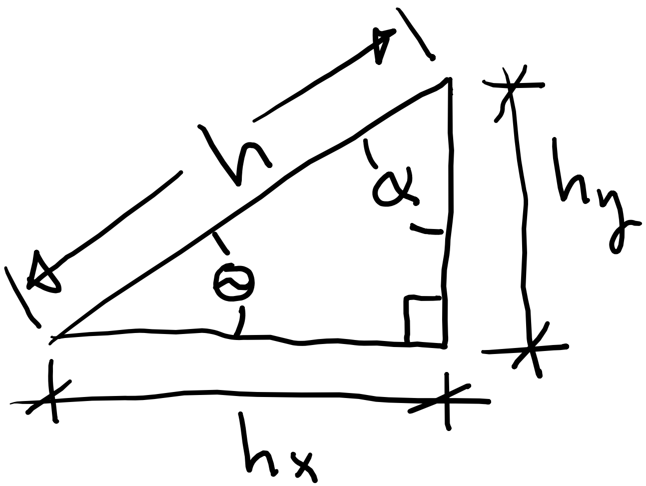

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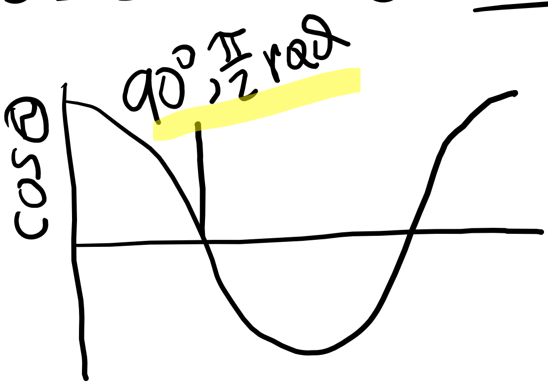

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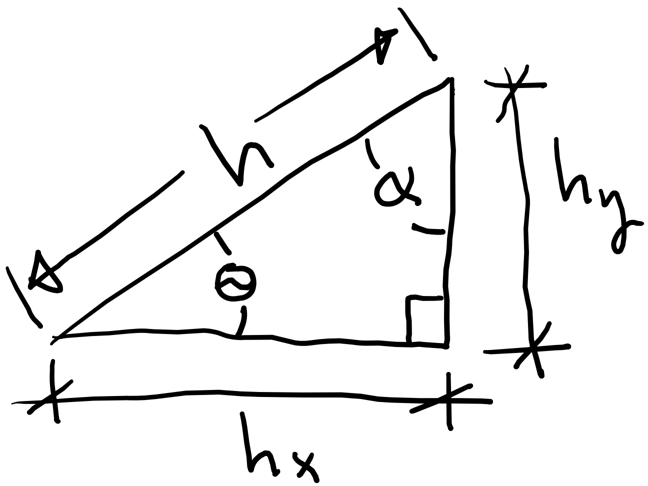

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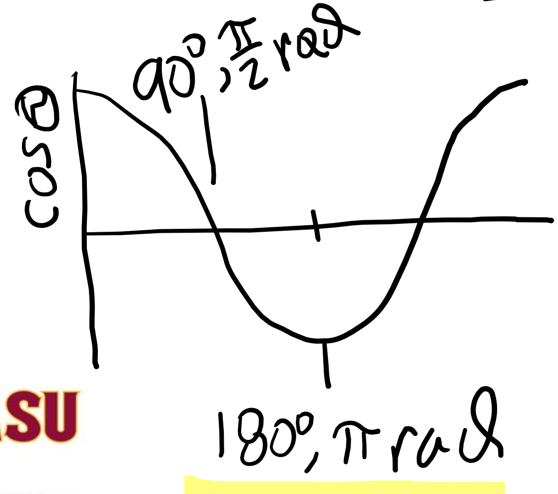

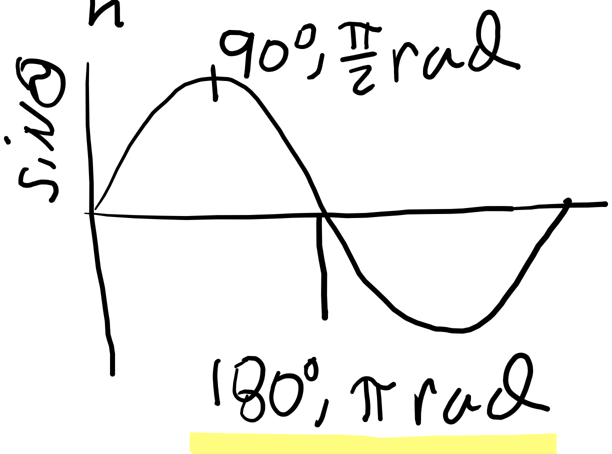

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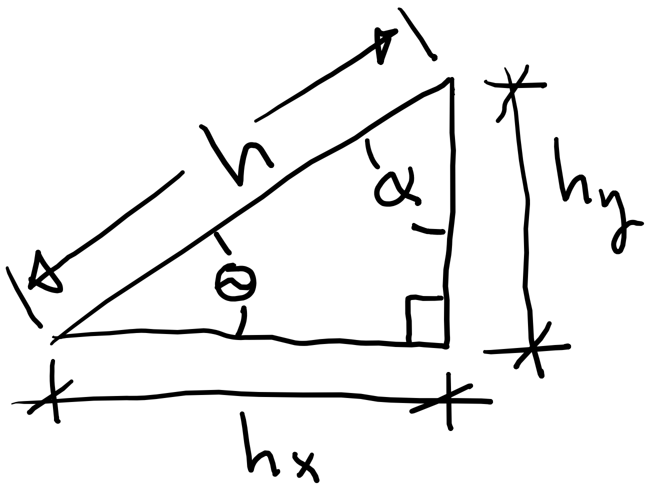

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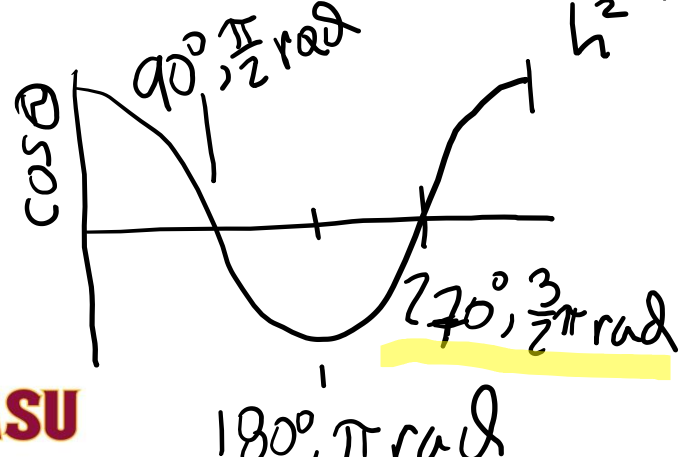

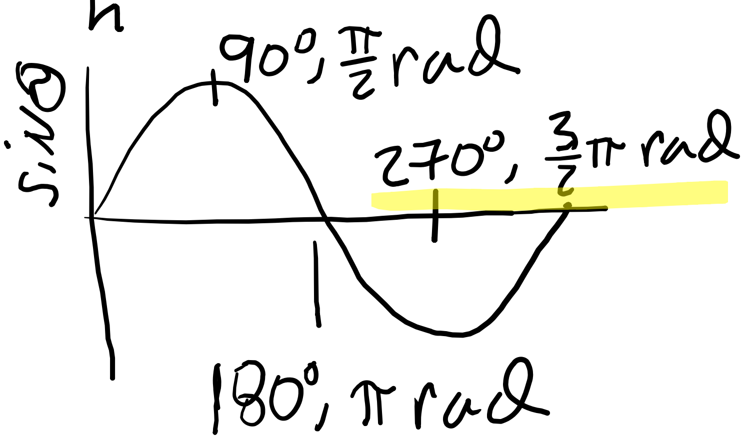

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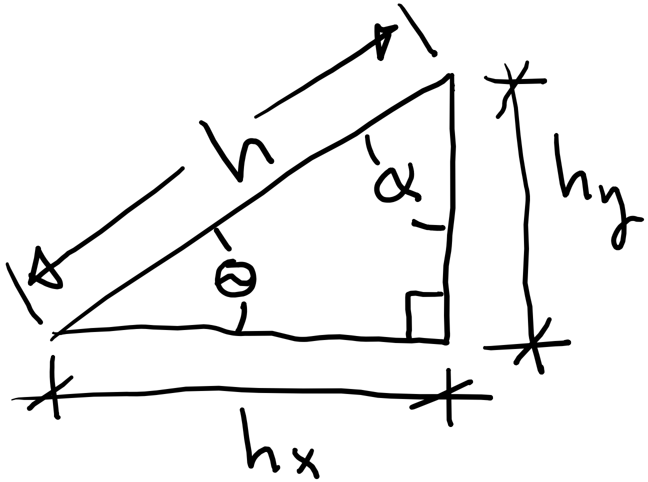

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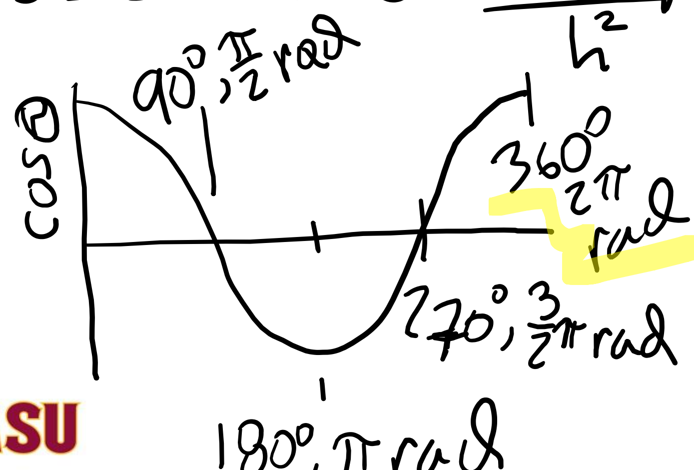

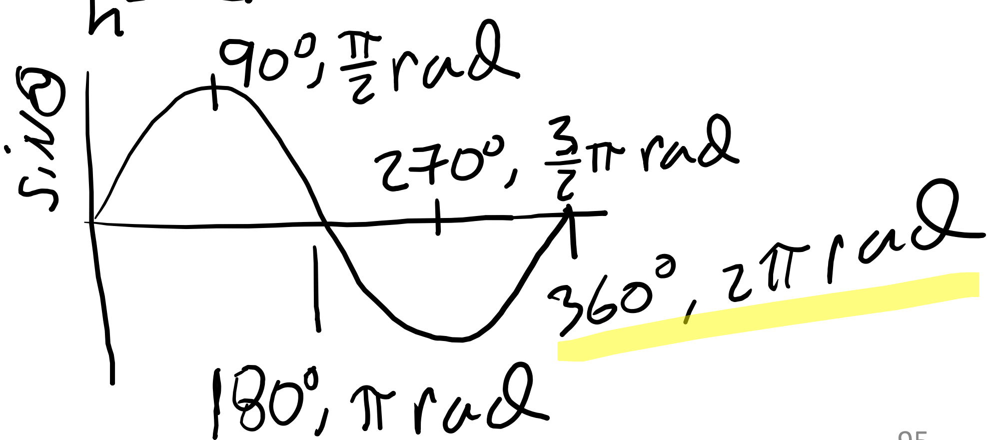

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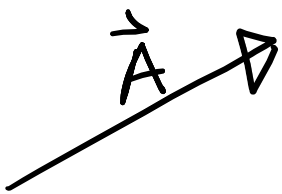

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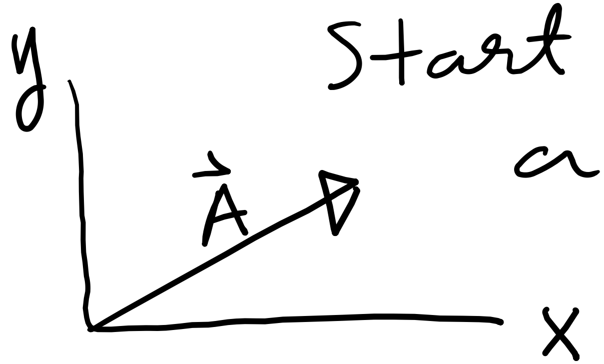
Start with some vector. Impose a coordinate system



Components of vectors

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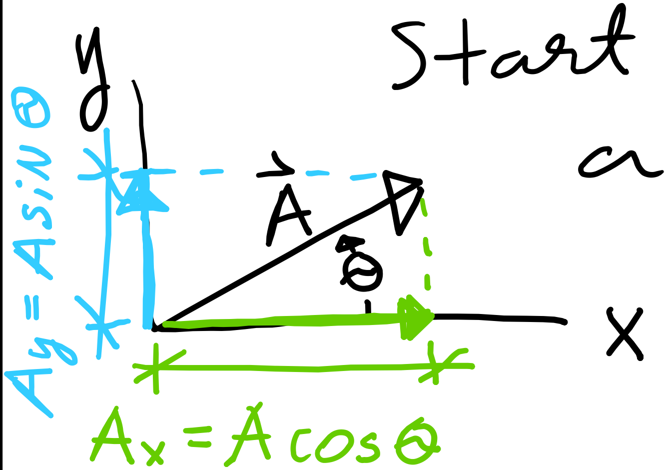

Start with some vector. Impose a coordinate system.

Break up vector into x and y components

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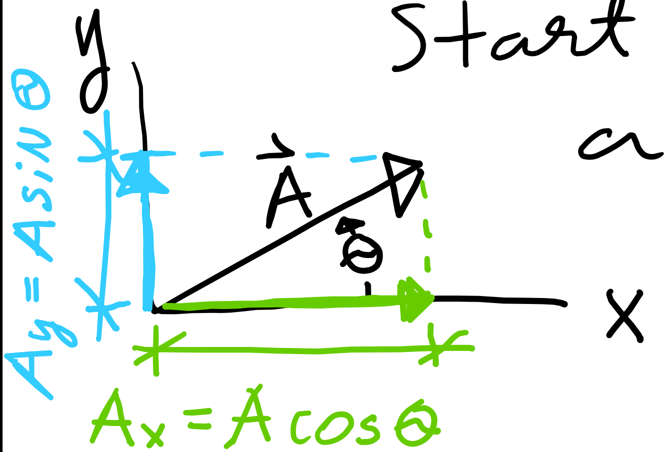

Break up vector into x and y components

Note: Components are NOT vectors

A_x & A_y are scalars

Components of vectors

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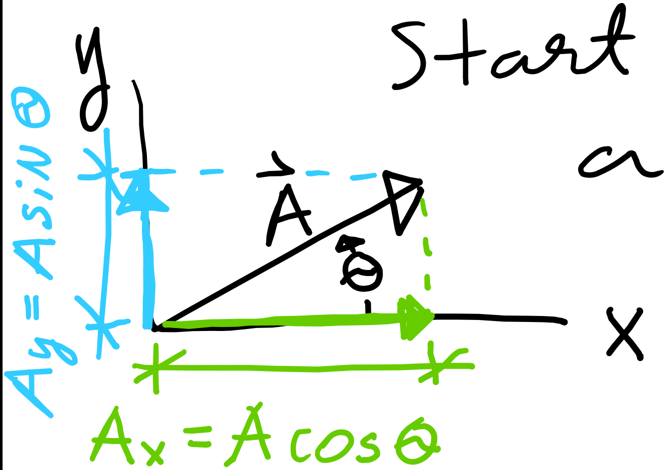

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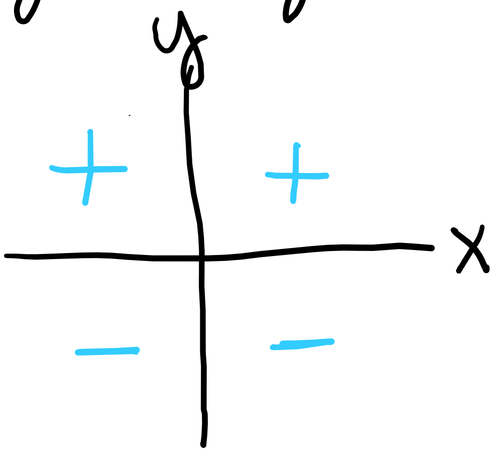
Signs

Signs

Sign of y



Signs

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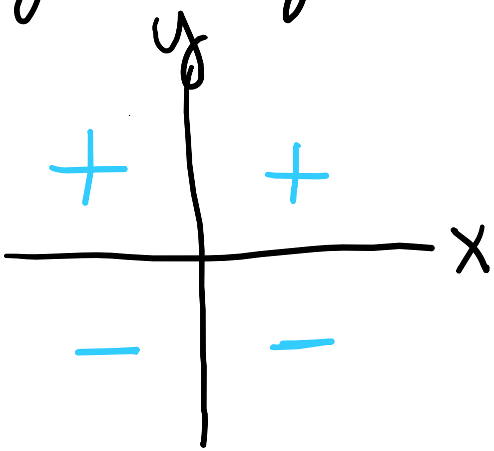
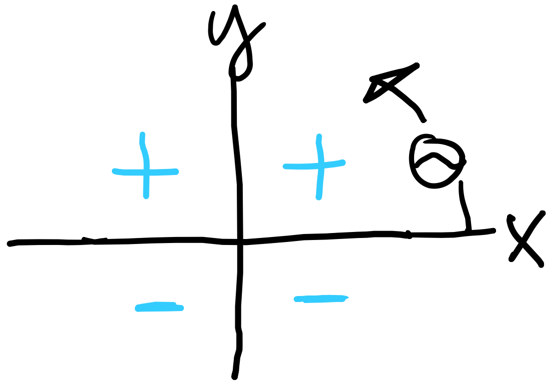


Sign of sin θ



Signs

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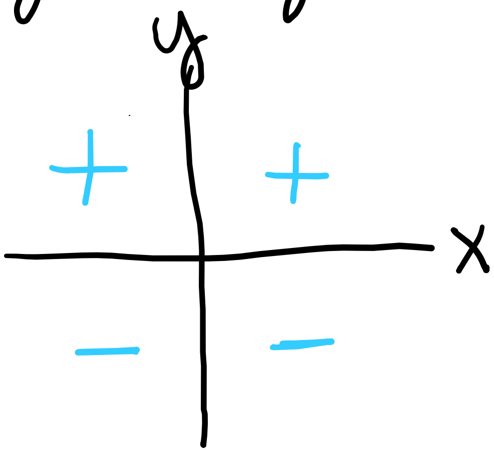
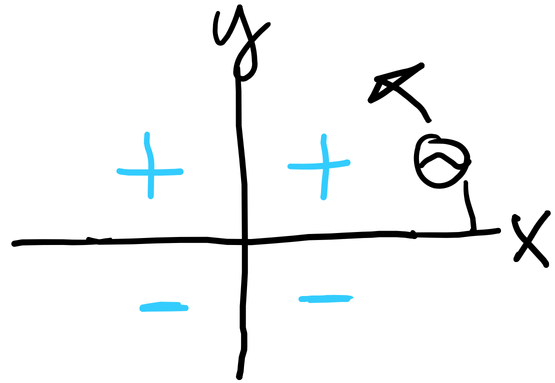
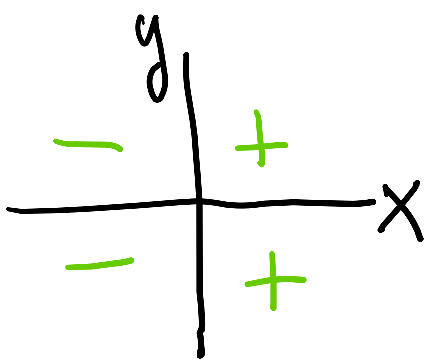


Sign of sin θ

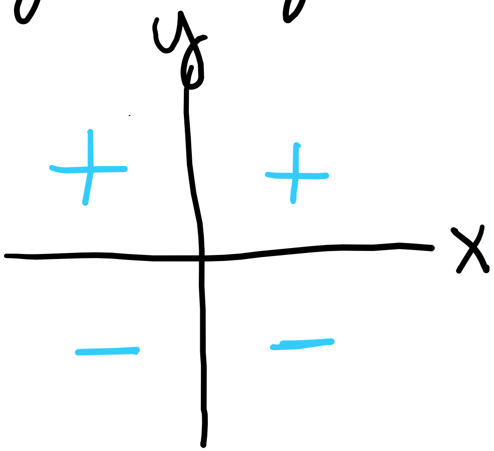


Sign of x

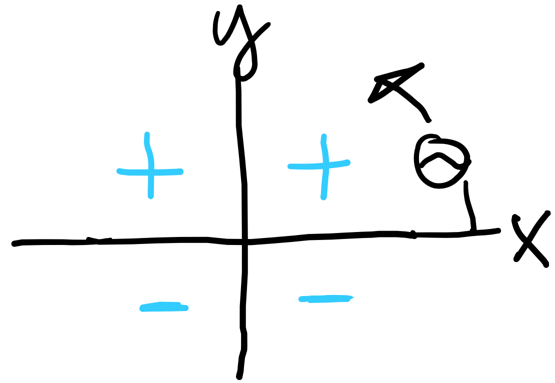


Signs

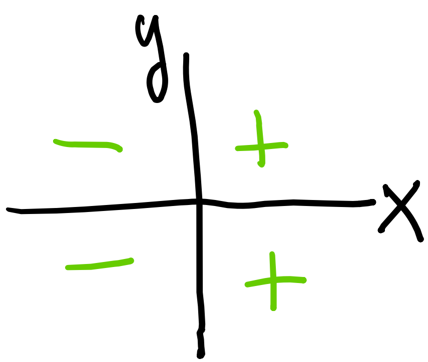
Sign of y



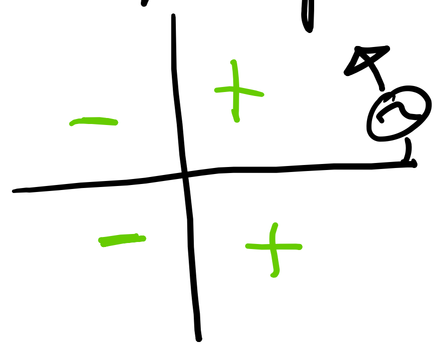
Sign of $\sin\theta$



Sign of x

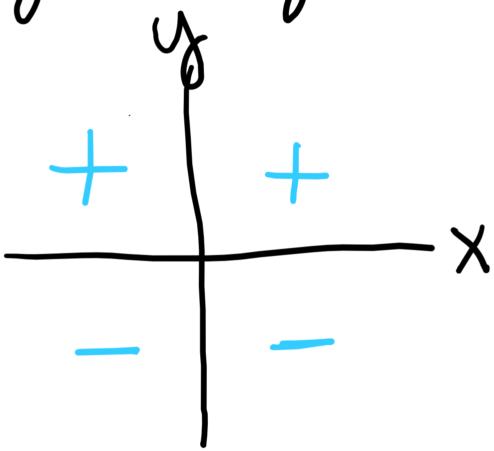


Sign of $\cos\theta$

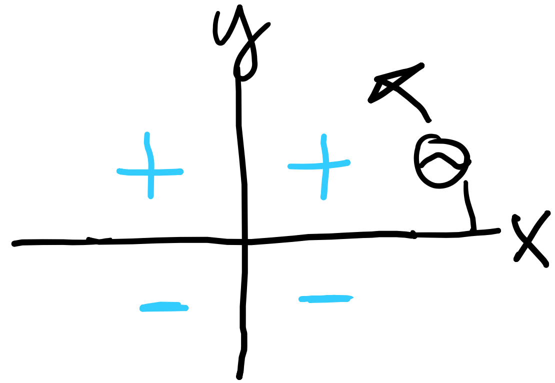


Signs

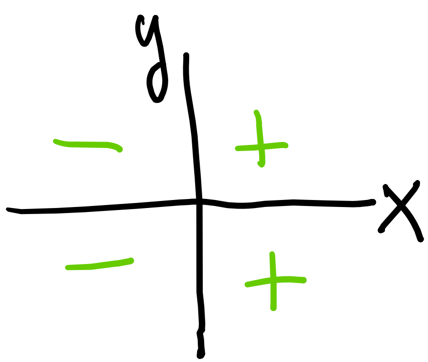
Sign of y



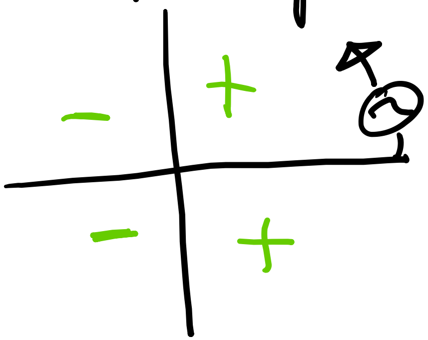
Sign of sin θ



Sign of x



Sign of cos θ



Note: θ is measured from x-axis in counterclockwise direction.

If you break up vector $\vec{A}$ using θ measured from positive x-axis in counterclockwise direction as

$$A_x = A\cos\theta \;\;\&\;\; A_y = A\sin\theta,$$

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the signs of each component will work out as desired.

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the signs of each component will work out as desired.

If you measure θ differently you just have to be careful

Finding vector mag & direction from components

Finding vector mag & direction from components

Given A_x & A_y

Finding vector mag & direction from components

Given A_x & A_y Then $A = \sqrt{A_x^2 + A_y^2}$

Finding vector mag & direction from components

Given A_x & A_y Then $A = \sqrt{A_x^2 + A_y^2}$

And $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

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So $\tan(45^\circ) = \tan(45^\circ + 180^\circ) = \tan(225^\circ)$

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Make a sketch to be sure

If given $A_x = 1$ & $A_y = 1$

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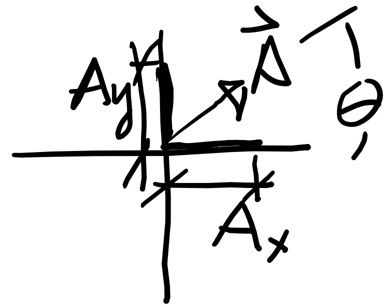
then



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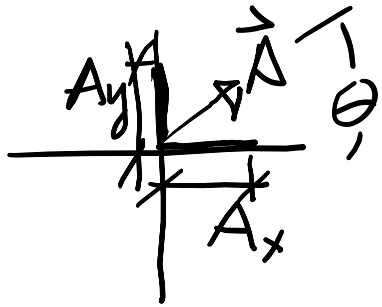
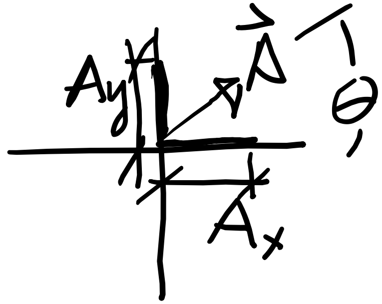


$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$

If given $A_x = 1$ & $A_y = 1$

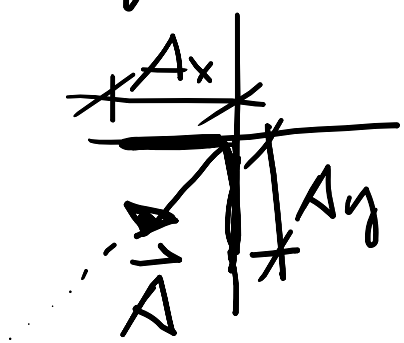
then



$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$

If given $A_x = -1$ & $A_y = -1$

then



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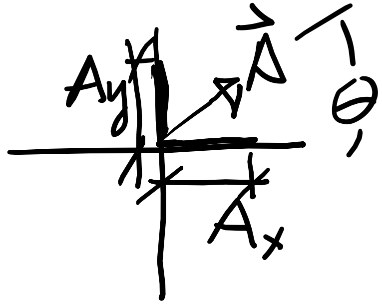

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45°$

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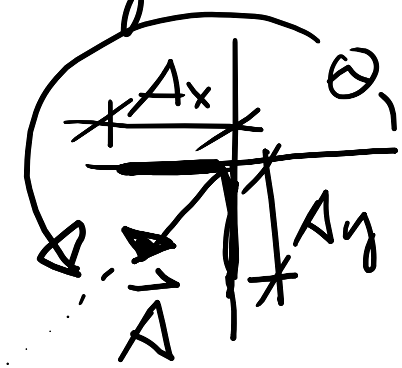
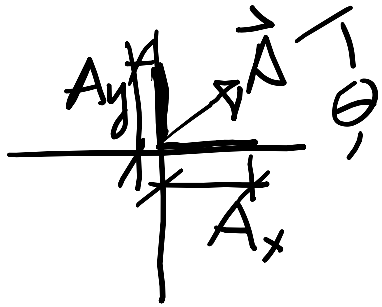


$\tan^{-1}\left(\frac{A_y}{A_x}\right)$

If given $A_x = 1$ & $A_y = 1$

then



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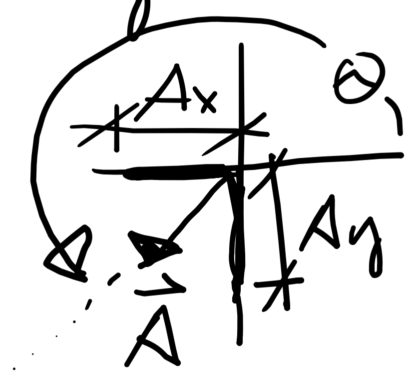
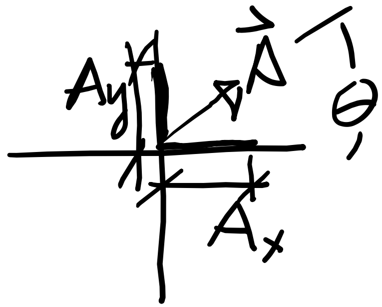


$\tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan\left(\frac{-1}{-1}\right)$

If given $A_x = 1$ & $A_y = 1$

then



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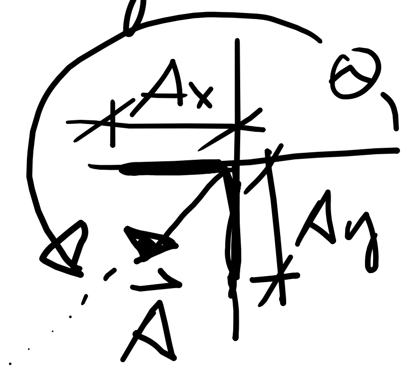

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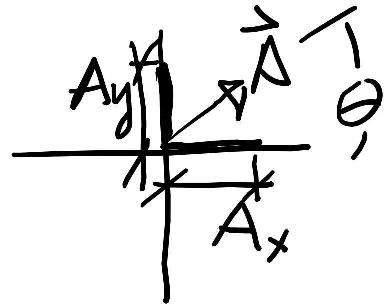

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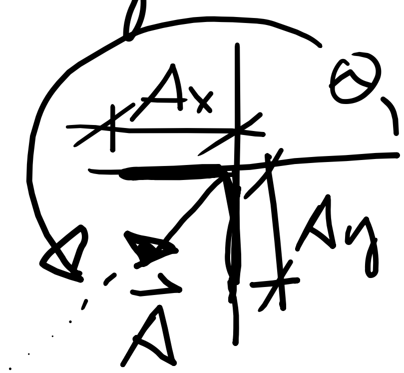

$\tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan\left(\frac{-1}{-1}\right)$

$= 45^\circ$

In this case

$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) + 180^\circ$

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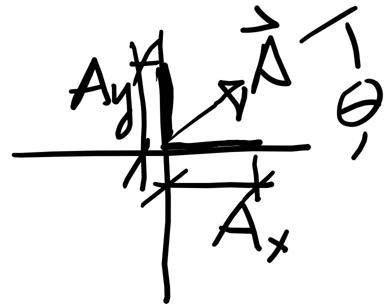

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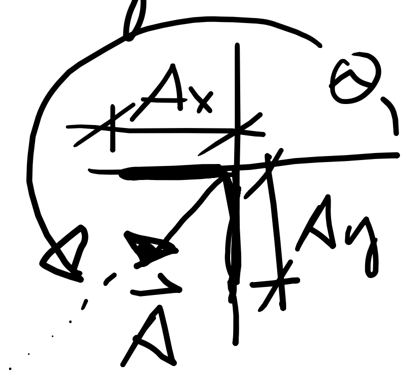

$$\tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan\left(\frac{-1}{-1}\right) = 45^\circ$$

In this case

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) + 180^\circ$$

So $\theta = 45^\circ + 180^\circ = 225^\circ$

Multiplication by a scalar

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If $\vec{D} = c\vec{A}$

Multiplication by a scalar

If $\vec{D} = c\vec{A}$ then $D_x = cA_x$

& $D_y = cA_y$

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Using components to calculate vector sum

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If $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

Multiplication by a scalar

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Using components to calculate vector sum

If $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$ then

$R_x = A_x + B_x + C_x + D_x$ & $R_y = A_y + B_y + C_y + D_y$

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3d magnitude

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3d magnitude

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$